Hopping Magnetoconduction and the Random Structure in Quasi One-Dimensional Inversion Layers

Rajiv K. Kalia
Argonne National Laboratory, Argonne, Illinois 60439

and

Weige Xue and Patrick A. Lee
Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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Conductance fluctuations due to variable-range hopping in 1D metal-oxide-semiconductor field-effect transistors are investigated in the presence of a magnetic field. With an increase in magnetic field, the Zeeman effect shifts the fluctuations to lower or higher chemical potentials. These shifts reflect the relative populations and occurrence of hopping from singly and doubly occupied sites. Combined with density-of-states measurements, they can provide an estimate of the intrasite Coulomb repulsion in 1D metal-oxide-semiconductor field-effect transistors. The orbital effect on conductance fluctuations is also discussed.

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Electronic transport measurements in 1D metal-oxide-semiconductor field-effect transistors (MOSFET's) reveal unusually large fluctuations in conductance with variations in gate voltage.[1,2] According to Lee,[3] this random structure is a manifestation of Mott's variable-range hopping (VRH) conduction. The original motivation for this explanation was that MOSFET wires are typically 30–70 times the localization length. Since a Mott hop covers several localization lengths, only a few hops are required to traverse the entire sample. In the VRH model, each hop is exponentially activated and the resistance of the sample is determined by a critical hop at the percolation threshold. Since the resistors have a log-normal distribution, variations of chemical potential can change the sample resistance by orders of magnitude. A recent study of resistance fluctuations as a function of sample size[4] reveals that the average of the logarithm of the resistance increases as $(\ln 2\alpha L)^{1/2}$ while the relative magnitude of the fluctuations decreases as $(\ln 2\alpha L)^{-1/2}$. The resistance fluctuations are therefore inherent in the VRH model and not just due to finite-size effects. The similarity between resistance fluctuations in the VRH model and those observed experimentally suggests that the experiments are probing something fundamental, namely, a critical hop between a pair of localized states. This presents the exciting possibility of investigating the effect of magnetic field on an individual hop between a pair of localized states. Experiments on the effect of magnetic field on resistance fluctuations in 1D MOSFET's are in progress.[5]

The influence of magnetic field on variable-range hopping conduction enters through the Zeeman shift and changes in wave functions of localized states. With an increased magnetic field, we find that the Zeeman effect rigidly moves conductance fluctuations to lower or higher values of chemical potential; on the other hand, the orbital effect does not cause any systematic shifts in the fluctuation spectrum. Rigid shifts due to the Zeeman effect reflect the nature of the dominant hopping process which, in turn, is related to the relative population of singly and doubly occupied sites in the system. Together with density-of-states measurements, this information can be used to estimate the value of the intrasite Coulomb repulsion in 1D MOSFET's.

Let us first discuss the effect of the Zeeman shift on conductance fluctuations. We consider a system consisting of N equally spaced sites,[6] each characterized by the same localization length, α^{-1}, and a random energy drawn from a uniform distribution function between $\pm W/2$. The Hamiltonian of our system is

$$\mathcal{H}=\sum_{i,\sigma}(E_i-\sigma\mu_B H)n_{i\sigma}+U\sum_i n_{i\uparrow}n_{i\downarrow}, \qquad (1)$$

where $\{E_i\}$ are the random-site energies, U is the intrasite Coulomb repulsion between electrons with opposite spins, $\mu_B=e/2mc$ is the Bohr magneton, and $n_{i\sigma}$ is the number operator for an electron with magnetic moment $\sigma=\pm 1$ (in units of μ_B) at the ith site.

In the absence of spin-flip scattering, there are four kinds of phonon-assisted hopping processes: (1) from a singly occupied site to an unoccupied site, (2) from a doubly occupied site to an unoccupied site, (3) from a singly occupied site to another singly occupied site with opposite spin, and (4) from a doubly occupied site to a singly occupied site. The net hopping rate, Γ_{ij}, is a sum of the hopping rates for these four processes[7]:

$$\Gamma_{ij}=\sum_{\sigma=\pm 1}\{\gamma_{ij}^{(1)}\exp[-\beta(E_i-\sigma\mu_B H-\zeta)]+\gamma_{ij}^{(2)}\exp[-\beta(2E_i+U-2\zeta)]+\gamma_{ij}^{(3)}\exp[-\beta(E_i+E_j-2\zeta)]+\gamma_{ij}^{(4)}\exp[-\beta(2E_i+E_j+U+\sigma\mu_B H-3\zeta)]\}/Z(E_i)Z(E_j), \quad (2)$$

where $\gamma_{ij}^{(s)}$ $(s=1\text{–}4)$ are the intrinsic hopping rates and $Z(E)$ is the grand-partition function. $\gamma_{ij}^{(s)}$ depend on the overlap of wave functions and the difference in energies between the initial and final states.

