Renormalization Flow in Lattice QED

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An investigation of pure U(1) gauge theory is made based on block-spin transformations for configurations of lattice size 16^4 down to size 8^4 and 4^4. Even operators (five types in the first three representations) and ten renormalized couplings are determined on these smaller lattices. This allows the mapping of the renormalization flow. An interesting fixed-point structure and results on the leading critical exponent are found.

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An understanding of the critical structure of compact QED has remained one of the challenges of lattice gauge theory. Although the pure U(1) gauge theory may be implemented on a computer in a straightforward way, the results have been controversial in many aspects.

The one-parameter action of Wilson[1] with the single plaquette term leads to a two-phase structure with a phase transition (PT) near $\beta=1$, classified originally as second-order type,[2] which is expected from rigorous results.[3] Further Monte Carlo studies in a two-parameter space for the action

$$S=\beta\sum_P \mathrm{Tr}\, U_P+\gamma\sum_P \mathrm{Tr}\, U_P^2, \tag{1}$$

where U_P denotes the usual product of U(1) link fields around the plaquette, showed the existence of a complex critical structure with a line of PT's crossing the Wilson axis ($\gamma=0$).[4] At sufficiently large γ a metastability signal with a substantial action gap at the transition was observed[4] and a high-statistics study[5] revealed a two-state signal for $\gamma\geqslant 0$ and suggested that the type of the PT might change from first to second order below the Wilson axis near $\gamma=-0.11(5)$.[5]

Such a tricritical point (TCP) would be of utmost interest: It would indicate the existence of a second relevant parameter. This might open the way for a nontrivial continuum limit of scalar QED (cf. Hausenfratz[6]). Gupta *et al.*[7] applied Monte Carlo renormalization-group (MCRG) techniques[8,9] in order to determine the leading critical exponent of the PT. Along the Wilson axis they find a crossoverlike increase of ν when approaching the PT from the hot side; they do not divide into hot and cold histories, and obtain a value for ν of ≈ 0.42.

A serious defect in the usual MCRG approach is the uncertainty about the RG flow—whether one really moves towards the fixed point (FP) and where it is situated. The possibility of following the flow in coupling-constant space has been suggested,[10,11] and Swendsen's method has been successfully used in spin models[10] and in field theory.[12] The method may be readily generalized to gauge theories and recently has been applied to U(1) gauge theory by Burkitt.[13]

Here I present results obtianed in a high-statistics study of pure U(1) gauge theory for lattice size 16^4 at various values of γ and β close to the corresponding PT's (cf. Table I). I also study Villain's heat-kernel action[14,15] near the critical point.

On the 16^4 lattice a clear two-state signal at the PT was observed for all values of γ (cf. Table I), even for $\gamma=-0.2$ and for the Villain action (cf. Grösch *et al.*[16]). Monopoles play a crucial role in the U(1) transition.[7,15,17,18] Monopole loops closed as a result of the periodicity of the boundary conditions may be in part responsible for the metastability signal, at least for $\gamma\leqslant 0$. At $\beta=1.0105$, $\gamma=0$, I observed no tunneling for at least 70 000 MC iterations, neither from the hot nor from the cold branch; the tunneling frequency rises for negative γ by a factor of 5–10. I consider it important to perform a simulation in a definite phase, and discarded results obtained in runs where tunneling on 16^4 occurred.

A block-spin transformation (BST) with a scale factor of 2 has been introduced. The link on the smaller-size lattice is $V_{x,\mu}/|V_{x,\mu}|$, where $V_{x,\mu}$ is constructed from the sum over paths of lengths 2 and 4 connecting the corresponding sites of distance 2 on the larger lattice:

$$V_{x,\mu}=U_{x,\mu}U_{x+\mu,\mu}+\sum_{\substack{\pm\nu\\ \nu\perp\mu}} U_{x,\nu}U_{x+\nu,\mu}U_{x+\nu+\mu,\mu}U^*_{x+2\mu,\nu}. \tag{2}$$

This BST leaves the necessary properties of gauge invariance intact. BST's to size 8^4 and 4^4 have been performed and fifteen operators have been measured on these smaller-size lattices. They may be written as real parts of products of link fields along the boundary of geometric objects, i.e.,

S_1, single plaquette,

S_2, double plaquette,

S_3, bent double plaquette,

S_4, twisted bent double plaquette,

S_5, planar 2×2 loop,

TABLE I. Values of the couplings ($\beta=\beta_1$, $\gamma=\beta_6$) on 16^4 near or at the critical point, the average plaquette action $\langle S\rangle$, the type of RG flow observed, and the leading relevant and irrelevant eigenvalues of the linearized BST. Where the λ_{irr} come out complex I give the absolute size and indicate the entry by an asterisk. Entries 13–15 are results for the Villain action.

