

Self-Consistency Constraints on Turbulent Magnetic Transport and Relaxation in a Collisionless Plasma

P. W. Terry, P. H. Diamond, and T. S. Hahm

Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712

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Novel constraints on collisionless relaxation and transport in drift-Alfvén turbulence are reported. These constraints arise as a result of the effects of mode coupling and incoherent fluctuations as manifested by the proper application of self-consistency conditions. The result that electrostatic fluctuations *alone* regulate transport in drift-Alfvén turbulence follows directly. Quasilinear transport predictions are discussed in light of these constraints.

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Transport caused by turbulent magnetic fluctuations is considered to be an important agent for relaxation and confinement degradation in magnetically confined plasmas. Previous investigations of turbulent magnetic transport typically have utilized quasilinear models of fluctuation dynamics, and have neglected self-consistent field effects.¹⁻⁴ In this Letter, novel constraints on magnetic transport in fully developed collisionless plasma turbulence are described. These constraints arise from the role of self-consistency conditions (i.e., quasineutrality and Ampère's law) in models of the dynamics of drift-Alfvén microturbulence which are more complete than quasilinear theory. In particular, it is argued that the self-consistency imposed by Ampère's law, along with proper consideration of the role of mode coupling and incoherent fluctuations in the dynamics of relaxation, together lead to the conclusion that transport and relaxation in drift-Alfvén turbulence are regulated by the *electrostatic* fluctuations. Previous transport models are then reconsidered in light of these constraints. Throughout this Letter, it is assumed that the drift kinetic equation (DKE) governs electron dynamics, and that ion dynamics is described by a warm, low-frequency response.

The DKE relates the dynamics of phase-space density fluctuations (δf) to the relaxation of the average distribution function ($\langle f \rangle$) through the expression

$$\int dv_{\parallel} d^3x \partial(\delta f^2)/\partial t = - \int dv_{\parallel} d^3x \partial(\langle f \rangle^2)/\partial t.$$

Predictions of plasma transport and relaxation are thus direct consequences of the nature of the fluctuation dynamics. In particular, it is noteworthy that in fully developed Vlasov turbulence, shear stresses generate granular, incoherent fluctuations which are macroparticlelike, localized, phase-space "blobs," analogous to fluid eddies rather than to waves.^{5,6} Under certain rather general conditions, such blobs can even support localized self-trapping potentials (i.e., positive for elec-

trons) and hence have lifetimes which exceed the average correlation time.⁷⁻⁹ In general, the incidence of incoherent fluctuations is indicative of the dynamical significance of mode-coupling processes. Thus, magnetic transport driven by fully developed turbulence *cannot* be described by quasilinear theory, which intrinsically neglects the effects of mode coupling and localized fluctuations.

Here, two related models of incoherent drift-Alfvén fluctuation dynamics and induced transport are described. The first is concerned with the evolution of an isolated phase space blob \tilde{f} in a drift-Alfvén system. In the second model, statistical averaging is used to construct a Lenard-Balescu turbulent collision integral for the relaxation of $\langle f \rangle$ due to "fully developed" (i.e., many blobs and collective resonances) drift-Alfvén microturbulence. While the statistical model is more representative of fully developed turbulence, the isolated-blob model helps develop physical intuition. Both yield qualitatively similar insights into self-consistency constraints on relaxation and transport.

In the first model, an isolated, localized, electron phase-space density blob \tilde{f} with velocity u_{\parallel} at position \mathbf{x}_0 is considered. The blob has correlation length Δv_{\parallel} in velocity (of order of the trapping width) and $\Delta x, \Delta y, \Delta z$ in position space, where $\Delta z \sim L_{\parallel}$ is the parallel length scale for the system. Such blobs have been characterized as maximum-entropy Bernstein-Greene-Kruskal equilibria. Studies of their self-consistent structure including the background plasma dielectric shielding response show these blobs to be long-lived, self-sustaining structures⁷⁻⁹ whose role in relaxation is described by the DKE. This equation states that for a background distribution $\langle f \rangle = \langle f(v_{\parallel}, x) \rangle$, \tilde{f} evolves according to

$$\int dv_{\parallel} d^3x \frac{\partial \tilde{f}^2}{\partial t} = -2 \int dv_{\parallel} d^3x \frac{d(\tilde{f}\langle f \rangle)}{dt}. \quad (1)$$

