Spectroscopic Strength of $1f_{7/2}$ Transitions Deduced from the Reaction $^{51}V(e,e'p)^{50}Ti$

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The spectral function for the knockout of $1f$ protons from ^{51}V leading to the $(1f_{7/2})^2$ quadruplet in ^{50}Ti has been measured in a high-resolution $(e,e'p)$ experiment. In the distorted-wave impulse approximation the $1f$ spectroscopic strength observed in the $0^+, 2^+, 4^+, 6^+$ quadruplet of ^{50}Ti is 1.11(14). This is considerably lower than predicted by shell-model calculations. In addition the rms radius of the $1f_{7/2}$ orbit, 4.20(14) fm, is deduced. With this rms radius, recent $^{51}V(d,^3He)^{50}Ti$ data yield a similar low value of the $1f$ strength.

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Absolute spectroscopic factors form an important test for nuclear-structure models that are based on the assumption of nucleons moving in a mean field. The $(e,e'p)$ knockout reaction provides a tool to obtain these quantities. In plane-wave impulse approximation the $(e,e'p)$ cross section is proportional to the square of the Fourier transform of the bound-state wave function. This implies that the strength (spectroscopic factor) and the radial scale (orbit radius) can be deduced. Thus, whereas the determination of spectroscopic factors in pickup reactions such as $(d,^3He)$ is mainly sensitive to the radius of the bound-state wave function, with the $(e,e'p)$ reaction the wave function can be sampled in a wide range of coordinate space by varying the missing-momentum variable of the reaction. Earlier $(e,e'p)$ experiments within this mass region have been carried out by Mougey *et al.*[1] with an energy resolution of about 1 MeV and by Nakamura *et al.*[2] with an energy resolution of about 7 MeV. In this Letter the transitions in the reaction $^{51}V(e,e'p)^{50}Ti$ leading to the 0^+, 2^+, 4^+, and 6^+ excited states in ^{50}Ti are discussed. In the extreme single-particle model these transitions result from knockout of a $1f_{7/2}$ proton from the $(1f_{7/2})^3_{J=7/2}$ valence proton configuration of ^{51}V. In order to obtain spectroscopic factors the data are analyzed in distorted-wave impulse approximation (DWIA), in which the final-state interaction (FSI) between the knocked-out proton and the residual nucleus is calculated with an optical model. The various error sources and model assumptions in the analysis are investigated in detail in order to assess the absolute calibration of the spectroscopic information. Finally, the impact of the radial information obtained in the $(e,e'p)$ reaction on the analysis of the reaction $^{51}V(d,^3He)^{50}Ti$ is discussed.

The experiment has been performed with the electron accelerator MEA and the dual-spectrometer setup at NIKHEF-K (see de Vries *et al.*[3]). The experimental conditions are similar to those of the proton knockout experiment on ^{90}Zr.[4] The $(e,e'p)$ cross sections were obtained in parallel kinematics in which the proton with momentum $\mathbf{p}'$ is detected in the direction of the

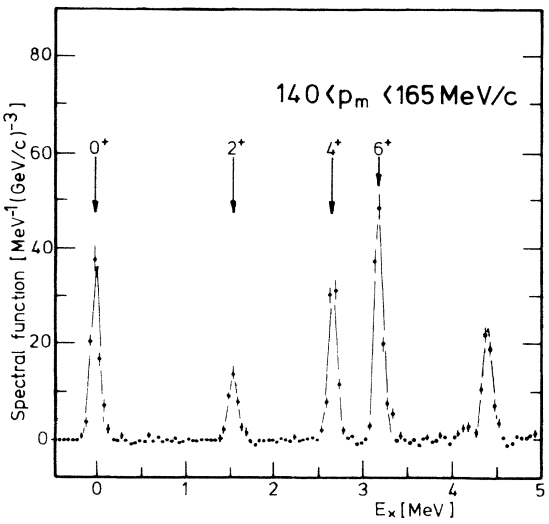

FIG. 1. Experimental spectral function for the reaction $^{51}V(e,e'p)^{50}Ti$ as a function of the excitation energy ($E_x = E_m - E_m^{g.s.}$). Various known states are indicated by their spin and parity.

