Nonlocal Electron Heat Transport by Not Quite Maxwell-Boltzmann Distributions

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We have made numerical calculations with a new nonlocal fluid treatment of Coulomb collisional electron transport which self-consistently accounts for the nonthermal high-energy electrons arising from the spatial transport of thermal electrons whose range is not short compared with the temperature scale length. Heat fluxes associated with steep gradients are reduced from classical, while ahead of a temperature front there is preheating which exceeds classical.

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The need for an efficient self-consistent treatment of high-flux electron transport in laser-irradiated plasmas has been acute for some time. Intense research has identified and resolved important issues in the Coulomb collisional theory.[1,2] It is now understood that the failure of classical transport occurs when the mean free path of the high-energy heat-carrying electrons is not much smaller than the temperature scale length. This implies that the transport fluxes are not locally determined. In fact, idealizations of systems of interest have been successfully described by a fully multigroup (in energy) diffusion (in space) treatment.[2] This reduced Fokker-Planck theory accounts for the nonthermal distribution induced at high energy by the transport itself; this in turn acts to alter the transport from classical. Heat fluxes associated with steep temperature gradients are reduced from classical, while ahead of a temperature front there is preheating which exceeds classical. This understanding has been gained by analysis of state-of-the-art numerical calculations still too inefficient for application to more than a few problems.

In practice *ad hoc* fluid schemes have been employed to treat high-flux electron transport. The most robust of these, flux limitation, limits the heat flux to a fraction of its so-called free-streaming value, $Q_{fl} = F_{fl} n(T/m)^{1/2} T$; however, in the context of the Coulomb collisional theory, this prescription is unphysical in that it is pointwise local. Recently a new fluid scheme possessing nonlocal phenomenology has been employed and normalized against certain first-principles calculations.[3]

Here we report numerical calculations with a nonlocal fluid treatment of Coulomb collisional transport which self-consistently accounts for the nonthermal high-energy electrons arising from the spatial transport of thermal electrons whose range is not short compared to the temperature scale length. The associated Fokker-Planck calculation has been carried out analytically and permits the first-principles formulation of a nonlocal (in space) description of the evolution of the plasma temperature. Our theory requires that the number and energy of the nonthermal electrons be small, but their number and energy fluxes are not restricted and may dominate the total fluxes. Indeed, we have recovered in detail the results of previous fully multigroup diffusion laser-absorption and heat-transport calculations.[2] Our transport scheme is suitable for implementation in one-dimensional laser-plasma simulation codes and is expected to be efficient enough for general utility.

The reduced Fokker-Planck equation which governs the electron distribution in position and total energy is[2]

$$\frac{\partial f_0}{\partial t} - \frac{\partial e\phi}{\partial t}\frac{\partial f_0}{\partial \epsilon} - \frac{1}{v}\frac{\partial}{\partial x}\frac{v^2\lambda_{90}}{6}\frac{\partial f_0}{\partial x} = \frac{df_0}{dt_{ee}}. \quad (1)$$

Here $mv^2/2 = \epsilon + e\phi(x,t)$ and the scattering mean free path is

$$\lambda_{90} = (mv^2)^2/2\pi e^4 (NZ^2 \ln\Lambda_{ei} + n \ln\Lambda_{ee}).$$

Equation (1) neglects the hydrodynamics of the background ions as slow on the transport time scale.

The electron number and energy

$$\{n, \tfrac{3}{2}nT\} = 4\pi \int_{-e\phi}^{\infty} \frac{de\,v}{m}\left\{1, \frac{mv^2}{2}\right\} f_0$$

obey the associated moments of Eq. (1)

$$\frac{\partial}{\partial t}\{n, \tfrac{3}{2}nT\} + \frac{\partial}{\partial x}\{\Gamma, Q\} + \{0, e\Gamma E\} = 0. \quad (2)$$

Electron-electron collisions, d/dt_{ee}, conserve both number and energy of the species. The electrostatic potential is determined by the requirement of quasineutrality, $n = ZN$. In the present application this becomes the vanishing of the number flux, $\Gamma(e\phi) = 0$.