Taylor expanding $\langle f \rangle$ around x_0 to first order in $(x - x_0)$ and noting that, for drift-Alfvén turbulence,

$dx/dt = c\hat{E}_\theta/B_0 + v_\parallel \hat{B}_r/B_0$, it follows that

$$\int dv_\parallel \int d^3x \frac{\partial}{\partial t} (\tilde{f}^2) = -2 \frac{\partial \langle f \rangle}{\partial x} \Big|_{x_0, u_\parallel} \left\{ \frac{c}{B_0} \langle \hat{E}_\theta \hat{n}_e \rangle_b - \frac{\langle \hat{B}_r \hat{J}_{\parallel e} \rangle_b}{|e| B_0} \right\}. \quad (2)$$

Here, $\hat{E}_\theta = -\nabla_\theta \hat{\phi}$, $\hat{B}_r = \nabla_\theta \hat{A}_\parallel$, where $\hat{\phi}$ and \hat{A}_\parallel are the electrostatic and the parallel component of the vector potentials, respectively. Also, $\hat{n}_e = \int dv_\parallel \tilde{f}$, $\hat{J}_{\parallel e} = -|e| \int dv_\parallel v_\parallel \tilde{f}$, $\langle \rangle_b$ denotes an average over the blob volume, and energy scattering has been ignored for convenience. With quasineutrality ($\hat{n}_e = \hat{n}_i$) and Ampère's law ($\hat{J}_{\parallel e} = -\nabla_\perp^2 \hat{A}_\parallel$, for negligible ion current), Eq. (2) implies that

$$\int dv_\parallel d^3x \frac{\partial \tilde{f}^2}{\partial t} = -2 \frac{\partial f_0}{\partial x} \Big|_{x_0, u_\parallel} \left\{ \frac{c}{B_0} \langle \hat{E}_\theta \hat{n}_i \rangle_b + \frac{\langle \hat{B}_r \nabla_\perp^2 \hat{A}_\parallel \rangle_b}{|e| B_0} \right\}. \quad (3)$$

Self-consistency constraints regulate the relaxation mechanisms. In particular, since $\hat{\mathbf{B}} = \nabla \hat{A}_\parallel \times \mathbf{n}$ and $\nabla \cdot \hat{\mathbf{B}} = 0$, it follows that $\langle \hat{B}_r \nabla_\perp^2 \hat{A}_\parallel \rangle_b = -\langle \partial_r (\hat{B}_r \hat{B}_\theta) \rangle_b$, which ultimately contributes only surface terms of $O(\Delta x/L_x) \ll 1$. Hence, in this simple drift-Alfvén system, magnetic fluctuations do not result in evolution of \tilde{f} nor in the relaxation of $\langle f \rangle$. The above argument is similar to that used to establish the ambipolarity of magnetic transport.¹⁰ However, here Ampère's law (with $\hat{J}_{\parallel i} = 0$) and the granularity (i.e., localization in phase space) of \tilde{f} imply that the transport processes associated with the relaxation of *all moments* of $\langle f \rangle$ are similarly constrained over scales of Δx , the radial correlation length, i.e.,

$$\langle \hat{B}_r \hat{J}_{\parallel e} \rangle_b \approx -\langle \hat{B}_r \hat{B}_\theta \rangle_b \Big|_{x_0 - \Delta x}^{x_0 + \Delta x} \rightarrow O(\Delta x/L_x).$$

It is instructive to note that the familiar quasilinear result

$$\int dv_\parallel d^3x \partial/\partial t (\Delta f^2) = \int dv_\parallel d^3x D (\partial f_0/\partial x)^2$$

(here D is the quasilinear diffusion coefficient for magnetic turbulence) can be recovered by the discarding of fluctuation granularity by replacement of \tilde{f} in the right-hand side of Eq. (1) with $f^{(c)}$, the linear coherent response. This observation is further evidence that the constraint on magnetic transport and relaxation discussed above arises as a consequence of self-consistency (Ampère's law) and the granularity of \tilde{f} .