momentum transfer $\mathbf{q}$ ($\mathbf{p}' \| \mathbf{q}$ and $|\mathbf{q}| < |\mathbf{p}'|$). Two proton kinetic energies ($T_p = 70,100$ MeV) were used in order to gauge the energy dependence of the FSI. For kinematical reasons the $T_p = 70$ MeV data are restricted to missing momenta $p_m < 170$ MeV/c, whereas the $T_p = 100$ MeV data cover the region $140 < p_m < 300$ MeV/c. The spectral function $S^{\text{expt}}(E_m, p_m)$ was obtained by dividing the coincidence cross section by a kinematical factor and the off-shell electron-proton cross section, for which the current-conserving prescription of de Forest was used.[5] A typical missing-energy spectrum is shown in Fig. 1. A careful estimate of all experimental uncertainties resulted in a total systematic error on S^{expt} of 5% and in a 1-MeV/c uncertainty in the determination of p_m. In Fig. 2 the measured momentum distributions, $\rho^{\text{exp}}(p_m) = \int_{\Delta E_m} S^{\text{expt}}(E_m, p_m)\, dE_m$, are shown for the transitions to the $0^+, 2^+, 4^+$, and 6^+ states at the two proton kinetic energies. The shapes of the measured distributions are quite similar, which indicates that the corresponding orbital radii are nearly equal. The relatively small difference between the measured momentum distributions for the two proton energies in the overlap region ($p_m \approx 150$ MeV/c) is due to the difference in distortions. The momentum distribution summed over the four transitions is in qualitative agreement with the results from an earlier $(e,e'p)$ experiment[2] carried out in Tokyo with a 7-MeV energy resolution.

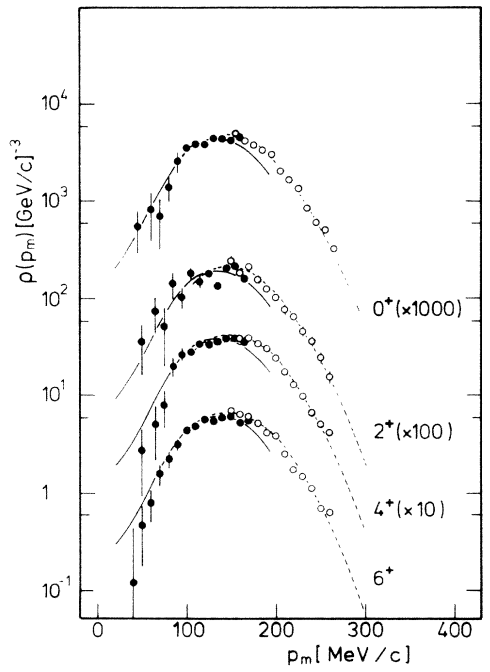

FIG. 2. Measured momentum distributions for the $1f$ transitions in the reaction ^{51}V$(e,e'p)^{50}$Ti. The curves represent DWIA calculations for ejected proton energies of 70 MeV (solid curve) and 100 MeV (dashed curve).

In order to extract absolute spectroscopic factors the distortions of the electron waves in the Coulomb field of the nucleus and of the knocked-out proton in the field of the residual nucleus should be calculated. In the DWIA the theoretical momentum distribution is given approximately by

$$\rho_\alpha^{\text{th}}(p_m, \mathbf{p}') = S_\alpha |\psi_\alpha^D(p_m, \mathbf{p}')|^2, \quad (1)$$

