Skyrmions with ρ and ω Mesons as Dynamical Gauge Bosons

Ulf-G. Meissner,[a] Norbert Kaiser, Andreas Wirzba,[b] and Wolfram Weise
Institute of Theoretical Physics, University of Regensburg, D-8400 Regensburg, West Germany
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We extend the Skyrme model to incorporate the vector mesons ρ and ω in a way consistent with the non-Abelian anomalies of QCD and vector-meson dominance. The vector mesons are treated as composite gauge bosons of a hidden $SU(2)_V \otimes U(1)$ symmetry. We investigate the bulk properties of hedgehog-type Skyrme solitons in the baryon number $B=1$ sector of this model.

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In the limit of a large number of colors N_c QCD effectively reduces to a nonlinear meson theory.[1,2] For two massless flavors with underlying $SU(2)_L \otimes SU(2)_R$ chiral symmetry, and in the extreme low-energy, long-wavelength limit, the theory is expressed in terms of the Goldstone pion fields alone. A minimal effective Lagrangean which synthesizes these features and guarantees the existence of stable soliton solutions is the Skyrme model.[3,4] It involves the 2×2 unitary field $U(x)=\exp[i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)/f_\pi]$ and combines the nonlinear σ model given by the Lagrangean $\mathcal{L}_0=(f_\pi^2/4)\mathrm{Tr}\{\partial^\mu U\,\partial_\mu U^\dagger\}$ with a fourth-order stabilizing term. Baryons arise as topological solitons with the hedgehog *Ansatz* $\boldsymbol{\pi}=\hat{\mathbf{r}}F(r)$ and the boundary values $F(0)=\pi$ and $F(\infty)=0$.

Recent developments[5,6] have pointed to the importance of vector mesons in a more complete meson-based description of hadron physics down to length scales of about 0.5 fm. Such an extension is suggested by the phenomenological success of the vector-meson-dominance model in describing electromagnetic interactions of hadrons. It is also motivated by the role of vector mesons in boson-exchange models of the nucleon-nucleon force.

We follow here Bando *et al.*,[7] Igarashi *et al.*,[8] and Fujiwara *et al.*,[9] who started from the observation that the nonlinear σ model has a hidden $[SU(2)_V]_{\mathrm{local}}$ gauge symmetry. The corresponding gauge boson is identified with the ρ meson. This scheme has a free parameter which is fixed such that the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation $m_\rho^2=2g^2f_\pi^2$ holds, where g is the $\rho\pi\pi$ coupling constant determined by $\rho\to\pi\pi$ decay. It is then shown[7-9] that the vector-meson dominance of photon couplings follows naturally with no additional assumptions.

The ω meson can be generated by extending the gauge group to $SU(2)_V\otimes U(1)$. Here the anomalous part of the effective Lagrangean, i.e., the Wess-Zumino term, plays the essential role in providing the ω couplings. We work in a minimal scheme which has neither a direct $\rho\omega$ coupling nor an A_1 field.

The aim of the present Letter is to show that such a scheme, with parameters completely determined in the meson sector, leads to satisfactory results also in the nontrivial (soliton) sector with baryon number $B=1$.

Let us consider the following Lagrangean in terms of the matrix-valued variables $\xi_L(x)$ and $\xi_R(x)$ connected to the hidden left-right symmetry of the nonlinear σ model. In the unitary gauge $U=\xi\xi$ (i.e., $\xi_L^\dagger=\xi_R=\xi$)

$$\mathcal{L}=\frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U\,\partial^\mu U^\dagger]-\frac{f_\pi^2}{2}\mathrm{Tr}[(\mathcal{D}_\mu\xi)\xi^\dagger+(\mathcal{D}_\mu\xi^\dagger)\xi]^2-\frac{1}{2g^2}\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}]+\tilde{\mathcal{L}}_{\mathrm{WZ}},\tag{1}$$