In the second model, relaxation and transport due to fully developed collisionless drift-Alfvén turbulence is examined with use of standard statistical turbulence theory. The relaxation of $\langle f \rangle$ is governed by the averaged Vlasov equation

$$\frac{\partial \langle f \rangle}{\partial t} = \frac{c}{B_0} \frac{\partial}{\partial r} \left\langle \left[\nabla_y \left(\hat{\phi} - \frac{v_\parallel}{c} \hat{A}_\parallel \right) \right] f \right\rangle, \quad (4)$$

$$\Gamma_\Gamma = \sum_{\mathbf{k}, \omega} \text{Re} \left\{ i \frac{c}{B_0} k_y \left[\left[R_{\mathbf{k}, \omega}^\phi \langle \phi^2 \rangle_{\mathbf{k}, \omega} - \frac{v_\parallel}{c} R_{\mathbf{k}, \omega}^A \langle \hat{A}_\parallel^2 \rangle_{\mathbf{k}, \omega} + R_{\mathbf{k}, \omega}^A \langle \hat{\phi} \hat{A}_\parallel \rangle_{\mathbf{k}, \omega} - \frac{v_\parallel}{c} R_{\mathbf{k}, \omega}^\phi \langle (\hat{A}_\parallel \hat{\phi}) \rangle_{\mathbf{k}, \omega} \right] + \left[\langle \hat{\phi} \tilde{h} \rangle_{\mathbf{k}, \omega} - \frac{v_\parallel}{c} \langle \hat{A}_\parallel \tilde{h} \rangle_{\mathbf{k}, \omega} \right] \right\}, \quad (6)$$

where we have neglected the velocity-space nonlinearity for simplicity. With the inclusion of fluctuations $f^{(c)}$ which are phase coherent with the potentials $\hat{\phi}$ and \hat{A}_\parallel and incoherent fluctuations \tilde{f} , the right-hand side of Eq. (4) can be represented as a Lenard-Balescu turbulent collision integral (LBTCl).⁵⁻⁷ The coherent fluctuations drive (quasilinear) diffusion and the incoherent fluctuations give rise to a collisionless drag. Equation (4) thus describes the slow-time-scale evolution of $\langle f \rangle$ due to averaged fast-time-scale fluctuations in the fields $\hat{\phi}$ and \hat{A}_\parallel due both to collective oscillations and to incoherent fluctuations associated with mode coupling in fully developed turbulence. The purpose of this part of the paper is to report effects on predicted transport and relaxation rates which follow from the imposition of self-consistency constraints relating incoherent and coherent fluctuations (equivalently, granularity and plasma response) in the turbulent transport equation.

When we extract the adiabatic distribution $\langle f \rangle e\hat{\phi}/T_e$ from the total fluctuating distribution $(\tilde{f} - \langle f \rangle e\hat{\phi}/T_e = \hat{h})$ the evolution of $\langle f \rangle$ is driven by the radial phase space flux: $\partial \langle f \rangle / \partial t = \partial \Gamma_\Gamma / \partial r$, where

$$\Gamma_\Gamma = \sum_{\mathbf{k}, \omega} \text{Re} \left\{ i \frac{c}{B_0} k_y \left\langle \left(\hat{\phi} - \frac{v_\parallel}{c} \hat{A}_\parallel \right) \hat{h} \right\rangle_{\mathbf{k}, \omega} \right\}. \quad (5)$$

