Relative $3s$ Spectroscopic Strength in ^{206}Pb and ^{208}Pb Studied with the $(e,e'p)$ Knockout Reaction

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A high-resolution $(e,e'p)$ measurement has been performed to determine the relative occupation probability of the $3s_{1/2}$ proton orbits in ^{206}Pb and ^{208}Pb. The knockout strength to the ground state of ^{205}Tl is 31(3)% smaller than that of ^{207}Tl. Appreciable fragmentation of $l=0$ knockout strength is observed in ^{205}Tl, whereas all $l=0$ strength in the excitation range up to 5.5 MeV resides in the ground state of ^{207}Tl. Up to this energy the summed $3s$ strength is found to be 16(9)% smaller in ^{206}Pb than in ^{208}Pb.

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Fractional occupation numbers of single-particle orbits of the order of unity are basic to the validity of the nuclear shell model. Unfortunately, occupation numbers are not directly measurable. For example, by single-nucleon transfer reactions one measures overlap integrals between the initial- and final-state wave functions that are related in a model-dependent way to the occupation numbers. Only if this model description as well as the reaction theory for transfer reactions is good enough can reliable occupation numbers be extracted from experimental data.

It has recently been argued[1,2] for the doubly closed-shell nucleus ^{208}Pb that short-range and tensor correlations give rise to a depletion of normally completely filled orbitals close to the Fermi surface. Estimates of this depletion are as large as 30%.

Most of the available information on the proton occupation numbers in the lead isotopes comes from single-proton transfer reactions[3,4] and more recently from electron-scattering experiments.[1,5] Results of the latter experiments are interpreted as being consistent with fractional occupation numbers of around 0.7. Additional spectroscopic information can be obtained from high-resolution $(e,e'p)$ knockout experiments. The $(e,e'p)$ knockout and the single-proton pickup reactions complement each other, since they probe different parts of the bound-state wave function and proceed via different reaction mechanisms. The theoretical description of both reactions is not yet accurate enough to determine absolute spectroscopic factors.

Recently, precise relative spectroscopic factors for $3s$ proton removal have been measured with the (d^3,He) reaction[4] relating the systems ^{206}Pb-^{205}Tl and ^{208}Pb-^{207}Tl. This reaction is sensitive to the asymptotic tail ($r \approx 10$ fm) of the $3s$ wave function and thus the extracted spectroscopic factors are a measure of the asymptotic normalization of this wave function. In this Letter we report results of a measurement of the relative $3s$ proton knockout strength in the lead isotopes ^{206}Pb and ^{208}Pb with the $(e,e'p)$ reaction. With this technique one probes the $3s$ proton wave function $\Psi_{3s}(r)$ through its Fourier-Bessel transform in momentum (p_m) space. In the plane-wave impulse approximation (PWIA), the reaction yield is proportional to the momentum distribution $\rho(p_m) = |\int j_0(p_m \times r) r^2 \Psi_{3s}(r)\, dr|^2$. This expression becomes extremely simple at $p_m = 0$; then the integrand becomes $r^2\Psi_{3s}(r)$, which has a dominant peak at $r \approx 6.5$ fm, where the third maximum of $\Psi_{3s}(r)$ is located. Hence at low p_m, the $(e,e'p)$ reaction yields a measure for the strength of the $3s$ wave function in the surface region of the Pb nucleus. At higher momenta the cross section becomes also sensitive to the other two maxima of Ψ_{3s}, which are located in the nuclear interior.

The experiment was performed with the linear accelerator MEA of NIKHEF-K at incident electron energies of 410 and 350 MeV. By employment of a dispersion-matching technique and by use of two high-resolution spectrometers,[6] a missing-energy resolution of 135 keV was achieved (see Fig. 1), which allows separation of the $\frac{1}{2}^+$ ground states from the first excited $\frac{3}{2}^+$ states in both isotopes. Data were taken on enriched ^{206}Pb and ^{208}Pb targets in parallel kinematics[7] with a fixed momentum $(\mathbf{p})$ of the detected proton ($T_p = 100$ MeV). Thus, in PWIA a proton with initial (missing) momentum $\mathbf{p}_m$ $(= \mathbf{p} - \mathbf{q})$ is knocked out of the nucleus parallel to the momentum transfer $\mathbf{q}$. Coincidence cross sections were measured for both