where $f_\pi=93$ MeV is the weak pion decay constant. Furthermore, $\mathcal{D}_\mu=\partial_\mu-iV_\mu$ is the pertinent covariant derivative induced by the $SU(2)\otimes U(1)$ gauge group with $V_\mu=(g/2)\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu+(g/2)\omega_\mu$ as the associated gauge fields (ρ and ω mesons), and $F_{\mu\nu}$ is the corresponding non-Abelian field tensor $F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu-i[V_\mu,V_\nu]$. Thus, we have enlarged the hidden symmetry of Ref. 7 to $SU(2)\otimes U(1)$ with equal gauge couplings for $SU(2)$ and $U(1)$. This leads to the relation $m_\rho^2=m_\omega^2=2g^2f_\pi^2$ (KSRF), generated in the second term of Eq. (1). The term $\tilde{\mathcal{L}}_{\mathrm{WZ}}$ stems from the action

$$\tilde{\Gamma}_{\mathrm{WZ}}=10C\int_{M_4}\mathrm{Tr}\{(\hat{a}_L^3\hat{a}_R-\hat{a}_R^3\hat{a}_L)-(\hat{a}_L\hat{a}_R\hat{a}_L\hat{a}_R)\},\tag{2}$$

where $\hat{a}_{L(R)}=D\xi_{L(R)}\xi_{L(R)}^\dagger=\mathcal{D}^\mu\xi_{L(R)}\xi_{L(R)}^\dagger\,dx^\mu$ are gauge-covariant differential one-forms defined in terms of the variables $\xi_{L(R)}$ as introduced above, and their covariant derivatives $\mathcal{D}^\mu$ as defined above (for details see Ref. 9). The constant C is given by $-iN_c/240\pi^2$ with N_c the number of colors ($N_c=3$). We have restricted ourselves to the first two terms of Eq. (4.5) of Ref. 9 with a special choice of their coefficients, $C_1=-C_2=10C$ (the solution to the homogeneous Wess-Zumino anomaly equation) for the following reasons: First, if one electromagnetically gauges the Wess-Zumino term with U embedded in $SU(3)$ one finds one-photon couplings of the form

$(e/2)A_\mu B^\mu$. To ensure vector-meson dominance on the one-photon level in the isoscalar channel, one has to cancel this term. The action given in (2) has this feature; this can easily be checked from Eq. (6.2) of Ref. 9. Second, the action (2) incorporates isoscalar vector-meson couplings to the topologically conserved baryon current

$$B^\mu = (1/24\pi^2)\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\{U^\dagger \partial_\nu U\, U^\dagger \partial_\alpha U\, U^\dagger \partial_\beta U\}.$$

Third, in the limit that the ρ field is expressed in terms of the ξ's [cf. Eq. (8) of Ref. 7], it exactly reduces to a Lagrangean of the form[10]

$$\tilde{\mathcal{L}}_{\mathrm{WZ}} = g_\omega \omega_\mu B^\mu, \tag{3}$$

with the coupling constant g_ω related to the gauge coupling g by

$$g_\omega = (N_c/2)g, \tag{4}$$

in agreement with the results of Kaymakcalan, Rajeev, and Schechter[5] and of Meissner and Zahed[6] (for $N_c = 3$). The restriction to these terms is vital if one wants to discuss vector-meson dominance based on mesons coupled to conserved currents. In other words, this is the simplest way of coupling the ω meson to the skyrmion stabilized by ρ mesons, consistent with the Wess-Zumino conditions. For $g = 5.85$ one obtains the physical ρ- and ω-meson masses $m_\rho = 770$ MeV $\simeq m_\omega$. It is evident that the Lagrangean (1) should give rise to stable soliton solutions even without the *ad hoc* Skyrme fourth-order term.

