Finite-Temperature Behavior of Lattice QCD with Wilson Fermion Action and Its Implication on Spectroscopic Studies

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Finite-temperature behavior of lattice QCD is studied with the Wilson fermion action and use of the Langevin technique for treating quarks dynamically. It is found that the transition zone from low- to high-temperature behavior does not cross the line of critical hopping parameter, but rather continues down to the strong-coupling limit. Practical implications for spectroscopic simulations at small quark masses are discussed.

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Formulating fermion action on a lattice suffers from the well-known difficulty of spectral doubling,[1] which forces usage of an action either with doubling but some remnant of chiral symmetry or with explicit breaking of chiral symmetry but no doubling problem. Widely used examples corresponding to these alternatives are the Kogut-Susskind action[2] and the Wilson action.[3] For the study of properties related to chiral symmetry, the former has been preferred. On the other hand, one would opt for the latter for spectroscopic studies since only up and down quarks may reasonably be treated as degenerate. It is an important question how these actions with quite different properties might lead to the behavior in the continuum limit which should be universal. This is particularly acute for the finite-temperature behavior for light-quark masses since chiral symmetry is believed to play an important role there. Such studies are also important for spectroscopic simulations which should be done on a lattice large enough to avoid finite-temperature effects.

While many finite-temperature studies have been made with the Kogut-Susskind action,[4] few studies can be found with the Wilson action, except for some earlier attempts[5] employing the low-order hopping-parameter expansion. This is partly because the fate of chiral symmetry is the least-understood aspect of the behavior of QCD at finite temperature and yet the most important for practical applications. Another reason, however, is that to draw physical conclusions one has to find the critical hopping parameter where the pion mass vanishes, and this task requires spectroscopic calculation which is quite time consuming in the presence of dynamical quarks. The purpose of the present Letter is to study systematically the finite-temperature behavior of QCD with the Wilson action in the whole (β,K) plane. Dynamical quark loops are fully taken into account by the Langevin method.[6,7] We shall show that the phase structure is very different from what one naively anticipates from the finite-temperature behavior observed with the Kogut-Susskind action. In particular, we show that for any value of the coupling β including the strong-coupling limit, the system exhibits a behavior characteristic of the high-temperature phase (namely, large values of the Polyakov line, energy density, etc.) as the hopping parameter K approaches its critical value K_c. With the Kogut-Susskind action the region of β showing the characteristics of the confining behavior is separated from that of the deconfinement by a sharp transition and the location of the transition seems to converge to a nonvanishing value of β as the quark mass $m_q \rightarrow 0$.[4] This has an important implication on spectroscopic studies with the Wilson fermion, which should be done at zero temperature; with the Wilson action finite-size effects resembling finite-temperature effects are severe near K_c for any lattice size and value of the coupling constant, whereas such effects might be easier to control for the Kogut-Susskind action inasmuch as one only has to work with β below the transition point at $m_q \cong 0$.

For our study we take the Wilson action

$$S = S_g + \sum_n \{ -\bar{\psi}_n\psi_n + K\sum_\mu [\bar{\psi}_n(1-\gamma_\mu)U_{n\mu}\psi_{n+\hat{\mu}} + \bar{\psi}_n(1+\gamma_\mu)U_{n-\hat{\mu},\mu}\psi_{n-\hat{\mu}}]\}, \quad (1)$$

where S_g is the single plaquette gauge action with the coupling $\beta = 6/g^2$. The gauge group is SU(3) and the number of flavors is taken to be $N_f = 4$ to compare with the case of the Kogut-Susskind action. The method of calculation is the same as described by Fukugita, Oyanagi, and Ukawa,[8] except that the quarks obey the antiperiodic boundary condition in the temporal direction. We employ a rather small lattice of $5^3 \times 3$ since the lattice size should not be crucial for the study of the phase structure of the system. The Langevin updates, over ten time units with the time step $\Delta\tau = 0.01$, are made at a point of (β, K) before moving to different points, and the averages of physical quantities are taken over the last five time units. The data are taken at 120 different combinations of (β, K) to gain an overall picture of the behavior of the system. We have checked at several values of (β, K) that the result does not change appreciably when we employ a spatially larger lattice, $8^3 \times 3$, or decrease the time step to $\Delta\tau = 0.002$. We have also made a separate spectroscopic analysis on a $6^3 \times 12$ lattice to determine the critical hopping parameter K_c where pion mass vanishes.

Figure 1 shows the Polyakov line $\langle \Omega \rangle \equiv \langle \frac{1}{3} \mathrm{Tr}(\prod U_{n4}) \rangle$ as a function of β at fixed values of K. A rapid crossover is seen at all the values of K shown, but it gradually becomes less pronounced with increasing K. For $K = 10^{-4}$ the rapid change occurs at $\beta \cong 5.5$, which agrees with β_c for the pure gauge system with $N_t = 3$.[9] The transition region is shifted towards smaller β with increasing K. Another noticeable feature is that for $K \gtrsim 0.1$, $\langle \Omega \rangle$ takes a finite value at β's even below the transition region, and this residual value of $\langle \Omega \rangle$ increases as K increases. This is in contrast to the case with the Kogut-Susskind action, where $\langle \Omega \rangle$ is very small, typically $\langle \Omega \rangle \lesssim 10^{-2}$ below the transition region.[4] It can easily be seen from the hopping-parameter expansion that this finite $\langle \Omega \rangle$ arises from the Wilson mass term that breaks the chiral symmetry.