The number and energy fluxes, Γ and Q, are

$$\{\Gamma, Q\} = -4\pi \int_{-e\phi}^{\infty} \frac{d\epsilon}{m} \left\{1, \frac{mv^2}{2}\right\} \frac{v^2 \lambda_{90}}{6} \frac{\partial f_0}{\partial x}. \quad (3)$$

If df_0/dt_{ee} is dominant in Eq. (1), then the distribution takes its local-thermodynamic-equilibrium form, the Maxwell-Boltzmann,

$$f_0 \to f_{\mathrm{MB}} = [n/(2\pi T/m)^{3/2}] \exp[-(\epsilon + e\phi)/T].$$

Employing f_{MB} in Eqs. (3) yields classical transport wherein $Q \propto -\partial T/\partial x$; this closes the fluid Eqs. (2).

The transport itself upsets the MB distribution whenever the scale length of interest, the temperature gradient L, is not much longer than the stopping length, $\lambda_s = (2\lambda_{90}\lambda_\epsilon/3)^{1/2}$. This may be seen by estimating $df_0/dt_{ee} \sim (v/4\lambda_\epsilon) f_0$ and comparing it with the spatial diffusion term $\sim (v\lambda_{90}/6L^2) f_0$. Here $\lambda_\epsilon = (mv^2)^2/2\pi e^4 n \ln\Lambda_{ee}$ is the energy-loss mean free path. It is instructive to recognize $\lambda_s = (2\lambda_\epsilon/3\lambda_{90})^{1/2}\lambda_{90}$ as the displacement associated with random walking the path length λ_ϵ in steps of λ_{90}. On account of the strong energy dependence of λ_s, it is the distribution of the higher-energy electrons that is most readily altered by the transport. Inspection of Eq. (3) reveals that these higher-energy electrons are the ones which carry the fluxes and so must in turn be accounted for in calculating the transport.

Thus motivated, we write $f_0 \simeq f_{\mathrm{MB}} + \delta f$ and solve for δf at high energy where Eq. (1) becomes

$$-\frac{1}{v}\frac{\partial}{\partial x}\frac{v^2\lambda_{90}}{6}\frac{\partial f_{\mathrm{MB}}}{\partial x} - \frac{1}{v}\frac{\partial}{\partial x}\frac{v^2\lambda_{90}}{6}\frac{\partial \delta f}{\partial x} = \frac{2mv^3}{\lambda_\epsilon}\frac{\partial \delta f}{\partial \epsilon}. \quad (4)$$

Here f_{MB} is annihilated by d/dt_{ee}, and we have neglected the thermalization of high-energy electrons on the bulk as small by $T\partial \ln\delta f/\partial\epsilon$ compared with the energy loss retained. Also, the temporal variation of the distribution and potential are assumed slow. Equation (4) makes explicit that the source (in energy, possibly negative) of nonthermal electrons δf is the spatial transport of thermal electrons f_{MB}.

We solve Eq. (4) in the limit $\epsilon > -e\phi$ so that $mv^2/2 \sim \epsilon$ and find

$$f_0 = \int d\xi' \int_\epsilon^\infty d\epsilon' \frac{\exp\{-[(\xi-\xi')^2/(\epsilon'^4 - \epsilon^4)]\}}{[\pi(\epsilon'^4-\epsilon^4)]^{1/2}} \frac{f_{\mathrm{MB}}(\xi',\epsilon')}{T(\xi')}. \quad (5)$$

Here $d\xi = dx/\tilde{\lambda}_s$ and we use $\tilde{\lambda} = \lambda/(mv^2)^2$. We have not yet taken account of spatial boundary conditions. We have also passed a factor of $Z^{1/2}$ through $\partial/\partial x$ in defining ξ and thus require that the ionization state vary more slowly than the temperature.

