Observation of a Fréedericksz Transition in Superfluid ^{3}He-A

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We have observed an instability of the uniform orbital texture of a slab of ^{3}He-A in a perpendicular magnetic field (Fréedericksz transition) in two different experiments with slab thicknesses of 105 μm and 2 mm. The theoretically predicted threshold fields for the two experiments are slightly larger than the observed values but have the same temperature dependence. Behavior characteristic of the uniform texture was obtained only when the A phase was entered by cooling from the normal phase. Very different behavior was observed when the A phase was entered by warming from the B phase.

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The Fréedericksz transition[1] in a nematic liquid crystal occurs when a thin slab of liquid of spatially uniform texture is subjected to a magnetic field in a direction such that the alignment favored by the field is perpendicular to that preferred by the boundaries. The analogy between superfluid ^{3}He-A and a nematic liquid crystal suggests that a similar transition should occur in the orbital texture of dipole-locked superfluid ^{3}He-A contained between two parallel plane surfaces when a magnetic field is applied perpendicular to the surfaces.

We have observed this transition in two different experiments cooled by nuclear demagnetization at a pressure of 29.3 bars. In the first, a disk of ^{3}He with nominal thickness 100 μm and diameter 8.38 mm was contained within a torsional oscillator; the changes in texture associated with the transition affected the resonant frequency and damping of the oscillator via the anisotropy of the normal-fluid density and viscosity. In the second experiment, the ^{3}He disk had a thickness of 2 mm and a diameter of 10.7 mm; textural changes were detected via their effect on the attenuation and velocity of pulses of 15.15-MHz zero sound. The experimental cells were normally cooled through T_c with a magnetic field in the plane of the disk, to encourage formation of the uniform texture. Lanthanum-cerium magnesium nitrate thermometry was used in both experiments, with the superfluid transition temperatures of Mitchell *et al.*[2] and the reduced temperature of the B-A transition of Greywall[3] used to calibrate the thermometers.

The free-energy density for dipole-locked ^{3}He-A in the presence of a magnetic field can be written as[4]

$$f=\tfrac{1}{2}K_S(\operatorname{div}\mathbf{l})^2+\tfrac{1}{2}K_B(\mathbf{l}\times\operatorname{curl}\mathbf{l})^2+\tfrac{1}{2}K_T(\mathbf{l}\cdot\operatorname{curl}\mathbf{l})^2+\tfrac{1}{2}\Delta\chi(\mathbf{l}\cdot\mathbf{H})^2,$$

where $\Delta\chi$ is the susceptibility anisotropy and K_S, K_B, and K_T are the dipole-locked splay, bend, and twist bending energy coefficients. The parallel planes bounding the fluid are taken to be at $z=0$ and $z=d$, and the field is parallel to $\hat{\mathbf{z}}$. We characterize the texture by the angle $\theta(z)$ between $\mathbf{l}$ and the z axis. θ satisfies the Euler-Lagrange equation

$$2(K_s\sin^2\theta+K_B\cos^2\theta)\theta''=-\sin(2\theta)[(K_S-K_B)\theta'^2+\Delta\chi H^2],$$

with the boundary conditions $\theta=0$ at $z=0$ and $z=d$.

For $H<H_\mathrm{F}$, where

$$H_\mathrm{F}=(\pi/d)(K_B/\Delta\chi)^{1/2}, \tag{1}$$

the only solution is the spatially uniform texture $\theta=0$. For $H>H_\mathrm{F}$, the texture is distorted, and for Ginzburg-Landau values ($K_S=K_B$) of the bending energy coefficients the distorted solution is symmetrical about $z=d/2$ and for $z<d/2$ is given by

$$\frac{\pi H}{H_\mathrm{F}}\frac{z}{d}=\int_0^{\theta(z)}\frac{d\theta'}{(\sin^2\theta_0-\sin^2\theta')^{1/2}}=F(\sin^{-1}[\sin\theta(z)/\sin\theta_0]\backslash\theta_0), \tag{2}$$

where θ_0, the value of θ at $z=d/2$, is evaluated by putting $z=d/2$ and $\theta(z)=\theta_0$ in Eq. (2). The notation for the elliptic integral $F(\Phi\backslash\alpha)$ is that of Abramowitz and Stegun.[5] As H increases through H_F, θ_0 increases initially with infinite slope, $\theta_0=2[(H/H_\mathrm{F})-1]^{1/2}$. For $H\gg H_\mathrm{F}$, $\cos\theta_0=4\exp(-\pi H/2H_\mathrm{F})$, and θ_0 tends asymptotically to $\pi/2$. The threshold field for the transition has previously been calculated by Fetter.[6]