Entry	β	γ	$\langle S\rangle$	RG flow type	λ_{max}	λ_{irr}
1	0.912	0.15	0.6197(3)	a	8.11(5)	0.55(10)
2	0.912	0.15	0.6696(1)	c	2.92(4)	0.87(9)*
3	0.970	0	0.5367(1)	a	1.10(50)	0.35(12)
4	1.010	0	0.6206(7)	a	7.94(5)	0.61(10)*
5	1.0103	0	0.6212(4)	a	7.34(15)	0.88(20)
6	1.0105	0	0.6228(5)	a	6.97(11)	0.41(11)
7	1.0105	0	0.6534(4)	b	4.57(5)	0.57(5)
8	1.020	0	0.6673(4)	c	2.13(21)	0.80(6)
9	1.050	0	0.6910(1)	c	2.30(56)	0.99(5)*
10	1.1185	−0.15	0.6582(22)	a	5.27(3)	0.50(24)*
11	1.121	−0.15	0.6692(45)	a	6.75(5)	0.52(16)
12	1.158	−0.20	0.6771(21)	a	6.35(5)	0.54(9)
13	1.160	−0.20	0.6986(8)	b	3.72(5)	0.68(13)
14	$\beta_V=0.643$		−0.3122(10)	a	6.06(4)	0.54(4)
15	$\beta_V=0.644$		−0.3060(18)	a	6.49(2)	0.64(9)
16	$\beta_V=0.645$		−0.2916(12)	b	3.87(6)	0.55(17)*

for representation of charge one, and S_6–S_{10} and S_{11}–S_{15} correspondingly for charge two and three; the representation for charge n amounts to taking the real part of the nth power of the variables. These operators are possible contributions to the general form of the action

$$S=\sum_i \beta_i S_i,\quad S_i=\sum_x S_{i,x},\tag{3}$$

where $S_{i,x}$ are contributions to interaction type i from site x.

Furthermore I determine the values of couplings β_1–β_{10} (corresponding to S_1–S_{10}) on the 8^4 and 4^4 lattices according to a generalization of Swendsen's method.[10] As discussed elsewhere in more detail[12,13] this amounts to a comparison of the measured expectation values $\langle S_i\rangle$ with the conditional expectation values $\langle S_i'\rangle$. This procedure allows one to determine the coupling constants of the effective action on 8^4 and 4^4 lattices obtained by BST's from 16^4.

The resulting flow structure in the 10-parameter subspace of even couplings turns out to map the RG flow remarkably consistently. A first step in this direction has been done independently already in Ref. 13, albeit with consideration of only β (β_1) and γ (β_6), fewer start points on 16^4 (only $\gamma=0$), and less statistics.

As for the details of the MC simulation I refer to Lang.[19] Typically several thousand updates from cold or hot starts have been discarded before starting to perform BST's and subsequent measurements. Five iterations separated consecutive BST's; bunches of several hundred such measurements were used to obtain a new and improved estimate of β_i. Several such bunches finally led to convergent values of β_i. The total number of 16^4 iterations for each value (β,γ) varies between 20 000 and 90 000. The statistics gathered and the time spent at various values of the parameters (β,γ) on the 16^4 lattice varies. About half of the computer time has been spent on the Wilson line ($\gamma=0$). More runs at further (β,γ) values confirmed the general scenario, but will not be presented here because of comparatively smaller statistics.