To investigate the nontrivial sector of (1), we will specialize to hedgehog skyrmions, with $U(r) = \exp\{i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)\}$. As a consequence of the hedgehog symmetry and the intrinsic parity of the vector mesons, the most general forms for the ρ and ω fields are

$$g\rho^{\mu,a}(\mathbf{r}) = -\epsilon^{\mu ja}\hat{r}^j G(r)/r\delta^{\mu i}, \tag{5}$$

$$\omega^\mu(\mathbf{r}) = \omega(r)\delta^{\mu 0}. \tag{6}$$

Here $G(r)$ and $\omega(r)$ are continuous functions of r. In terms of the functions $F(r)$, $G(r)$, and $\omega(r)$, the energy functional is obtained as

$$E = 4\pi\int_0^\infty dr\, r^2\left\{\frac{f_\pi^2}{2}\left[F'^2 + \frac{2\sin^2 F}{r^2}\right] + \frac{2f_\pi^2}{r^2}[G-(1-\cos F)]^2 + m_\pi^2 f_\pi^2(1-\cos F) + \frac{1}{2g^2r^4}[2r^2G'^2 + G^2(G-2)^2] - \frac{1}{2}\omega'^2 - \frac{1}{2}m_\omega^2\omega^2 + \frac{g_\omega}{2\pi^2}\omega\frac{\sin^2 F}{r^2}F'\right\}$$

$$\equiv E_\pi + E_{\pi\rho} + E_\pi^m + E_\rho + E_\omega + E_{\pi\omega}. \tag{7}$$

Finiteness of the energy and baryon number $B = 1$ impose the following boundary conditions

$$F(0) = \pi, \quad F(\infty) = 0,$$
$$G(0) = 2, \quad G(\infty) = 0, \tag{8}$$
$$\omega'(0) = 0, \quad \omega(\infty) = 0.$$

The functions F, G, and ω can now easily be obtained by functional minimization. For $f_\pi = 93$ MeV, $g = 5.85$, and $m_\pi = 0$, they are shown in Fig. 1. In Fig. 2, $F(r)$ and $G(r)$ are plotted in comparison with the results of Igarashi *et al.*[8] Since part of the repulsion in our model is due to the ω, $F(r)$ and $G(r)$ extend further out in space. This means that the soliton will have a larger (isoscalar) rms radius as compared to the one found in Ref. 8. In Table I we give the skyrmion mass m_{sk} derived from Eq. (7), together with the rms radius. Comparing with the results of Ref. 8, we see that the mass has increased by $\sim 40\%$ due to the additional ω meson-induced repulsion.

The hedgehog root-mean-square radius r_H (rms radius), i.e., the baryonic charge radius $r_H^2 = \int r^2 \times B^0(\mathbf{x})\, d^3x$, $r_H \sim 0.50$ fm, appears to be small if compared with the nucleon charge radius. However, one should note that our model incorporates vector-meson dominance if coupled to the photon field, so that part of the (isoscalar) charge radius is given by the propagating ω meson. One finds[11] that the isoscalar form factor is

$$G_{I=0}(\mathbf{q}) = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\int d^3r'\,\frac{\exp(-m_\omega|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|}B_0(\mathbf{r}') = \frac{m_\omega^2}{m_\omega^2+\mathbf{q}^2}\tilde{B}_0(\mathbf{q}), \tag{9}$$

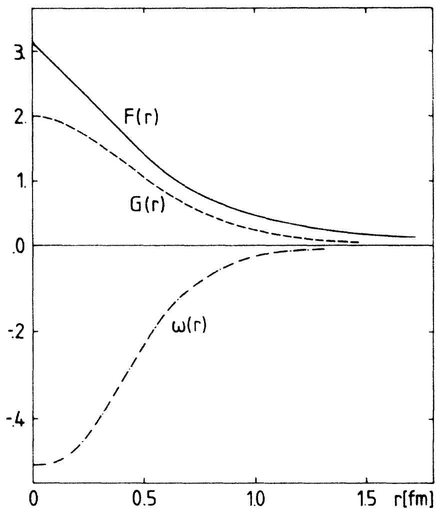

FIG. 1. The chiral angle $F(r)$, and the vector-meson profiles $G(r)$ and $\omega(r)$ for $g=5.85$, $f_\pi=93$ MeV, and $m_\pi=0$. Notice the different scale for the ω meson.