The motion of the torsional oscillator was driven and detected electrostatically at a frequency close to the resonant frequency (~ 1333 Hz). The resulting charge variation was amplified by use of a current preamplifier and then phase-sensitively detected, and the bandwidth and resonant frequency were deduced. In the normal state, the bandwidth versus resonant-frequency plot for the torsional oscillator was in excellent agreement with the theoretical prediction; the slip length calculated by Einzel *et al.*[7] was used to determine the small finite-mean-free-path correction. Values of ηT^2 obtained from the normal-state measurements agreed with those of Carless, Hall, and Hook[8] at 29.3 bars if a plane spacing of 105 μm was assumed; this value was used for subsequent analysis.

Figure 1 shows the resonant frequency and bandwidth of the oscillator as functions of magnetic field for the A phase at $T/T_c = 0.832$. The sharp change in resonant frequency associated with the Fréedericksz transition at $H = H_{\rm F}$ is clearly visible. The bandwidth decreases slightly at the transition, but increases in larger fields. Analysis of the results for the uniform texture gives $\rho_{n\perp}/\rho = 0.876$ and $\eta_{44}/\eta_c = 0.225$, where $\rho_{n\perp}$ and η_{44} are the appropriate normal-fluid density and viscosity for this texture (we use standard crystallographic notation for the viscosity

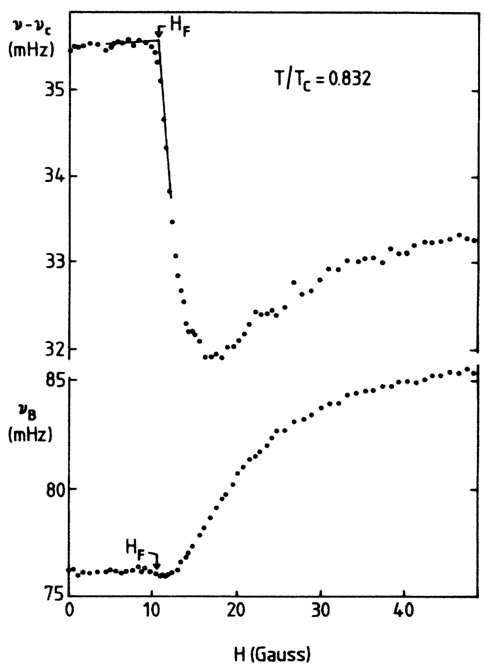

FIG. 1. Resonance-frequency shift ($\nu_R - \nu_c$) from the frequency ν_c at the superfluid transition, and bandwidth ν_B for the torsional oscillator, as functions of applied magnetic field. At the Fréedericksz transition, ν_B first decreases slightly, a feature which is more marked close to T_c.

coefficients). This value of $\rho_{n\perp}/\rho$ agrees well with that obtained by Berthold *et al.*[9] For the limit of large field, we tentatively assume that the texture can be approximated as l uniform along a particular direction in the plane of the disk; the alignment of l by flow should be slight, since at our velocity amplitude of about 30 μm/s the flow-alignment energy is only about 1% of the field energy at $H_{\rm F}$. In this case, the anisotropic hydrodynamics has an analytic solution, and fitting this to the high-field limit in Fig. 1 gives $\rho_{n\parallel}/\rho = 0.925$ and $\eta_{66}/\eta_c = 0.198$. This gives rather less than the factor-of-2 anisotropy in ρ_s expected near T_c, but the degree of agreement can be considered encouraging because (i) we have ignored the thin boundary layer in which l must turn normal to the surface; and (ii) with a purely vertical field, there are no strong forces to make the direction of l uniform in the plane of the disk.