Choosing the chemical potential ζ between $\pm W/2$, we consider hopping of electrons between reservoirs attached to the two ends of the sample. Following Ambegaokar, Halperin, and Langer,[8] in the presence of a small electric field and net current between sites i and j is a product of the conductance, $e^2\beta\Gamma_{ij}$, and the net electrochemical potential difference between these sites. The exact total current and conductance of the system can be obtained by solving the linear equations arising from Kirchoff's law. Alternatively, because of the exponential variation in Γ_{ij} it is reasonable to approximate the total resistance of the system by the smallest resistance R_c such that the resistors $R_{ij}\leq R_c$ form a percolation network connecting the reservoirs.[3,8] The pair of sites with resistance R_c represents the weak link in the percolation path.

Percolation calculations were performed for a 1000-site system at several values of the magnetic field. In our simulations, $\alpha=0.02$ in units of spacing between sites, energy is measured in units of W so that $W=1$, and $U=0.04$ and 0.4. For given ζ and H values, we find that the contribution to the net hopping rate comes predominantly from one of the four hopping processes and that variations in ζ or H can change the nature of the hopping process. Hopping process (1) or (4) is dominant if the energies of both weak-link sites are close to ζ or $\zeta-U$; processes (2) or (3) contribute significantly when the energy of one of the weak-link sites is approximately $\zeta-U$ and the other close to ζ.

A magnetic field suppresses the occupation probability of antiparallel-magnetic-moment states, thereby reducing the hopping rates for processes (2) and (3): At sufficiently high fields ($\mu_B H \gtrsim 5kT$), hopping conduction occurs via processes (1) and (4). Figure 1(a) shows that an increase in magnetic field shifts some of the resistance fluctuations to lower values of chemical potential. This feature is related to hopping process (1) for the following reason: The population of antiparallel-magnetic-moment states is suppressed by the magnetic field and the hopping process is therefore dominated by parallel-magnetic-moment electrons with energy $E_i-\mu_B H$. Since the hopping rate depends on the separation of these energy levels from the chemical potential, an increase in H is equivalent to a reduction in ζ and the fluctuations associated with process (1) therefore shift to lower values of the chemical potential.

Figure 1(b) shows that some of the resistance fluctuations shift rigidly to higher values of ζ with an increase in magnetic field. This is due to hopping of electrons from doubly occupied sites to singly occupied sites. Since the population of singly occupied sites is dominated by parallel-magnetic-moment electrons, an electron hopping from a doubly occupied site has to have an antiparallel magnetic moment. Relative to ζ, the energy of the hopping electron, $E_i+\mu_B H$, increases with an increase in H and the fluctuations therefore shift to higher values of chemical potential. Besides these rigid shifts, a change in magnetic field can also alter the weak link so that the hopping process changes from type (1) to (4) or vice versa. All of these features are observed at both $U=0.04$ and 0.4. It should be noted that for $U=\infty$ the fluctuations arise only from process (1) and they shift to lower chemical potentials with increase in the magnetic field.[9]

The rigid shifts in conductance fluctuations become most obvious from the autocorrelation function,

$$C(\zeta,\Delta H)=\langle[\ln R(\zeta_0,H)-\langle\ln R(H)\rangle][\ln R(\zeta_0+\zeta,H+\Delta H)-\langle\ln R(H+\Delta H)\rangle]\rangle. \quad (3)$$

Figure 2 shows the behavior of $C(\zeta,\Delta H)$ in two different regions of ζ: Curve 1 is calculated over the range $\zeta=-0.1$ and $\zeta=0$, curve 2 over $\zeta=0$ and 0.2, and the averaging in $\langle\ln R\rangle$ is over corresponding regions of ζ. Between $\zeta=-0.1$ and 0, the fluctuation spectrum is dominated by hopping process (1) and, consequently, one finds a peak in C at $\zeta\simeq-\mu_B\Delta H$. On the other hand, hopping process (4) is dominant over the range $\zeta=0$ and 0.2 and, thus, there is a peak in C at $\zeta\simeq\mu_B\Delta H$. Clearly, if the correlation function C is calculated over the entire range of ζ, then the superposition of curves such as those shown in Fig. 2 will lead to a broad peak at $\zeta=0$.