where $B_0(\mathbf{r})$ is the baryon-number density and $\tilde{B}_0(\mathbf{q})$ its Fourier transform. The corresponding rms radius is

$$r_{I=0}^2 = -6\frac{dG_{I=0}}{d\mathbf{q}^2}\bigg|_{\mathbf{q}=0}$$

$$= \frac{6}{m_\omega^2} + r_H^2 = (0.79 \text{ fm})^2, \qquad (10)$$

in good agreement with the experimental value 0.78 fm. A similar relation will also hold for the isovector charge radius which involves the ρ meson, but here one needs to project onto states of good spin and isospin.[11] In the model without the ω (Ref. 8) the (hedgehog) axial-vector coupling constant g_A as extracted from the tail of the pion field $F(r)$ is much too small ($g_A=0.32$ for $g=5.85$ and $m_\pi=0$). We find the more reasonable value $g_A=0.81$. This result is not surprising since the strength of the pion field depends on the spatial extension of the source to which it couples.

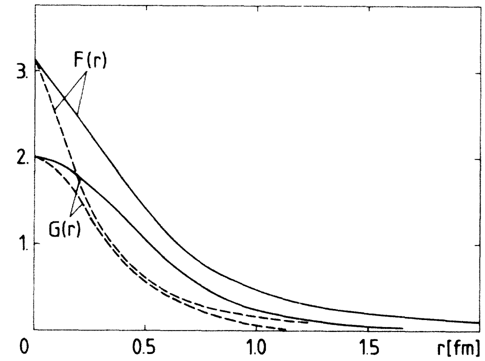

FIG. 2. The pion and ρ-meson profiles $F(r)$ and $G(r)$, respectively, compared with the results of Ref. 8 (dashed lines) for $m_\pi=0$, $g=5.85$.

We conclude that the nonlinear σ model, unified with ρ and ω mesons as gauge bosons of the hidden $SU(2)_V \otimes U(1)$ symmetry, has stable $B=1$ soliton solutions with satisfactory properties and size. Apart from the pion mass, the model has only two parameters: the pion decay constant f_π and the universal ρ-meson coupling constant g, both of which are uniquely fixed in the meson sector. The ω-meson coupling strength is determined by the Wess-Zumino term together with universality, with no additional parameter.

The repulsion induced by the ω meson stabilizes the soliton at an rms radius r_H of about 0.5 fm. This should be compared with the much smaller rms radii $r_H \lesssim 0.3$ fm obtained according to Ref. 8 in a model without an ω meson, in which the stabilizing mechanism is provided by the ρ meson alone.

We have pointed out that a soliton size of half a Fermi is quite reasonable: If the system is probed by a photon, the inherent vector-meson dominance implies that the apparent mean-square radius seen by the photon is not r_H^2, but $r_H^2 + r_V^2$ with $r_V^2 = 6/m_V^2 \simeq (0.62 \text{ fm})^2$

TABLE I. Various contributions to the skyrmion mass (M_{sk}) and size (r_H), as defined in (7), for $m_\pi=0$ (chiral limit) and $m_\pi=138$ MeV. For comparison, the results of Ref. 8 are also given.

	This work		Following Ref. 8	
	$m_\pi=0$	$m_\pi=138$ MeV	$m_\pi=0$	$m_\pi=138$ MeV
E_π (MeV)	819	773	418	400
E_π^m (MeV)	...	40	...	8
E_ρ (MeV)	356	372	525	537
$E_{\pi\rho}$ (MeV)	29	33	104	112
E_ω (MeV)	−228	−255	...	...
$E_{\pi\omega}$ (MeV)	455	509	...	...
M_{sk} (MeV)	1431	1473	1048	1057
r_H (fm)	0.52	0.49	0.28	0.27

because of the intermediate vector meson of mass m_V connecting the photon with the soliton. The combined effect brings one close to the observed nucleon electromagnetic radii.

In the limit of infinite vector-meson masses m_V, but keeping g/m_V fixed, the Lagrangean (1) reduces to the standard Skyrme model with the fourth-order term related to the ρ meson and an additional sixth-order term $(g_\omega/m_\omega)^2 B_\mu B^\mu$.

It is obvious that by taking this limit one ignores important physics at length scales $\sqrt{6}/m_V \simeq 0.6$ fm.

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[a]Address after January 1, 1987: Massachusetts Institute of Technology, Center for Theoretical Physics, Cambridge, MA 02139.

[b]Present address: NORDITA, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark.

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