Dynamic Scaling of Eden-Cluster Surfaces

Kardar, Parisi, and Zhang[1] recently predicted that the dynamic critical exponent of the surface thickness in the Eden growth process[2] in $1+1$ dimensions is $z=\frac{3}{2}$, that three dimensions are the critical dimension, and that the surface remains smooth in four and more dimensions. We spent many hours on a Cray-2 computer to check these predictions on large systems.

Figure 1 shows our effective critical exponent $\beta=\chi/z=1/(2z)$ which describes how the thickness $W \propto t^{\beta}$ increases at intermediate times. Later β reaches its asymptotic limit $\beta=0$ corresponding to a time-independent equilibrium thickness for fixed L, where L is the spatial extent of our square lattice. We see a complicated behavior,[3] with the effective exponent for the longest times close to $\frac{1}{3}$. Thus it seems possible that the asymptotic exponent is $\beta=\frac{1}{3}$ corresponding to $z=\frac{3}{2}$. Some earlier simulations[1] were hampered by size effects or statistical errors to give reliable results; Ref. 3 noted that $\beta=0.3$ should be a lower bound only.

Also, we confirm for strips up to $L=8192$ that $\chi=\frac{1}{2}$, i.e., in equilibrium the thickness varies as $L^{\chi}=L^{1/2}$. Details and additional results will be given separately.[4]

The dynamic exponent β on the simple-cubic and the four-dimensional hypercubic lattice behaves similarly for short and intermediate times up to $t \sim 2^9$ where β is rising again; limitations in computer time and memory even on the Cray-2 prevented a detailed study of this increase of β for longer times. For the equilibrium width (exponent χ) we observe that the trend found in previous studies (logarithmic in three and constant in four dimensions) no longer continues for larger systems (up to linear spatial extent $L=1024$ and $L=128$, respectively), and that the width increases faster for large L, Fig. 2. Thus our jump of more than two orders of magnitude in computational power leads to the suggestion that perhaps some of the surface problems in two dimensions but not necessarily those in the three- and four-dimensional Eden models are understood theoretically.

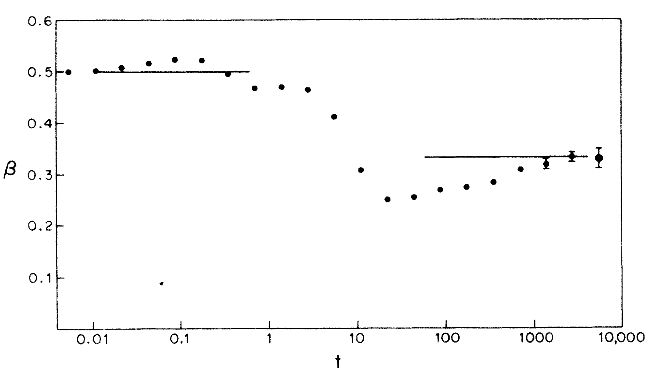

FIG. 1. Variation of effective dynamic exponent $\beta=d(\log W)/d(\log t)$ with time t in $1+1$ dimensions, for lengths $L \geqslant 2^{20}$ on the square lattice.

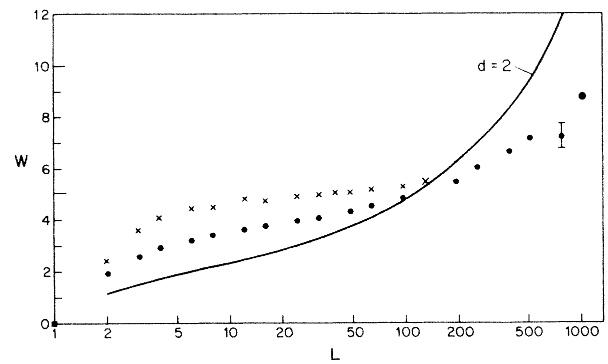

FIG. 2. Equilibrium thickness W for flat surfaces of linear dimension L in $2+1$ (dots) and $3+1$ (crosses) dimensions, $10^4 < t_{\max} < 10^6$.

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