The plane of textural distortion can be made definite by application of a magnetic field with a component parallel to the slab boundaries. The well-defined plane is that containing the horizontal field component and the normal to the boundaries. The response of the oscillator to a varying magnetic field has been measured for a fixed parallel component and also for fields at various constant angles to the slab normal. A full numerical solution of the anisotropic hydrodynamics, currently in progress, should enable us to obtain complete information on the anisotropy of the normal-fluid density and viscosity from these measurements. A feature of our results is that in a field at 45° to the slab normal, the increase in damping and associated reversal of the frequency shift observed in large perpendicular fields are absent, confirming that these effects are associated with l at a small angle to the plane of the disk.

The texture obtained on warming from the B phase responded in a very different way to an applied perpendicular field, whether or not a field was applied in the plane of the disk at the time of the transition. The resonant frequency started to decrease at zero field and, although there was a knee in the resonant-frequency versus field curve at about the Fréedericksz transition field, the decrease in resonant frequency associated with it was small and the subsequent increase in larger fields was often sufficient to give a resonant frequency higher than in zero field. These observations suggest that the low-field texture obtained after warming from the B phase to the A phase is anomalous: either a disordered texture or a different ordered texture. The response of the anomalous texture to a magnetic field in the plane of the slab was comparable to that of the uniform texture, namely, no observable change in damping and a very small frequency shift. We have no certain explanation for this surprising result, but one possibility is that we have created a uniform **d** texture with many small regions of **l** parallel and antiparallel to **d**.

In the ultrasonic experiment, the plane surfaces of the slab were the surfaces of the quartz crystals used for generation and detection of the ultrasonic pulses. Typically, pulses of duration 2.8 μs, spacing 5 s, and amplitude 1.25 V were applied to the transmitter crystal, and the amplified signal from the receiver crystal was detected by integration of the output of a double-balanced mixer used as a phase-sensitive detector. The quadrature response was also measured by insertion of a quarter wavelength of cable in the reference channel.

The ultrasonic experiment was constructed in order to detect and measure the inertia associated with rotation of **l** by the method suggested by Eastop, Hall, and Hook.[10] We have not yet succeeded in creating a texture that precesses with a precessing magnetic field, as required for that experiment, and we suspect that textural motion is inhibited by the presence of the side walls of the slab; the aspect ratio of the ^{3}He cavity is necessarily rather small.

The response of the detected sound signal to a field applied perpendicular to the slab (i.e., vertically) is shown in Fig. 2. The symmetry of these results (and the lack of response to a reversed field) suggests that there is a trapped vertical field of about 1.1 G. Asymmetry in the response to a horizontal field suggests that this trapped field has a horizontal component of 0.11 G. The full curves in Fig. 2 have been fitted to the data by taking the Fréedericksz field, the trapped vertical field, and the sound anisotropy coefficients as fitting parameters. This fit gives a trapped field of 1.116 G for the data shown; the distribution of values for all data gives 1.04 ±0.04 G. Our interpretation of this parameter as a trapped field is strengthened by the fact that it shows no dependence on either temperature or the field in the demagnetization solenoid, and by the observation that when the vertical component is compensated by an applied vertical field the resulting texture is independent of an applied horizontal field, as expected for the uniform texture. As in the torsional oscillator experiment, very different results were obtained when the A phase was entered by warming from the B phase.

Rounding of the theoretically sharp Fréedericksz transition in the ultrasonic experiment is largely accounted for by the trapped horizontal field of 0.11 G, a factor which was allowed for in calculation of the theoretical curves of Fig. 2. Response times, instrumental or textural, are not likely to have produced rounding in either experiment, since we observed no significant difference between upward and downward field sweeps. Other possible contributions to rounding are nonuniformity of slab thickness, magnetic field, or texture, and superflow in the cell, particularly that resulting from heating due to the sound pulses in the ultrasonic experiment.