Coherent and incoherent nonadiabatic density components are substituted into Eq. (5) in order to obtain diffusion and drag operators. By definition, $\hat{h}^{(c)}$, the usual coherent component, can be written as $\hat{h}^{(c)} = R_{\mathbf{k}, \omega}^\phi \hat{\phi}_{\mathbf{k}, \omega} + R_{\mathbf{k}, \omega}^A \hat{A}_{\parallel \mathbf{k}, \omega}$, where R^ϕ and R^A are generalized nonlinear electron coherent response functions. The coherent ion response is hydrodynamic, $\omega > k_\parallel v_T$, so that $n_{i\mathbf{k}, \omega} = R_{\mathbf{k}, \omega}^{\text{ion}} \hat{\phi}_{\mathbf{k}, \omega}$, in accord with the usual drift-Alfvén model. Substituting $\hat{h} = \hat{h}_e^c + \tilde{h}_e$ into Eq. (5) yields

where the first four terms constitute the usual quasilinear diffusion operator, containing electrostatic, magnetic, and off-diagonal terms, respectively. The last two terms constitute the drag operator, and are induced by incoherent fluctuations. Note that the first quasilinear term and first drag term govern all trans-

port moments arising from $\mathbf{E} \times \mathbf{B}$ motion ($\langle \hat{E}_\theta \hat{f} \rangle$), whereas the second quasilinear term and second drag term govern all transport moments arising from magnetic flutter ($\langle \hat{B}_r \hat{f} \rangle$).

The fluctuations $\hat{h}_{\mathbf{k},\omega}^\xi$ and $\tilde{h}_{\mathbf{k},\omega}$ are related by Ampère's law and quasineutrality, which respectively imply that

$$d_{\mathbf{k},\omega}^{A,A} \hat{A}_{\parallel \mathbf{k},\omega} + d_{\mathbf{k},\omega}^{A,\phi} \hat{\phi}_{\mathbf{k},\omega} = -\frac{4\pi|e|}{c} \int dv_{\parallel} v_{\parallel} \tilde{h}_{\mathbf{k},\omega} = -\frac{4\pi}{c} \tilde{J}_{\parallel \mathbf{k},\omega}$$

and that

$$d_{\mathbf{k},\omega}^{\phi,\phi} \hat{\phi}_{\mathbf{k},\omega} + d_{\mathbf{k},\omega}^{\phi,A} \hat{A}_{\parallel \mathbf{k},\omega} = -\int dv_{\parallel} \tilde{h}_{\mathbf{k},\omega} = -\tilde{n}_{\mathbf{k},\omega}.$$

Here $d^{A,A}$, $d^{A,\phi}$, $d^{\phi,A}$, and $d^{\phi,\phi}$ are dielectric tensor elements obtained from velocity moments of the electron and ion coherent phase-space density responses. It is thus possible to express \hat{A}_{\parallel} and $\hat{\phi}$ in terms of \tilde{J}_{\parallel} and \tilde{n} , as

$$\hat{A}_{\parallel \mathbf{k},\omega} = \mathcal{L}_{\mathbf{k},\omega}^{-1} \left[d_{\mathbf{k},\omega}^{\phi,\phi} \frac{4\pi}{c} \tilde{J}_{\parallel \mathbf{k},\omega} - d_{\mathbf{k},\omega}^{A,\phi} \tilde{n}_{\mathbf{k},\omega} \right], \quad (7a)$$

$$\hat{\phi}_{\mathbf{k},\omega} = \mathcal{L}_{\mathbf{k},\omega}^{-1} \left[d_{\mathbf{k},\omega}^{A,A} \tilde{n}_{\mathbf{k},\omega} - d_{\mathbf{k},\omega}^{\phi,A} \frac{4\pi}{c} \tilde{J}_{\parallel \mathbf{k},\omega} \right]. \quad (7b)$$