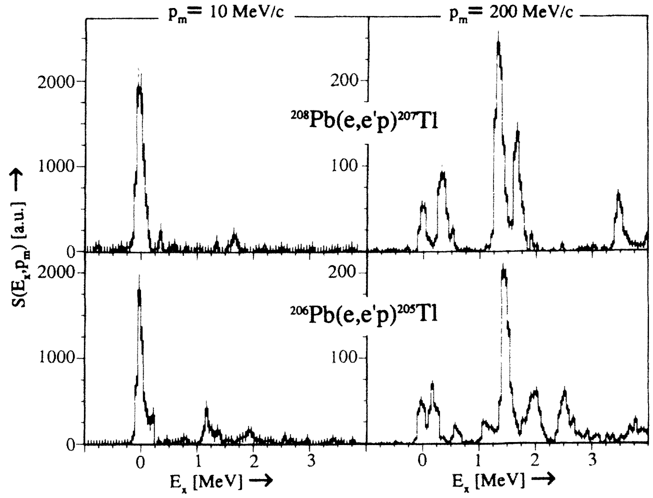

FIG. 1. The experimental spectral function vs excitation energy for ^{206,208}Pb at low and high values of the missing momentum.

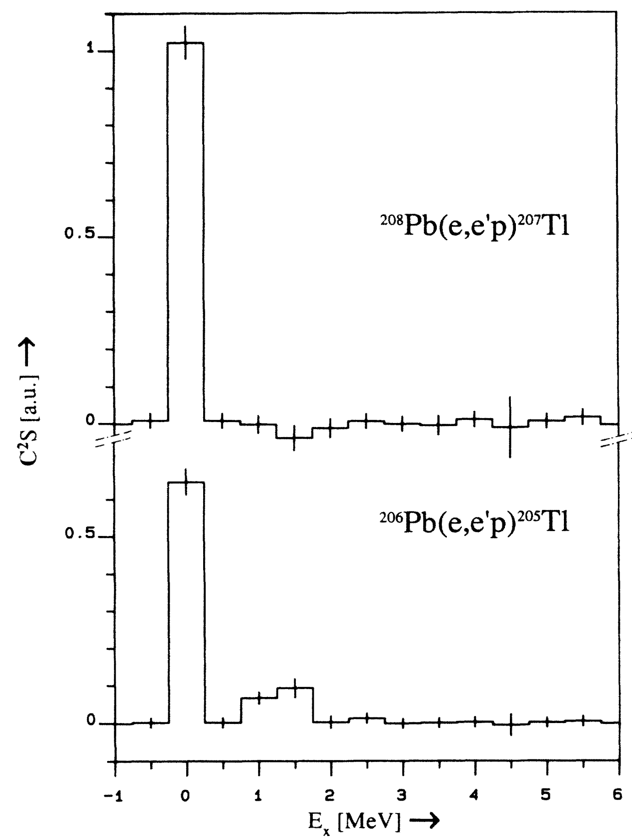

FIG. 2. Fragmentation of the $3s_{1/2}$ proton-hole strength in ^{205,207}Tl.

isotopes with p_m centered at 15, 100, and 200 MeV/c with a 50 MeV/c momentum acceptance. At $p_m = 15$ MeV/c the shape of the $3s_{1/2}$ momentum distribution is almost flat and independent of details of the final-state interaction and of the detailed structure of the bound-state wave function. Moreover, there is hardly any strength of higher-l knockout expected at low p_m and the counting rate for $3s_{1/2}$ knockout is at its maximum. The two other kinematics were used to check the p_m dependence of the ratio of the cross sections and to determine the contributions of higher-l strength.

The coincidence efficiency was determined with the reaction ^{1}H$(e,e'p)$ to be 99.6(8)%. The target thicknesses were determined from elastic electron scattering at $q = 1.75$ fm^{-1} by comparison with elastic cross sections measured elsewhere.[5,8] Accidental coincidences ($< 6\%$) were subtracted from the total yield, the radiative tails were unfolded, and corrections were made for dead-time effects ($< 15\%$). By addition of all relevant error contributions quadratically a total systematical error on the cross-section ratio of 2% was obtained. The measured coincidence cross sections were converted to a spectral function $S(E_x,p_m)$ representation[9]; see Fig. 1. Integration of the spectral function over the excitation energy (E_x) yields the momentum density $\rho(p_m)$ for each transition.

The quantity of interest in the present experiment is the ratio of the spectral functions of ^{206}Pb and ^{208}Pb for $l=0$ knockout strength. We can obtain this ratio either by performing an l decomposition of the spectral functions integrated over some excitation-energy interval, or by determining $\rho_i(p_m)$ for the individual transitions in the spectral function from peak fitting. The results of the first method are independent of the location and nature of the transitions in the excitation-energy spectrum, but rely on the calculation of momentum densities in the distorted-wave impulse approximation (DWIA). The second method is only possible for well-separated levels ($\Delta E_x > 150$ keV).