where S_α is the spectroscopic factor and $\psi_\alpha^D(p_m, \mathbf{p}')$ is the Fourier transform of the overlap between the distorted outgoing proton wave function and the bound-state wave function $\phi_\alpha(r)$. The distorted proton waves were calculated in a Woods-Saxon–type optical potential which represents the interaction with the residual nucleus. The factorized form of Eq. (1) is only correct if the spin-orbit term in the optical potential is neglected. In the actual calculations this is treated exactly in an unfactorized approach.[6] The potential parameters were taken from a systematic study of their energy dependence in the proton energy region between 80 and 180 MeV.[7] A correction for nonlocality has been applied.[8] The bound-state wave functions were calculated in a Woods-Saxon potential ($V \approx 56$ MeV, $a_0 = 0.65$ fm, $V_{\text{s.o.}} = 7$ MeV) that produces the correct binding energy. In addition, a nonlocality correction to the bound-state wave function was applied.[8] The Coulomb distortion of the electron wave functions is accounted for by employment of an effective momentum transfer in the "high-energy approximation" ($\Delta e = \Delta e' = 1.16 Z\alpha / R_c$).[9] The enhancement of the calculated momentum distributions ($\sim 10\%$) induced by this Coulomb distortion has been assumed to be p_m independent.

By comparison of the data with the calculated momentum distributions at both proton kinetic energies the spectroscopic factors and the orbit radii are obtained (see Table I). In order to obtain a proper fit the radial parameter (r_0) of the bound-state potential had to be increased from $r_0 = 1.286$ fm for the 0^+ transition to $r_0 = 1.331$ fm for the 6^+ transition. Since the increase in r_0 is accompanied by an increase in the

TABLE I. Spectroscopic factors and rms radii of the $1f_{7/2}$ wave functions obtained with the reaction ^{51}V$(e,e'p)^{50}$Ti. Only statistical errors are given.

Level (J^π)	E_x (MeV)	rms[a] (fm)	$S(1f_{7/2})$	$S(2p_{3/2})$
0^+	0.000	4.19(6)	0.30(2)	···
2^+	1.554	4.20(10)	0.13(1)	< 0.008
4^+	2.675	4.20(6)	0.27(2)	< 0.005
6^+	3.199	4.22(5)	0.41(3)	···

[a] $a[(A-1)/A]^{1/2}$ correction factor [A. E. L. Dieperink and T. de Forest, Jr., Phys. Rev. C **10**, 543 (1974)] has been applied to transform the obtained radii (relative to the $A-1$ system) to a system with a potential fixed in the laboratory.

binding energy the rms radii of the bound-state wave functions remain virtually constant. The fitted momentum distributions are also shown in Fig. 2. Before discussing the deduced $1f$ strength, the possible presence of $l=1$ knockout strength in the momentum distributions of the 2^+ and the 4^+ states should be considered. Such strength would reflect $2p$ admixtures in the wave functions of the ^{51}V ground state and of the 2^+ and 4^+ states in ^{50}Ti. The upper limit for such strength (see Table I) is very small, which is in agreement with the result of a shell-model calculation in which the configuration space is extended to include $(1f_{7/2}^{-1}\,2p_{3/2})$ and $(1f_{7/2}^{-1}\,1f_{5/2})$ admixtures.[10]

The total $1f$ strength for the quadruplet amounts of 1.11, which is only 37% of the sum-rule limit. In order to assign an error to this value various model assumptions should be considered. The calculated absorption factor η due to the FSI depends on the proton energy ($\eta_{70}=0.38$, $\eta_{100}=0.52$) and on the assumed parametrization of the optical potential. Comparison to a different parametrization at 65 MeV[11] and to a potential with a stronger absorptive part in the nuclear interior [still giving a good fit to the (p,p) data[12]] resulted in a model error of 6% due to the treatment of the FSI. The effect of a 1-MeV/c uncertainty in the experimental missing momentum translates into an error of 4% in the spectroscopic factor. Uncertainties in the treatment of the Coulomb distortion contribute 1% to the error. With addition of all model errors including the experimental systematic error of 5%, a total systematic error of 9% is assigned to the spectroscopic factors. Including the statistical error, we arrive at a total $1f$ strength for the four transitions of 1.11(14).