Setting $\mathcal{L} = d^{\phi,A} d^{A,\phi} - d^{A,A} d^{\phi,\phi} = 0$ determines the eigenfrequencies of the system. Equations (7a) and (7b) indicate that the collective plasma response shields incoherent density and current fluctuations. This shielding mechanism underlies the relationship between $\hat{h}_{\mathbf{k}}^\xi$ and $\tilde{h}_{\mathbf{k}}$, which follows from Eqs. (7a) and (7b) and the definition of $\hat{h}_{\mathbf{k}}^\xi$. Note that Eqs. (7a) and (7b) assume that the collective resonances (where $\text{Re} \mathcal{L}_{\mathbf{k},\omega} = 0$) are nonlinearly oversaturated or stable.

Self-consistency constraints are imposed on the LBTCI by substitution of Eqs. (7a) and (7b) into Eq. (6) for each $\hat{\phi}$ and \hat{A}_{\parallel} . For the electron response functions $R_{\mathbf{k},\omega}^A$, $R_{\mathbf{k},\omega}^\phi$ which multiply the diffusion and off-diagonal terms, moderate or weak spectral broadening ($\Delta\omega \leq \omega$) is assumed, allowing these functions to be written in standard fashion with use of the ballistic propagator $\pi\delta(\omega - k_{\parallel} v_{\parallel})$:

$$R_{\mathbf{k},\omega}^\phi = -\frac{|e|}{T_e} \langle f \rangle (\omega - \omega_{*e}^T) i\pi\delta(\omega - k_{\parallel} v_{\parallel}),$$

$$R_{\mathbf{k},\omega}^A = -R_{\mathbf{k},\omega}^\phi (v_{\parallel}/c).$$

Similarly, the correlation function is written as $\langle \tilde{h}^2 \rangle_{\mathbf{k},\omega} = 2\pi\delta(\omega - k_{\parallel} v_{\parallel}) \langle \tilde{h}^2 \rangle_{\mathbf{k}}$. The drag terms are multiplied by the unit operator $\mathcal{L}^{-1} \mathcal{L} = \mathcal{L}^{-1} (d^{A,\phi} d^{\phi,A} - d^{\phi,\phi} d^{A,A})$. The dielectric tensor elements appearing in this unit operator have electron contributions which are $4\pi|e|/c$ times the v_{\parallel} moment of $R_{\mathbf{k},\omega}^A$ and $R_{\mathbf{k},\omega}^\phi$ in the case of $d^{A,A}$ and $d^{A,\phi}$, respectively; and the v_{\parallel} average (zeroth moment) of $R_{\mathbf{k},\omega}^A$ and $R_{\mathbf{k},\omega}^\phi$ in the case of $d^{\phi,A}$ and $d^{\phi,\phi}$. If we note these contributions, inspection of the LBTCI shows that the magnetic

“flutter” diffusion term $v_{\parallel}^2 \langle (\hat{B}_r/B_0)^2 \rangle \delta(\omega - k_{\parallel} v_{\parallel})$ is exactly canceled by the electron-electron piece of the $\langle \hat{A}_{\parallel} \hat{h} \rangle$ drag term,

$$\mathcal{L}^{-1} (d^{A,\phi} \text{Im} d^{\phi,A} - d^{\phi,\phi} \text{Im} d^{A,A}) \omega / k_{\parallel} c \langle \hat{A}_{\parallel} \tilde{h} \rangle$$

($d^{\phi,A}$ and $d^{A,A}$ contain electron contributions only). Similarly, inspection shows that the electrostatic diffusion term ($\langle \hat{\phi}^2 \rangle$) is exactly canceled by the electron-electron piece of the $\langle \hat{\phi} \hat{h} \rangle$ drag,

$$\mathcal{L}^{-1} (\text{Im} d_{(\text{elec})}^{A,\phi} d^{\phi,A} - \text{Im} d_{(\text{elec})}^{\phi,\phi} d^{A,A}) \langle \hat{\phi} \tilde{h} \rangle.$$