Equation (5) shows that nonthermal electrons at ξ and ϵ have come from all ξ' and all $\epsilon' > \epsilon$ by stopping transport. We observe that f_0 of Eq. (5) is determined by the spatial distribution of density and temperature in f_{MB}. We shall neglect the small number and energy, δn and $\delta 3nT/2$, of nonthermals in the fluid equation (2) so that the density and temperature of the plasma become those of the MB distribution. Thus substitution of f_0 of Eq. (5) into Eqs. (3) closes the fluid Eqs. (2) and naturally yields a nonlocal (in space) treatment of the transport.

In Eqs. (3) we have energy integrals of $\partial f_0/\partial x = (1/\tilde{\lambda}_s)\partial f_0/\partial\xi$; this differentiation acts on the Gaussian kernel in Eq. (5) for f_0. We replace $\partial/\partial\xi$ by $-\partial/\partial\xi'$ and integrate by parts in ξ' to cast the spatial derivative onto f_{MB}/T. The electric field enters explicitly upon differentiation of the exponential factor of f_{MB}. It is convenient to split the potential into local and nonlocal parts, $e\phi = e\phi_1 + e\phi_{\mathrm{nl}}$, where $\partial e\phi_1/\partial\xi = (T/n)\partial n/\partial\xi - (\frac{5}{2})\partial T/\partial\xi$. Finally $\epsilon > -e\phi$ is exploited by letting $-e\phi \to 0$ everywhere except under differentiation by ξ'.

The resulting double energy integrals have been computed to obtain

$$\{\Gamma, Q\} = -\frac{(\tilde{\lambda}_{90}/\tilde{\lambda}_\epsilon)^{1/2}}{4\pi(3m)^{1/2}} \int dx'\, nT^{-1/2}\{1,T\}\left[\frac{\partial T}{\partial x'}\{I,K\} - \frac{\partial e\phi_{\mathrm{nl}}}{\partial x'}\{J,L\}\right]. \quad (6)$$

Here we have made the transformation $d\xi'\partial/\partial\xi' = dx'\partial/\partial x'$. The nonlocal transport propagators $P = I$, J, K, and L are functions of only

$$\theta = \left|\int_x^{x'} [dx''/\tilde{\lambda}_s(x'')]/T^2(x')\right|,$$

which is the number of stopping lengths from the source point x' to the field point x at an energy equal to the source temperature.

The propagators have the following form:

$$P(\theta) = \theta^{2+2\beta}\int_0^\infty dy\, y^\beta \exp[-\theta^{1/2}y^{1/2} - 1/y] \int_0^1 dy' \frac{y'^\alpha}{(1-y')^{1/2}} \exp[y'/y(1-y')].$$

TABLE I. Coefficients for the nonlocal transport propagators described in the text.

P	α	β	$P(0)$	$-P'(0)$	A	γ	$\int_0^\infty d\theta\, P(\theta)$
I	0	0	48	$8\sqrt{\pi}$	1	1	$240\sqrt{\pi}$
J	0	$-\frac{1}{4}$	16	$8\sqrt{\pi}$	$4^{-1/5}$	$\frac{3}{5}$	$48\sqrt{\pi}$
K	$\frac{1}{4}$	$\frac{1}{4}$	$\dfrac{24\sqrt{\pi}\Gamma(\frac{5}{4})}{\Gamma(\frac{7}{4})}$	$16\sqrt{\pi}$	$4^{2/5}\Gamma(\frac{5}{4})$	$\frac{13}{10}$	$1152\sqrt{\pi}$
L	$\frac{1}{4}$	0	$\dfrac{96\sqrt{\pi}\Gamma(\frac{5}{4})}{\Gamma(\frac{7}{4})}$	$8\sqrt{\pi}$	$4^{1/5}\Gamma(\frac{5}{4})$	$\frac{9}{10}$	$192\sqrt{\pi}$

For small θ they fall off linearly, $P(\theta)\to P(0)+P'(0)\theta$, while for large θ they fall off exponentially in $\theta^{2/5}$,

$$P(\theta)\to 32(2\pi/5)^{1/2}A\theta^{\gamma}\exp[-(5/4^{4/5})\theta^{2/5}].$$

The constants α, β, $P(0)$, $P'(0)$, A, γ, and the propagator normalization integrals are shown in Table I.