Since $H_F \propto 1/d$, we plot in Fig. 3 values of $H_F d$ for the two experiments. The theoretical curves were calculated by use of Eq. (1). We use expressions for $\Delta \chi$ and K_B given by Leggett[11] and Cross.[12] Near T_c the

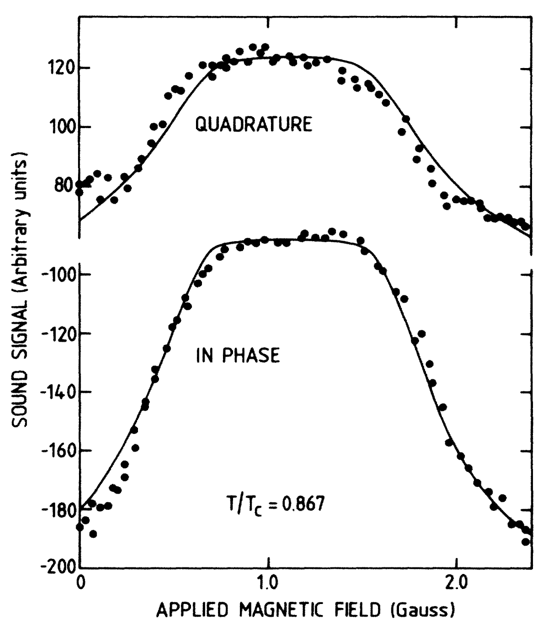

FIG. 2. In-phase and quadrature sound signals as functions of applied vertical magnetic field in the ultrasonic experiment. The curves are theoretical, fitted for a Fréedericksz field of 0.510 G, and a trapped field with vertical component 1.116 G and horizontal component 0.11 G, as described in the text.

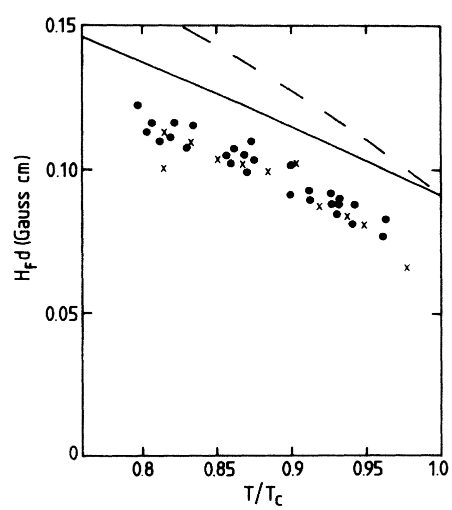

FIG. 3. Measured values of $H_F d$ as a function of reduced temperature for torsional oscillator experiment (circles) and ultrasonic experiment (crosses). The theoretical curves are explained in the text.

threshold field may be written as

$$H_F = (3\pi/4\sqrt{2}d)\rho\hbar^2\gamma/mp_F\chi_N,$$

where γ is the gyromagnetic ratio of ^{3}He and χ_N the normal-state susceptibility. Away from T_c the threshold field depends on the Fermi-liquid parameters F_1^s, F_0^a, and F_1^a, and is also sensitive to strong-coupling corrections to the energy gap. In calculation of the theoretical curves shown in Fig. 3, we have used the following data: $F_1^s = 13.08$,[3] $F_0^a = -0.758$,[3,13] $F_1^a = -0.8$,[14] $\chi_N = 10.3\times10^{-8}$.[13,15] The continuous theoretical curves were calculated for the energy gap predicted by the BCS theory, and the dashed curve was obtained by application of an enhancement factor of 1.3 to the BCS gap, as suggested by the specific-heat discontinuity measured by Greywell.[3]

The experimental values of $H_F d$ for both experiments lie below the theoretical curves. There is an uncertainty of around 5% in both experiments in the magnetic field measurement and, in addition, a possible error of around 3% in the estimate of d for the torsional oscillator experiment from detailed analysis of the normal-state data. The small aspect ratio of the ultrasonic cell and heat currents in this cell are likely to lead to a reduction in the transition field. At T_c the main uncertainty in the theoretical value of H_F is the estimated 4% accuracy of the χ_N measurement. It would be premature, therefore, to conclude that the discrepancy between theory and experiment in Fig. 3 is significant. We note that the temperature dependence predicted by the theory does seem to be reflected by the experimental results. It is worth noting that the agreement with theory for our magnetic-field–induced transition in a slab geometry is much closer than for the superflow-induced transition in bulk liquid studied by Bates *et al.*[16]

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