Let me start the discussion of the BST results with an important, I think, observation. Figure 1(a) exhibits the expectation values of operators S_2 and S_1 as obtained after one or two BST's on 8^4 or 4^4 lattices, respectively. One observes a remarkable coincidence of the results that actually come, as discussed above, from various start values of the 16^4 couplings; the values on the 16^4 lattices are in most cases away from the curve in Fig. 1(a). All results are consistent with a common functional relation between S_2 and S_1. Similar figures may be drawn for all other operators measured. This indicates a fast approach to the renormalized trajectory. Note that this trajectory and the position of its intersection with the critical surface are dependent on the particular form of BST chosen.

Thus one finds three regions with three types of RG-flow structure:

(a) Hot phase. Below the PT's and precisely at the PT's (on the hot branch) the system flows towards the trivial FP at $\beta_i=0$.

(b) Critical points. If one starts on the 16^4 lattice at the PT on the cold branch (cf. Table I) one stays critical and quickly approaches a FP on the critical surface where one observes a stable set of values for all measured operators when comparing 8^4 and 4^4 results.

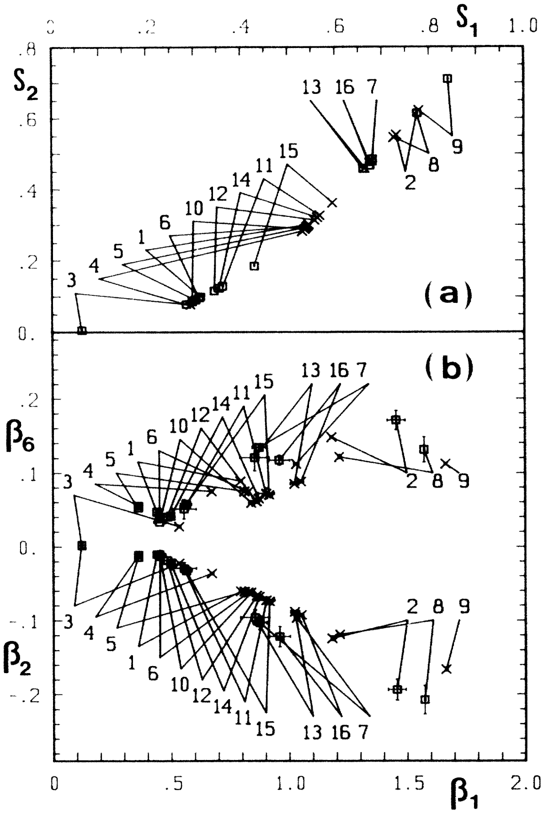

FIG. 1. (a) Expectation values of operator S_2 vs S_1 measured after one BST on 8^4 lattices (crosses) and after two BST's on 4^4 lattices (squares). The numbers denote the corresponding entries in Table I. Note the behavior at the fixed point $[S(8^4)=S(4^4)]$. (b) Like (a), but for the coupling constants β_2 and β_6 vs β_1.

(c) Cold phase. Starting on the cold side somewhat above the PT (e.g., at $\beta=1.02$, $\gamma=0$) one moves away from the critical surface towards larger couplings.

The behavior for the operators is confirmed by the results for the coupling constant determined for the 8^4 and 4^4 configurations. In Fig. 1(b) I exhibit the results for two, as it turned out, important couplings β_2 and $\beta_6 \equiv \gamma$ plotted versus $\beta_1 \equiv \beta$. As a result of the numerically much more delicate problem of their determination, the values have a larger numerical uncertainty. In the cold phase—where there are fewer results—the coupling constants are only marginally consistent with a common function form. A possible explanation may be the existence of redundant operators,[20] e.g., like those discussed for gauge theories.[21] Still the flow behavior and the results for the other couplings (not shown in the figure) follow the pattern of the operators. The actual numbers will be published elsewhere.[19]

Correlations of operators measured on configurations obtained from each other by BST's allow a determination of an estimate for the truncated, linearized BST called T.[8,9] At a FP of the BST the largest eigenvalues of T^* are related to the leading critical exponents through $\nu = \log s/\log\lambda$, where the scale factor of the BST is $s=2$ in this case. Only even operators have been considered (those contributing to the continuum action $F_{\mu\nu}^2$) and thus only the leading even eigenvalues may be determined. As was pointed out[22] there are various sources of systematical errors when solving the truncated equation for the truncated T; however, it turns out that the leading correction terms to the leading eigenvalue cancel.