The Zeeman effect enhances the average resistance of the system because the magnetic field suppresses the population of antiparallel-magnetic-moment states and consequently the rates for processes (2) and (3). At sufficiently high magnetic fields, processes (2) and (3) are completely suppressed and the fluctuations associated with processes (1) and (4) simply shift to lower and higher chemical potentials, respectively.

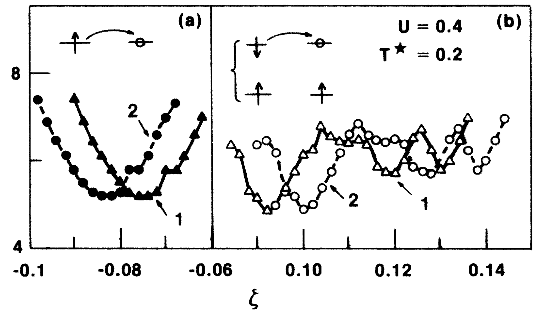

FIG. 1. Segments of resistance-fluctuation spectrum corresponding to Zeeman shifts of 0.032 (curve 1) and 0.04 (curve 2). In (a), curves 1 and 2 coincide if the former is shifted by $\zeta = -\mu_B \Delta H = -0.008$. However, an exactly opposite shift causes curve 2 to coincide with curve 1 in (b).

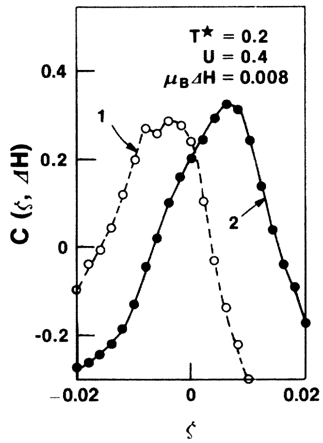

FIG. 2. Results for the correlation function $C(\zeta, \Delta H)$ due to the Zeeman effect at $T^* = T/T_0 = 0.2$ where $T_0 = \alpha W$.

This results in the saturation of the value of average resistance. The relative magnitude of fluctuations in $\ln R$ decreases slightly with an initial increase in H, but at higher values of H there is no further change aside from statistical variations. The net enhancement of the average resistance and the relative magnitude of resistance fluctuations are insensitive to the value of the intrasite Coulomb repulsion.

Next, let us discuss the influence of the orbital effect on conductance fluctuations. We consider a rectangular strip of length La, width Ma (a is the lattice constant) with $L \times M$ sites in the presence of a magnetic field perpendicular to the strip. The tight-binding Hamiltonian for this system is

$$\mathcal{H} = \sum_{lm} E_{lm} |lm\rangle\langle lm| + \sum_{l,m}\sum_{l'm'} V_{lml'm'} |lm\rangle\langle l'm'|, \tag{4}$$

where

$$V_{lml'm'} = \begin{cases} 1, \text{ if } m = m',\ l' = l \pm 1, \\ \exp[\pm i(e/\hbar) lHa^2], \\ \qquad\qquad \text{if } l = l',\ m' = m \pm 1. \end{cases} \tag{5}$$

$\{E_{lm}\}$ are random-site energies between $\pm W/2$, where W characterizes the degree of disorder in the system. Periodic boundary conditions are used in the y direction. We obtain the eigenvalues and eigenfunctions of this Hamiltonian for (50×20)- and (100×20)-site systems at $W = 8$ and $H = 0$, 0.2, 0.4, 0.6, and 0.8. (H is measured in units of $\hbar c/a^2 e$.) From the eigenfunctions we calculate the inverse participation ratios, which are proportional to the square of localization lengths, λ.[10] The latter form the basis of percolation calculations for 1D MOSFET's.