The l decomposition was carried out by integration of the spectral function over 0.5-MeV excitation-energy bins and subsequent fitting of the obtained $\rho_i(p_m)$ with an incoherent sum of calculated $\rho_\alpha(p_m)$ with $\alpha = 3s$, $2d$, $1g$, and $1h$. These densities were calculated with Woods-Saxon bound-state wave functions [$r_0 = 1.197(A-1)^{1/3}$ fm, $a_0 = 0.65$ fm, $V_{\text{s.o.}} = 6$ MeV, $E_{\text{sep}}(206) = 7.252$ MeV, and $E_{\text{sep}}(208) = 8.013$ MeV (Ref. 10)]. The distortion of the outgoing proton was calculated with a standard optical potential.[11] For each Pb isotope the largest $3s_{1/2}$ component was found in the knockout leading to the ground state of Tl (see Fig. 2). Significant $l=0$ strength was also found between 0.75 and 1.75 MeV in ^{205}Tl consistent with the known presence of two $\frac{1}{2}^+$ states at $E_x = 1.22$ and 1.44 MeV in ^{205}Tl.

In the second approach, the $l=0$ ground-state transition was integrated and the small contribution of the fitted low-energy tail of the first excited state at 0.35 (0.20) MeV for ^{207}Tl (^{205}Tl) was subtracted. In Fig. 3 the resulting ratio between the two ground-state strengths is shown as a function of p_m. Also shown is the ratio of the $3s_{1/2}$ DWIA calculations assuming equal spectroscopic factors for the two isotopes (dashed curve). By adjusting this curve to the data we obtain 0.69(3) for the ratio (R_0) of the $3s_{1/2}$ spectroscopic factors for the ground-state transition. As Fig. 3 shows, no significant p_m dependence of the ratio is observed. This indicates that the ratio of spectroscopic factors is also r independent. The same procedure has been applied to the $\frac{1}{2}^+$ state at 1.22 MeV in ^{205}Tl relative to the ground-state transition in ^{207}Tl. Here we find a scaling factor of 0.13(5). These results are consistent with the ones found with the l decomposition (see Table I).

In addition to these individual ratios the following integral quantities have been deduced from this experiment: (a) The fraction of the ground-state $3s$ strength relative to the strength integrated up to $E_x = 5.5$ MeV is $\alpha_{205}^2 = S_0^{206}/\int S^{206} = 0.78(4)$ and $\alpha_{207}^2 = S_0^{208}/\int S^{208} = 0.99(7)$; (b) the $3s$ strength in ^{206}Pb relative to the ground-state strength in ^{208}Pb is $\sum R_i = (S_0^{206} + S_{12}^{206})/S_0^{208} = 0.83(5)$; and (c) the ratio of the total strength integrated up to 5.5 MeV in both isotopes is $R_{\text{tot}} = \int S^{206}/\int S^{208} = 0.84(9)$.

From the results presented here we conclude that there is 22(4)% fragmentation of the $3s_{1/2}$ hole strength in ^{205}Tl and hardly any fragmentation in ^{207}Tl up to an excitation energy of 5.5 MeV. This is consistent with recent (d,^{3}He) results[4] and with shell-model calculations in a large configuration space.[12]

We find that the total $3s$ strength up to 5.5 MeV is 16(9)% smaller in ^{206}Pb than in ^{208}Pb. This effect might be due to a neutron rearrangement, which shifts $3s$ hole strength to much higher excitation energy in the case of the open neutron shell in ^{205}Tl, but not in the case of the closed shell in ^{207}Tl.

The relative strength R_0 is 10(5)% smaller in this experiment than in the (d,^{3}He) reaction[4]; see Table I. This can be explained by an unlikely large change (15%) of the most sensitive optical-model parameter W from one isotope to the other. A more likely explanation of this difference resides in the choice of the bound-state wave function, which is sampled in very different regions of r space in the two reactions. For instance the standard prescription for the $A^{1/3}$ dependence of the geometry of the mean field, in which the $3s_{1/2}$ proton is bound, in going from ^{206}Pb to the closed-shell nucleus ^{208}Pb, may be inadequate. If for instance the Woods-Saxon well radius r_0 is increased by 1% in ^{206}Pb, then a reanalysis shows that the R_0 values deduced from both reactions become consistent