The surviving terms in the LBTCI are (1) the electron-ion electrostatic drag term

$$\mathcal{L}^{-1} (\text{Im} d_{(\text{ion})}^{A,\phi} d^{\phi,A} - \text{Im} d_{(\text{ion})}^{\phi,\phi} d^{A,A}) \langle \hat{\phi} \tilde{h} \rangle,$$

(2) “nonresonant” magnetic and electrostatic drag operators,

$$\mathcal{L}^{-1} (d^{A,\phi} \text{Re} d^{\phi,A} - d^{\phi,\phi} \text{Re} d^{A,A}) \omega / k_{\parallel} c \langle \hat{A}_{\parallel} \tilde{h} \rangle$$

and

$$\mathcal{L}^{-1} (\text{Re} d^{A,\phi} d^{\phi,A} - \text{Re} d^{\phi,\phi} d^{A,A}) \langle \hat{\phi} \tilde{h} \rangle,$$

respectively, and (3) the cross diffusion terms $\langle \hat{A}_{\parallel} \hat{\phi} \rangle$ and $\langle \hat{\phi} \hat{A}_{\parallel} \rangle$, which, like the nonresonant drag operators, are also proportional to $\text{Re} d_{\mathbf{k},\omega}$. Consistent with the assumption of moderate spectral broadening, the susceptibilities $d_{\mathbf{k},\omega}$ are evaluated on collective resonance, $\text{Re} L = 0$, where $\text{Re} d_{\mathbf{k},\omega}$ is negligible on either the Alfvén or drift-wave branches. The collective resonance condition implies that the plasma shielding response is dominated by waves. With $\text{Re} d_{\mathbf{k},\omega} = 0$, both nonresonant drag operators and cross diffusion terms $\langle \hat{A}_{\parallel} \hat{\phi} \rangle$ and $\langle \hat{\phi} \hat{A}_{\parallel} \rangle$ vanish, so that only the electron-ion drag terms associated with $\mathbf{E} \times \mathbf{B}$ motion survive. Equation (6) reduces to

$$\Gamma_{\Gamma} = \sum_{\mathbf{k},\omega} -\frac{(c/B_0)k_y}{|k_{\parallel}|} 2\pi\delta(\omega - k_{\parallel} u_{\parallel}) [\text{Re} S_{\mathbf{k},\omega}], \quad (8a)$$

where

$$S_{\mathbf{k},\omega} = \mathcal{L}_{\mathbf{k},\omega}^{-1} \langle \hat{\phi} \tilde{h} \rangle_{\mathbf{k}} [d_{\mathbf{k},\omega}^{A,A} \text{Im} d_{\mathbf{k},\omega}^{\phi,(\text{ion})}], \quad (8b)$$

in the usual case of negligible ion current ($\hat{J}_{\parallel}^i \rightarrow 0$, $\text{Im} d_{\mathbf{k},\omega}^{A,(\text{ion})} \approx 0$). Finally, it should be noted that in

the electrostatic limit the LBTCI reduces to the form previously derived and evaluated.⁶

Equations (8a) and (8b) indicate that magnetic transport including quasilinear magnetic flutter transport [$\sim v_{\parallel}^2 \langle (\hat{B}_r/B_0)^2 \rangle_{\mathbf{k}} \delta(\omega - k_{\parallel} v_{\parallel})$] does not contribute to the relaxation of $\langle f \rangle$, and thus is not responsible for electron energy or momentum transport! Insight into this result can be gained by noting that $\partial \langle f_e \rangle / \partial t \sim \text{Im} d_{\mathbf{k}, \omega}^{\phi, \phi(\text{ion})}$, which indicates that electron phase-space relaxation is proportional to the dissipative ion coupling to the electrostatic potential. Indeed, if such dissipative ion coupling is absent, the LBTCI vanishes and $\langle f \rangle$ cannot relax. This result is analogous to those obtained using collisional and collisionless Lenard-Balescu equations for a one-dimensional electron-ion plasma.¹¹ In that system, constraints of momentum and energy conservation on the interaction (collisional or collisionless) of localized phase space fluctuations imply that like "particle collisions" leave the final state identical to the initial state, thus precluding relaxation of $\langle f \rangle$. This results in a similar cancellation of electron-electron terms, leaving $\partial \langle f \rangle / \partial t$ proportional to $\text{Im} \chi_i$, where χ_i is the ion susceptibility. Here, since $\hat{J}_{\parallel}^i \rightarrow 0$, the only electron-ion coupling occurs through $\text{Im} d_{\mathbf{k}, \omega}^{\phi, \phi(\text{ion})}$.