Starting with N ($= 1000$ or 2000) uniformly spaced sites along a wire, we assign each of them a random energy between ± 0.5 and a localization length from the distribution of localization lengths for that H value.[11] To compare with our previous results, we scale each λ at $H = 0$ by an amount f so that their average is still α^{-1}. The factor f is then used to scale the λ's at finite values of H. Calculations for the hopping probability between a pair of sites reveal that the dominant contribution involves the larger of the two localization lengths in question[7] because of the broad distribution of localization lengths. The hopping transport, or equivalently the resistor connecting a pair of sites, is given by the Miller-Abrahams expression.[12] Again, we obtain the percolation solution to this random-resistor network.[3,8]

In the presence of the orbital effect alone, the autocorrelation function $C(\zeta, \Delta H)$ has a peak at $\zeta = 0$ (see Fig. 3), but there are not any systematic shifts in the spectrum of resistance fluctuations. The average value of $\ln R$ decreases slightly with the initial increase in H, and then it saturates at larger values of H. The persistence of negative magnetoresistance into the localized regime has been discussed by Lee and Fisher.[13] The relative magnitude of the fluctuations is insensitive to changes in magnetic field and the average value of resistance continues to exhibit Mott's $T^{-1/2}$ behavior.

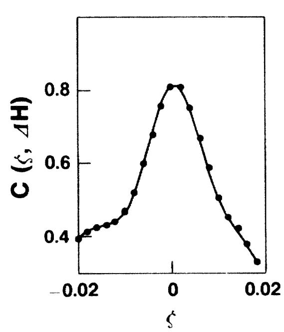

FIG. 3. Correlation function $C(\zeta, \Delta H)$ in the presence of the orbital effect at $T^* = 0.2$. Here, magnetic fields H and ΔH [see Eq. (3)] are 0.2 in units of $\hbar c/a^2 e$.

Let us turn to the experimental implications of our results. Orbital and Zeeman effects can be experimentally separated by the orientation of the magnetic field: If the field is parallel to the sample, we predict that the fluctuations in conductance arise because of hopping from either (a) singly occupied sites to unoccupied sites, or (b) doubly occupied sites to singly occupied sites. With an increase in magnetic field, the fluctuations associated with weak links of type (a) shift to lower while those corresponding to process (b) shift to higher gate voltages. If the magnetic field is perpendicular to the sample, both the orbital effect and the Zeeman shift influence conductance fluctuations. But systematic shifts in the conductance-fluctuation spectrum can be solely attributed to Zeeman shifts of occupied sites.

The present model does not entirely cover the situation in MOSFET's. The assumption of a step-function density of states makes hops from singly and doubly occupied sites equally probable, thereby causing the correlation function $C(\zeta,\Delta H)$ to be symmetric in ζ. The experimental situation is expected to be somewhat different because the tail in the density of states will lead to unequal hopping probabilities for processes (1) and (4) and make C asymmetric. Clearly, magnetoconductance measurements in ultranarrow MOSFET's can provide the relative populations of singly and doubly occupied sites near ζ and $\zeta-U$, respectively. To obtain the value of U, one needs additional information on the populations of singly and doubly occupied sites, e.g., from the measurement of density of states.[14]

In conclusion, we have discussed the effect of magnetic field on conductance fluctuations in the variable-range hopping model for finite 1D systems. Our results will be applicable to measurements of magnetoconductance in 1D MOSFET's.[5] We suggest that the experiments should be analyzed in terms of the autocorrelation function C, since it can directly differentiate between orbital and Zeeman effects as well as reveal the hopping process and the nature of localized sites. Systematics of magnetoconductance fluctuations combined with density-of-states measurements will be able to provide the intrasite Coulomb repulsion in 1D MOSFET's.

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[6]In variable-range hopping only those sites that are close to the chemical potential are important. Distances between such sites are uncorrelated because their energies are random. Thus it is not essential to assign random energies as well as random positions to sites in the system.

[7]This model has been used to study positive magnetoresistance due to the Zeeman effect in other systems; see, for example, H. Kamimura, Prog. Theor. Phys. **72**, 206 (1982), and references therein.

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[9]We thank A. B. Fowler for emphasizing the importance of including U.

[10]At $H=0$ the average inverse participation ratio is around 25 and the average localization length is around 5. For comparison, see A. McKinnon and B. Kramer, in *The Application of High Magnetic Fields in Semiconductor Physics,* edited by G. Landwehr, Springer Lecture Notes in Physics Vol. 177 (Springer, Berlin, 1983), p. 74.

[11]Since the width of quasi 1D MOSFET's is much less than the variable-range hopping length, it is reasonable to regard them as 1D systems. On the scale of hopping length d, the localization lengths can be considered independent of one another since they are much smaller than d. It is therefore reasonable to assign to each site along the wire a localization length, chosen randomly, from the distribution of localization lengths.

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