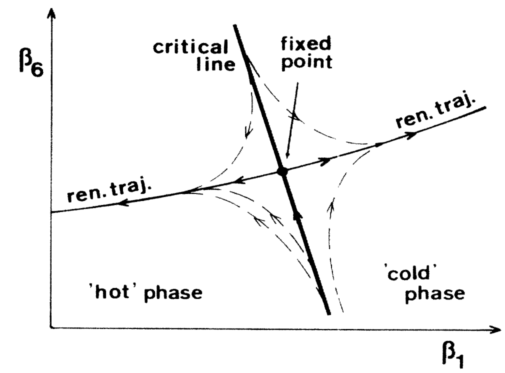

FIG. 2. Sketch of the RG-flow structure moved into the $(\beta=\beta_1, \gamma=\beta_6)$ plane for the sake of the discussion.

Table I gives the leading relevant and irrelevant eigenvalues. I consider only results for $T^{(2,1)}$, i.e., from correlations involving measurements after one (8^4) and two (4^4) BST's. I found that one has to include at least $S_{1\text{-}6}$ to achive stable eigenvalues which then change only slightly when including the other operators $S_{7\text{-}15}$. The errors give a measure of this dependence and are certainly underestimated. The average over the results for flow type (b) is $\lambda_{\max}=4.05 \pm 0.45$. At the FP I thus identify a leading critical exponent $\nu=0.50 \pm 0.04$, consistent with the value for a Gaussian theory. Away from the FP [cases (a) and (c)] the values of $1/\nu$ are proportional to the slope of the β function and may be used to estimate its shape.[19] A decrease of the leading relevant eigenvalue when approaching the PT from the hot side has also been observed in Ref. 7. A crossover behavior between the domain influenced by the tricritical exponents and that close to and at the critical point is typical for a nearby TCP.

I now summarize the results and conclude (cf. Fig. 2).

(1) Throughout the hot phase and even at the PT on the hot branch the RG flow drives the system towards the trivial FP. The correlation length is finite in this domain. The flow quickly approaches a unique trajectory which shows that there is just one relevant parameter. The two-state signal observed throughout may be a finite-size effect associated with periodically closed monopole loops.[7,16] The hot state at the PT

would then be an artifact of the MC simulation. As discussed, tunneling between the two states seldom occurred which made a distinction between hot and cold histories natural. Mixing of the results from the two states may lead to results interpolating between the values of ν obtained for flow types (a) and (b).

(2) Precisely at the PT (sensitive to the fourth digit of the coupling within the present statistics) on the cold branch of the two-state signal a FP is observed. The correlation length $\xi=1/a$ is infinite there; all dimensionless masses am_{phys} vanish. I have found this behavior for values $\gamma\leq 0$; from my results alone I cannot exclude that this behavior cannot be produced at $\gamma=0.15$ (the corerelation length may be large but finite there, indicating a true first-order transition).

(3) Already slightly above the critical point the RG flow leads away from the FP following again a unique trajectory: The points after two BST's are clearly at larger couplings than those after just one BST. If the RG flow really approaches a line of FP's (with a marginal operator), this line has to be outside the range of β values investigated and does not extend to the critical surface with its isolated FP (where no marginal eigenvalue was observed). We know that there is a massless state (photon) throughout the cold phase.[3,23] However, there may be further states with masses not equal to zero in the cold phase (i.e., $a\neq 0$). Such a feature is compatible with the present results. Investigations including further operators such as, e.g., monopoles[7,18] may improve our understanding in this respect.

(4) I find no signal of a TCP. Such a point lies on a tricritical submanifold in the multidimensional critical surface. One would expect that the RG flow on this surface behaves differently in its critical and in its tricritical part. In particular, there will be two FP's of the BST: one with one relevant parameter reached from the critical part of the singular line in the (β,γ) plane, and the other a tricritical FP with two relevant parameters. All observed RG flows approach a common trajectory and there is no indication of fundamentally different behaviors for the studied range of values. A possible TCP could be situated, however, at $\gamma>0$; as mentioned I did not observe flow type (b) for $\gamma=0.15$.

I consider the results in some respects quite surprising, but I think that they demonstrate the importance, feasibility, and usefulness of real-space RG studies for lattice gauge theories.

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