In Fig. 2 we show the corresponding gluon internal energy E_g normalized by the temperature T^4. It is apparent that E_g/T^4 stays at small values for β below the transition region. The rapid increase of E_g to a finite value in a narrow range of β signals a rapid liberation of gluons across the transition region.

We exhibit in Fig. 3 the location of the transition as deduced from our analysis. The transition region is operationally defined as the range of β over which the slopes of E_g/T^4 and $\langle \Omega \rangle$ as functions of β increase from their low values in the small-β region to the high values in the transition region. This in practice means that the lower end corresponds to the starting point of the rise and the upper end to the midpoint of the rapid increase. Moving the upper end to the point at which the rapid increase seems to terminate does not change the feature discussed below. We also show in Fig. 3 the value of K_c determined from the pion mass

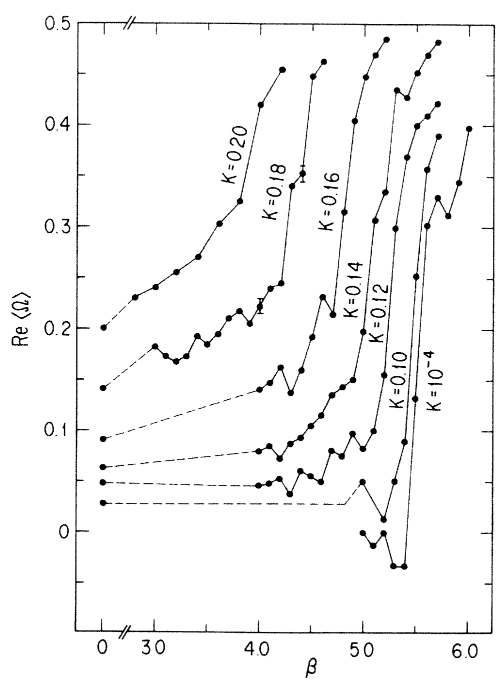

FIG. 1. Average of Polyakov line $\mathrm{Re}\langle \Omega \rangle$ as a function of β at fixed values of the hopping parameter K. Typical errors taking account of the autocorrelation in Langevin time are also shown.

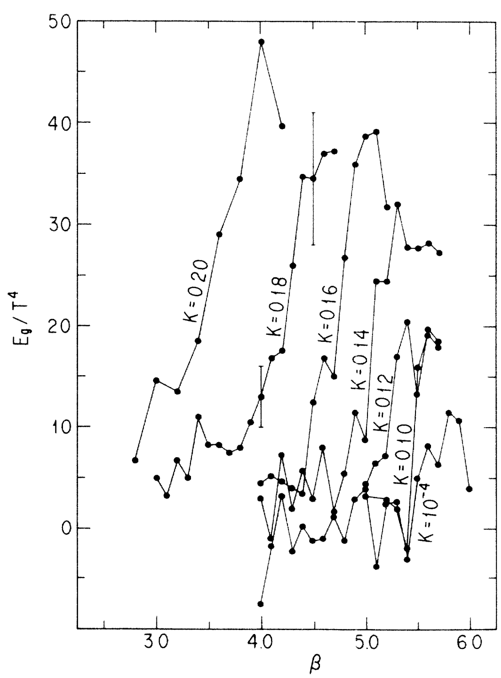

FIG. 2. Gluon internal energy E_g as a function of β at fixed values of K.

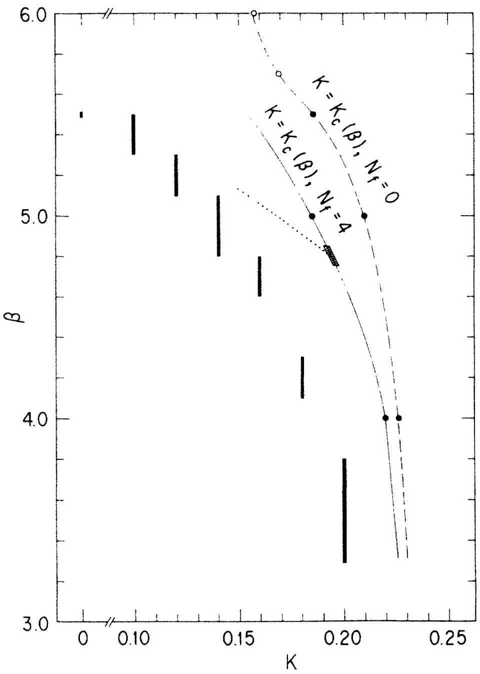

FIG. 3. Position of the transition zone from low- to high-temperature behavior. We also show the line of the critical hopping parameter $K = K_c(\beta)$ determined from our spectroscopic analysis (solid circles), together with those taken from Ref. 10 for the quenched case $N_f = 0$ (open circles). The dotted line starting from the shaded area shows the position of transition expected from the analysis with the Kogut-Susskind fermion (see Ref. 11).