The orbit radii are within the errors the same for the four transitions (Table I). If we include the systematic error of 2.5% which is mainly due to uncertainties in the treatment of the FSI, the average radius of the $1f_{7/2}$ proton orbit is 4.20(14) fm. This is in agreement with the result of a mean-field calculation using the density-matrix expansion (4.24 fm,[13] of a density-dependent Hartree-Fock calculation, and of a density-dependent Hartree-Fock-Bogoliubov calculation[14] (4.14 and 4.11 fm, respectively). Accurate values for the $1f_{7/2}$ orbit radius have been reported on the basis of (e,e') experiments. From the inelastic $C6$ excitation of the 6^+ state[15] in ^{50}Ti a radius of 4.29(3) fm is deduced and from a magnetic elastic scattering experiment[13] 4.06(5) fm after correcting for meson exchange currents. On the basis of our result it is impossible to give a preference to either result.

Comparison of the $1f$ strength obtained in this experiment with published results of $(d,{}^3\mathrm{He})$ proton pickup reactions[16] shows a large discrepancy (Table II). This is due to several reasons. The hadronic results are obtained from a local, zero-range DWBA calculation with a fixed radial parameter for all transitions ($r_0=1.20$ fm). In a more recent $(d,{}^3\mathrm{He})$ experiment[17] finite-range and nonlocality corrections for the bound state and the optical potential are applied (yielding a reduction of about 10%). In addition a fixed orbit size of 4.06 fm is assumed for the bound-state wave functions. Taking our central value of 4.20 fm, causes a reduction of the reported $(d,{}^3\mathrm{He})$ spectroscopic factors to 0.30, 0.15, 0.26, and 0.39 for the 0^+, 2^+, 4^+, and 6^+ transitions, respectively, which shows the sensitivity of the hadronic result for the radial scale of the bound-state wave function clearly. In any case it is clear that both the $(d,{}^3\mathrm{He})$ and the $(e,e'p)$ reaction yield a significant reduction of the $1f$ strength of approximately the same magnitude. This is in contrast with a shell-model calculation[10] (Table II).

In summary, the DWIA analysis of the $(e,e'p)$ reaction yields a surprisingly small fraction of the total $1f$ strength for the four studied transitions. With the orbit size deduced from this $(e,e'p)$ experiment and applying finite range and nonlocality corrections to the $(d,{}^3\mathrm{He})$ reaction, the $1f$ strength observed in both reactions is in qualitative agreement. On basis of the present analysis it cannot be excluded that part of the missing strength will be observed at higher excitation energies. However, the extra strength observed in the $(d,{}^3\mathrm{He})$ experiments up to 7 MeV is about 10% of the

TABLE II. Spectroscopic factors for $1f$ transitions to the $(1f_{7/2})^2$ quadruplet in ^{50}Ti.

	$(e,e'p)$	$(d,{}^3\mathrm{He})$		Theory	
Level	$S(1f_{7/2})$ [a]	C^2S [b]	C^2S [c]	$S(1f_{7/2})$ [d]	$S(1f_{7/2})$ [e]
0^+	0.30(2)	0.73	0.41	0.75	0.75
2^+	0.13(1)	0.39	0.20	0.42	0.34
4^+	0.27(2)	0.64	0.35	0.75	0.72
6^+	0.41(3)	1.05	0.53	1.08	1.07
Sum	1.11(4)	2.81	1.49	3.00	2.87

[a]Present experiment.
[b]Reference 16.
[c]Reference 17.
[d]A pure $(1f_{7/2})^n$ configuration.
[e]Shell-model calculation (Ref. 10) including 1p1h admixtures in a $(1f_{7/2})(2p_{3/2})(1f_{5/2})$ configuration space.

combined strength of the four transitions. In addition the experimental Coulomb sum rule, obtained from inclusive quasielastic electron scattering in the same mass region and integrated over a large excitation energy domain,[18] yields about 50–60% of the expected strength. This indicates that it is unlikely that much strength is located at higher excitation energies. In order to understand the observed reductions it is important to study the different prescriptions for the coupling between the virtual photon and the nucleon. Using data from a recent experimental test of the impulse approximation,[19] a relative deviation of $\leq 10\%$ of the value for the electron-proton coupling is observed. However, as in this experiment only the ratio between the longitudinal and transverse component is tested, a more pronounced effect on the absolute value of the electron-proton coupling cannot be *a priori* excluded.

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