The two models of collisionless drift-Alfvén dynamics, the isolated blob and fully developed turbulence models, respectively, give consistent, complementary insights into the effect of the same self-consistency constraints on relaxation and transport. In the case of an isolated blob, Ampère's law and the granularity (i.e., localization in phase space) of \tilde{f} lead to the result that $\partial \langle \tilde{f}^2 \rangle / \partial t \sim \langle \hat{B}_r \hat{J}_{\parallel e} \rangle \sim \langle \partial(\hat{B}_\theta \hat{B}_r) / \partial r \rangle \rightarrow 0$, to $O(\Delta x / L_x)$. In the case of fully developed turbulence, Ampère's law, quasineutrality, and the proper consideration of granular, incoherent fluctuations in the dynamics of $\langle f \rangle$ result in the cancellation of diffusive magnetic flutter terms (in the LBTCI) by electron-electron drag terms ($\sim \langle \hat{A}_{\parallel} \hat{h} \rangle$). Both results indicate that *transport and relaxation in drift-Alfvén turbulence are regulated by electrostatic fluctuations.*

It is interesting to reconsider theories of anomalous transport due to magnetic flutter in light of the discussion presented here. In particular, a recent paper¹ by Kadomtsev and Pogutse (KP) treats transport caused by small-scale, high-frequency ($\Delta x \sim c/\omega_{pe}$, $\omega \sim v_{Te}/qR$) electromagnetic turbulence. Tacitly assuming that the transport-causing small scales are nonlinearly driven by larger scales via cascading, KP then use quasilinear theory (with dissipation due to electron Landau resonance) and mixing-length estimates to derive the thermal conductivity $\chi_e \sim \epsilon (c^2/\omega_{pe}^2) v_{Te}/qR$. However, dissipative ion coupling is ignored throughout their analysis. Thus, by way of contrast, a parallel calculation following the discussion presented here yields the result that χ_e vanishes! The discrepan-

cy is due to the fact that KP invoke mode coupling to drive the transport-causing scales, but compute χ_e using quasilinear theory ignoring incoherent fluctuations. The theory of Carreras, Diamond, and co-workers⁴ also investigates magnetic-flutter transport effects. Their study is a treatment based on resistive magnetohydrodynamics and is not fully self-consistent. However, the present considerations do *not* apply to magnetic transport resulting from relaxation driven by collisional or macroscopic (i.e., resistive magnetohydrodynamics) turbulence and conclusions of the present analysis cannot be extended to such cases.

It is important to note that other restrictions apply as well to the discussion presented here. These results are not applicable to regimes with strong spectral broadening ($\Delta\omega > \omega$). In that case it is not possible to use a ballistic propagator in the particle responses and the cancellations described above are only partial. While magnetic transport would then be possible, it nevertheless could not be described by quasilinear theory but would require some sort of large-amplitude, self-consistent treatment. Furthermore, stationary turbulence is assumed throughout. Nonstationary turbulence (such as in the case of growing waves) permits the exchange of energy and momentum between waves and incoherent fluctuations, thus allowing different relaxation mechanisms. Finally, these considerations do not straightforwardly lend themselves to the study of magnetic transport induced by external perturbations, such as an applied helical coil.

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