analysis at several β's for both quenched and full QCD systems. We observe that the zone of transition (we simply call it the transition line hereafter) does not meet the line of critical hopping parameter $K = K_c(\beta)$ even at $\beta \cong 3.5$. This is rather different from what might be expected from the behavior of a system with Kogut-Susskind action. If the critical line $K = K_c(\beta)$ corresponds to vanishing quark mass $m_q = 0$ of the Kogut-Susskind action, then the results for the latter action[4] on an $8^3 \times 4$ lattice, scaled down to $N_t = 3$ taking account of empirical violation of scaling, lead to $\beta = 4.75$–4.85 as an estimate of the crossing point.[11] Of course, it does not make much sense to seek precise agreement of the values, since the two actions are expected to produce the same physics only in the continuum limit and we are obviously quite far from it. However, we observe here a signature that the physics is very different between the two actions. Figure 3 suggests that the transition line runs almost parallel with the critical line running down to the strong-coupling limit without crossing the critical line $K = K_c(\beta)$. Our data at $\beta = 0$ show that $\langle \Omega \rangle$ increases to a large value (e.g., $\mathrm{Re}\langle \Omega \rangle \cong 0.3$ at $K \cong 0.23$) toward K_c. This also supports the proposal that the phase transition or its remnant reaches the limit $\beta = 0$ without crossing the line $K = K_c(\beta)$.

This has several important implications. If the transition is really a phase transition separating two phases and running down to the strong-coupling limit without crossing the line $K = K_c(\beta)$, one cannot reach $K = K_c$ on a finite lattice; the critical line $K = K_c$ is always in the high-temperature phase.

Even if the transition turns into a mere crossover beyond some K, this still poses serious problems of practical importance. Since the rise of $\langle \Omega \rangle$ toward K_c mainly comes from the contribution of the Wilson mass term in the quark loop wrapping around the lattice, we expect a similar phenomenon when the spatial size of the lattice is not large instead of the temporal. Hence, our results for finite temperature may be translated into those on the effects of finite spatial extent in spectroscopic calculations. This implies that, even if the transition is a crossover, spectroscopic quantities may change rapidly in the transition region and those measured close to K_c reflect the dynamics at high temperature and not that at zero temperature required for spectroscopic studies. We emphasize that this cannot be avoided for any choice of the coupling since the transition region runs almost parallel to the critical line down to $\beta = 0$. Even away from the critical line $K = K_c(\beta)$, the fact that the value of the Polyakov line $\langle \Omega \rangle$ becomes sizable within the transition region means that the measured hadron masses might be substantially affected by the fake loop even rather far from K_c. Therefore, to make the spectroscopic calculation valid, we have to limit ourselves to the region where $\langle \Omega \rangle$ is small enough. This means that we need a lattice with a size much larger than what we naively expect to make a simulation for small quark masses.

In this Letter, we have not answered the question of whether or not there exists a real phase transition near $K = K_c$. What we can conclude from our analysis is that the transition becomes continuously weaker as the hopping parameter increases. This is in agreement with the prediction of the perturbative analysis[12] in K and also with the results of the simulation with the Kogut-Susskind action for heavy quarks.[4] For ligher quarks, however, it has been suggested that the transition may become stronger again with the Kogut-Susskind action,[13] but symptoms for such a behavior were not observed in our data with the Wilson fermion toward $K \rightarrow K_c(\beta)$. It is likely that the first-order phase transition observed for a pure gauge system disappears at some point on the line.[14] If this is the case, it is important to study at what point the phase transition disappears and whether the point moves toward the line of K_c for the weaker coupling. This is necessary if we should reconcile the finite-lattice result with the first-order chiral phase transition predicted in the continuum by a renormalization-group analysis.[15]

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[11]Extrapolation of β_c for the Kogut-Susskind action has given $\beta_c = 4.9 - 5.0$ at $m_q = 0$ on $8^3 \times 4$ lattice (Ref. 4). If we assume the scaling for T_c we expect $\beta_c(K = K_c) = 4.7$–4.8 for $N_t = 3$. If we take account of the scaling violation by using the actual value of $N_t \Lambda_L(\beta_c)$ for $N_t = 3$ and 4 in the quenched approximation, the expected $\beta_c(K = K_c)$ is shifted to 4.75–4.85. Alternatively we can use the direct data for β_c from the Kogut-Susskind analysis at $m_q = 0.1$ which gave $\beta_c \cong 5.1$ (see Fukugita and Ukawa, Ref. 4). At these values for m_q and β we found $m_\pi a \cong 0.77$ on a $6^3 \times 12$ lattice. A search for K in the Wilson fermion spectroscopic analysis yielding the same value of $m_\pi a$ and application of an analysis similar to that above, lead to $(\beta, K) \simeq (4.95, 0.175)$.

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