We have analyzed two limiting temperature profiles which illuminate the physics in play. First, in the limit of gradients much longer than the stopping length, the nonlocal heat flux reduces to classical because only x' near x contributes in the integrals of Eqs. (6). We let $dx'\to T^2\tilde{\lambda}_s\, d\theta$ and employ the propagator normalizations. Taking $\partial e\phi_{\rm nl}/\partial x\to 5\,\partial T/\partial x$ from $\Gamma=0$ yields the classical results,

$$\partial e\phi/\partial x\to (T/n)\partial n/\partial x+(\tfrac{5}{2})\partial T/\partial x$$

and

$$Q\to 25.532n(T/m)^{1/2}\lambda_{\rm MFP}\,\partial T/\partial x.$$

Here $\lambda_{\rm MFP}=T^2\tilde{\lambda}_{90}$.

Second, our nonlocal heat flux is naturally self-limiting. For a temperature step from hot to cold over a distance much shorter than a stopping length we find $\Delta e\phi_{\rm nl}\to 3\Delta T$ and

$$Q\to 1.285(\tilde{\lambda}_{90}/\tilde{\lambda}_\epsilon)^{1/2}(n/m^{1/2})(T_H^{3/2}-T_C^{3/2}).$$

Note that the temperature *within* the transition interval does not enter here; the limit of our nonlocal treatment is indeed nonlocal.

We have also solved the fluid equation (2) with the nonlocal fluxes of Eq. (6) numerically. Figure 1 shows some results of illuminating a stationary plasma with a constant intensity laser beam. To this end we have introduced the source κI in the energy equation.[4] The calculation was terminated when the heat flux into the overdense plasma equaled the absorbed laser flux so that the coronal plasma had reached a quasisteady condition.

In Fig. 1 we note that the signatures of the nonlocal transport are (1) a hot absorption layer, (2) a steep temperature gradient separating the laser- and conduction-heated plasma, (3) a nonisothermal corona, and (4) a preheat foot on the temperature profile. The reduced absorption recorded in the table is associated with (1) since $\kappa_c\propto T^{-3/2}$; this feature and (2) are typical of strongly flux-limited simulations required to reproduce results of laboratory experiments. Here (1), (2), and (3) are due to the stopping of electrons as they transport from the laser-heated plasma. The nonisothermal corona also results from nonlocal transport from there towards the colder, denser plasma. This physical phenomenon is also responsible for the preheat foot.

Figure 2 shows that including nonlinear reduction of the laser opacity[5] brings the absorbed energy, heat flux, and temperature all into agreement with previous direct solutions of Eq. (1).[2] Indeed, at high energy the underlying nonlocally determined distribution function, f_0 of Eq. (5), is in agreement as well.[2]

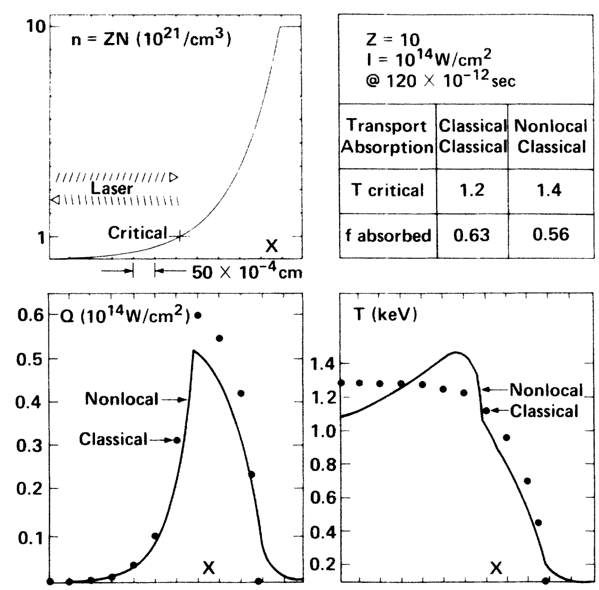

FIG. 1. Nonlocal transport gives significantly different results from classical in a calculation of a laser-heated plasma; classical absorption, $K=1.0$ (Ref. 4). Here a 1-μm wavelength laser is incident upon a 100-μm scale length plasma initially isothermal at 100 eV. The collision strengths are $\ln\Lambda_{ei}=2.5$ and $\ln\Lambda_{ee}=5$. We limit $\epsilon_{\rm osc}$ to $\sqrt{2}$ its vacuum value.