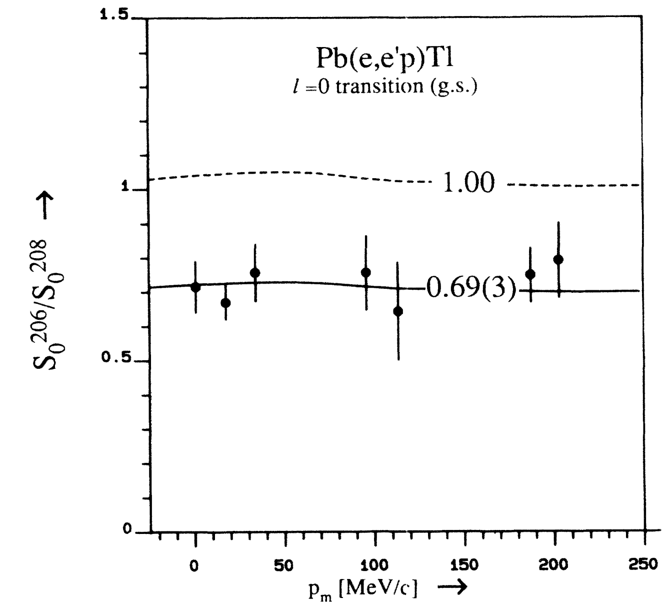

FIG. 3. Ratio of the $3s_{1/2}$ strengths for the transition to the ground states of ^{205}Tl and ^{207}Tl. The dashed DWIA curve corresponds to equal spectroscopic factors for the two isotopes. The solid curve is scaled to fit the experimental ratios.

TABLE I. $3s$ strength in ^{206}Pb relative to that of the ground state of ^{208}Pb; only statistical errors are given.

	$\int_{E_x} S_{206}/\int_0 S^{208}$				S_i^{206}/S_0^{208}	
	E_x interval (MeV)	($e,e'p$) l decomposition		E_x (MeV)	($e,e'p$) peak fitting	(d,^{3}He) Ref. 4
R_0	(−0.5, 0.5)	0.67(5)	R_0	0.0	0.69(3)	0.77(1)
R_{12}	(0.5, 1.5)	0.16(2)	R_1	1.22	0.13(5)	0.12(2)
			R_2	1.44	a	0.04(2)
$\sum R_i$	(−0.5, 1.5)	0.83(5)	$\sum R_i$			0.93(4)

[a]State not resolved in peak-fitting procedure.

[$(e,e'p)=0.67(3)$ and $(d,{}^3\mathrm{He})=0.67(1)$]. This observation, if substantiated by future measurements, makes the parallel investigation of the $(e,e'p)$ and $(d,{}^3\mathrm{He})$ reactions a method to determine the rms radii of individual shell-model orbits.

The observation that the ratio of the $3s$ ground-state wave functions is independent of r (for $r < 6.5$ fm) indicates that the bound-state wave functions are very similar for both isotopes. The present results are rather insensitive to the type of bound-state wave function. They remained stable ($< 3\%$) when either Hartree-Fock wave functions[13] or Woods-Saxon wave functions, used in the recent $(d,{}^3\mathrm{He})$ experiment,[4] were applied instead of the ones mentioned above. This demonstrates that the relative spectroscopic strength deduced in this $(e,e'p)$ experiment is insensitive to details of the bound-state wave functions.

When one combines the additional information from the absolute (e,e') charge-density-difference experiment[5] on the two isotones ^{205}Tl and ^{206}Pb with the present results, one may hope to approach the determination of absolute spectroscopic factors for the two isotopes. However, the question of whether the $3s$ proton contribution z to the charge-density distribution is an absolute number, that can be derived in a model-independent way from the (e,e') experiment, is still open. First, the influence of particle-hole admixtures on the charge-density difference is still insufficiently known. Second, the effect of a possible modification of the photon-proton coupling in the nuclear medium has been demonstrated recently,[14] which resulted in a better agreement with the shape and size of the experimental charge-density difference. The value of z near unity used in Ref. 14, however, contradicts the fragmentation of the $3s$ hole strength in ^{205}Tl observed in hadronic transfer reactions[3,4] and in the present $(e,e'p)$ experiment.

Further theoretical work is required to interpret the electron-scattering results in this mass region[1,5] as well as the fragmentation of the hole strength found in this experiment, before the question of fractional occupancy for the $3s$ proton orbit can be answered.

Finally, we plan to determine z directly from a relative $(e,e'p)$ measurement on ^{205}Tl and ^{206}Pb, since z is related via a sum rule with the difference between the total $3s_{1/2}$ strength in the two isotones.[4,15] A similar experiment will also be performed with the $(d,{}^3\mathrm{He})$ reaction.

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