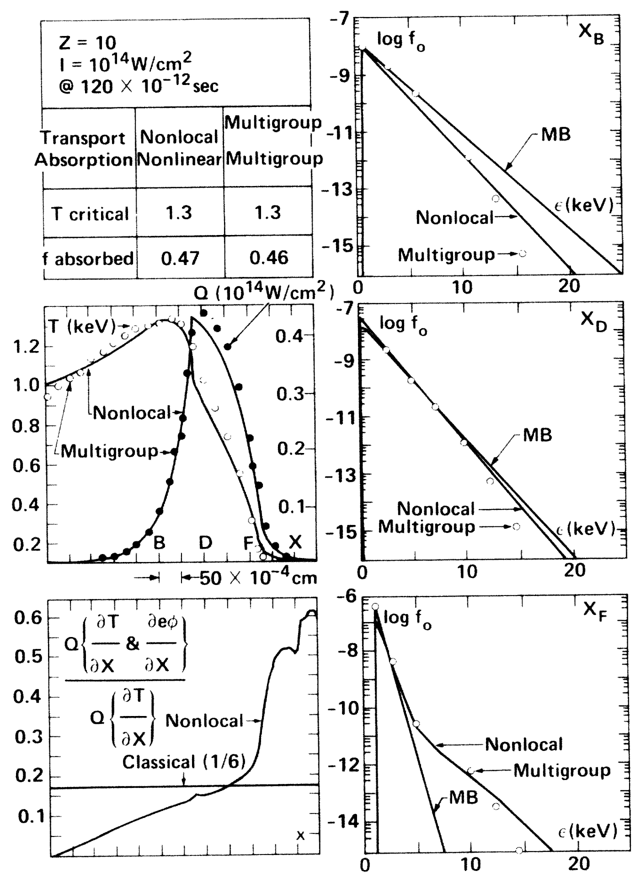

FIG. 2. Previous fully multigroup diffusion transport results (Ref. 2) are recovered by the nonlocal treatment when nonlinear reduction of the laser opacity (Refs. 4 and 5) is introduced in the calculation described in Fig. 1; here $K \sim 0.7$. The electron distribution is shown at the positions labeled on the (T,Q) plot. To show the role of the electric field we plot the ratio of the heat flux to that due to the temperature gradient alone.

In our treatment we have neglected the effect on transport of the nonthermal distribution induced at low energy by laser heating.[5] This heating is communicated to the high-energy transporting electrons via the temperature T; self-collisions mediate between laser heating at low energy and transport at high energy.

The effect of the nonlocally determined electric field is also displayed in Fig. 2. The field acts to reduce the heat flow from that due to the temperature gradient. The reduction is substantial and is not constant, being both larger and smaller than in the classical limit. In particular, less-than-classical reduction occurs in the preheat region because only a relatively small electric field arises there to establish zero total current against the small current of high-energy heat-transporting electrons.

We have checked that neglect of thermalization of high-energy electrons on the bulk electrons is justified.[6] Only modest errors of our nonlocal treatment are indicated.

Energy conservation in the laser-heated plasma calculations requires that there be no heat flux into the vacuum. This implies the condition $\partial f_0/\partial x = 0$ at the boundary; this is straightforward but tedious to implement as an (infinite) series of images. Provided that the entire system is many stopping lengths in extent, it is sufficient to include a single-image plasma on the vacuum side of the fluid calculation. In our calculations we extended the Γ and Q integrations over this image plasma to ensure that the fluxes vanished correctly at the boundary.

Implicit solution of the nonlocal transport equations requires inversion of a full matrix for the temperatures and potentials. This, together with setting up the underlying variables, represents a modest burden compared with the classical diffusion theory. The new physics obtained appears to be commensurate with its cost.

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