ac Free-Electron Laser

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A possible new source of tunable coherent radiation is analyzed. By the passing of a relativistic electron beam (relativistic factor γ_0) through a homogeneous transverse ac electric or magnetic field of frequency f_0, electromagnetic radiation of frequency $f \sim 2\gamma_0^2 f_0$ is generated by its coupling the negative-energy electrostatic beam modes with the high-frequency electromagnetic modes. Existing superconductor cavities which can supply ac fields of frequency $f_0 \gtrsim 10$ GHz with amplitude $E_{max} > 20$ MV/m may be used to lead to a new class of submillimeter or even visible generating devices.

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We analyze a new form of free-electron laser and check the validity of the ideas by computer simulation using a 1-2/2 dimensional, relativistic, electromagnetic particle code.[1] A relativistic electron beam with velocity $\mathbf{v}_0$ is propagated along the axis of a temporally oscillating (frequency f_0), spatially uniform, transverse electric or magnetic structure.[2] Temporal oscillations play the same role here as spatial oscillations in the conventional free-electron laser (FEL).[3] Relativistically, there is a correspondence between wave number and frequency, and between electric field and magnetic field. In the beam frame, the oscillating field appears to be an incident electromagnetic wave with frequency $f' = \gamma_0 f_0$ and wave vector $\mathbf{k}_0' = -\boldsymbol{\beta}_0\gamma_0\omega_0/c$, where $\omega_0 = 2\pi f_0$, $\boldsymbol{\beta}_0 = \mathbf{v}_0/c$, and $\gamma_0 = (1-\beta_0^2)^{-1/2}$. Therefore, as in conventional FEL, the radiation generation can be thought of as stimulated scattering of the incident pump wave seen by the electron beam. We will not consider the forward scattered waves[4] because they are shifted down in frequency and also have a much smaller growth rate than the backward scattered waves. For $\gamma_0 >> 1$, with reasonable f_0 the backward scattered wave would have frequency $\sim 2\gamma_0^2 f_0$. It is tunable by adjustment of the beam energy γ_0 and/or the pump frequency f_0. There exist superconductor cavities which can supply ac fields of frequency $f_0 \gtrsim 10$ GHz with amplitude $E_{max} > 20$ MV/m.[5] This is equivalent to a wiggler magnetic field of 700 G which is comparable to those in use and thus this method may be used to lead to a new class of submillimeter or even visible generating devices.

In the following, we first give the dispersion relation derived from a fluid theory; growth rates and efficiencies are obtained from this to compare with computer-simulation results. We then discuss the feasibility of this new form of free-electron laser (ac FEL) and compare its performance to more conventional FEL's.

Consider an unmagnetized, neutral, transversely homogeneous, unbounded electron beam with velocity $\mathbf{v}_0$ moving through a spatially uniform, transverse-oscillating electric field $\mathbf{E}_0\cos(\omega_0 t)$ (i.e., $\mathbf{E}_0 \perp \mathbf{v}_0$). Using fluid theory,[6,7] we derived the following dispersion relation for $\epsilon_0\omega_p/\omega_0 << \gamma_0\beta_0$:

$$D_{ES}(k,\omega_{ES})D_{EM}(k,\omega_{EM}) = F, \qquad (1)$$

where

$$D_{ES} = (\omega_{ES} - kv_0)^2/\omega_p^2 - (1+3k^2\lambda_D^2)/\gamma_0^3,$$

$$D_{EM} = (\omega_{EM}/\omega_p)^2 - (kc/\omega_p)^2 - 1/\gamma_0,$$

$$F = \frac{k^2c^2\epsilon_0^2}{4\gamma_0^5\omega_0^2}\left[1+\gamma_0^2\beta_0\left(\frac{kv_0-\omega_{ES}}{kc}\right)\right],$$

and $\omega_{EM} = \omega_{ES} + \omega_0$, $\epsilon_0 = eE_0/mc\omega_p$, $\omega_p = (4\pi n_0 e^2/m)^{1/2}$, $\lambda_D = (4\pi n_0 e^2/T)^{-1/2}$; ω_{EM} and ω_{ES} are respectively the electromagnetic and electrostatic wave frequencies, and m and T are respectively the electron rest mass and temperature. For the case that the electron beam is moving through a spatially uniform transverse-oscillating magnetic field $\mathbf{B}_0\cos(\omega_0 t)$, we got the same relation except that $\epsilon_0 = \beta_0 eB_0/mc\omega_p$. If both the transverse-oscillating electric and magnetic fields are allowed, we must use $\epsilon_0 = e(\mathbf{E}_0 + \boldsymbol{\beta}_0\times\boldsymbol{\beta}_0)/mc\omega_p$. As should be, our computer simulation gives the same results for the same ϵ_0. For $F << 1$, the normal modes (given by $D_{ES}=0$ and $D_{EM}=0$) are weakly coupled, so that the usual mode-matching conditions are satisfied; i.e., $\omega_{EM} - \omega_{ES} = \omega_0$, $k_{EM} - k_{ES} = 0$ in this case, which leads to $f \cong 2\gamma_0^2 f_0$. For each k near the normal mode k_n, we can solve Eq. (1) for the corresponding frequency and growth rate, which are respectively given by

$$\omega_{EM}^R = (H - \mu/2)\omega_p, \qquad (2)$$

$$\omega_i = \tfrac{1}{2}\omega_p(\bar{F}\gamma_0^{3/2}/H - \mu^2)^{1/2}, \qquad (3)$$

where

$$H = [(kc/\omega_p)^2 + 1/\gamma_0]^{1/2},$$

$$\mu = H - (\omega_0 + kv_0)/\omega_p + (1+3k^2\lambda_D^2)^{1/2}\gamma_0^{-3/2},$$

$$\bar{F} = \frac{k^2c^2\epsilon_0^2}{4\gamma_0^5\omega_0^2}\left[1+\gamma_0^2\beta_0\left(\frac{\omega_0 + kv_0 - \omega_{EM}^R}{kc}\right)\right].$$

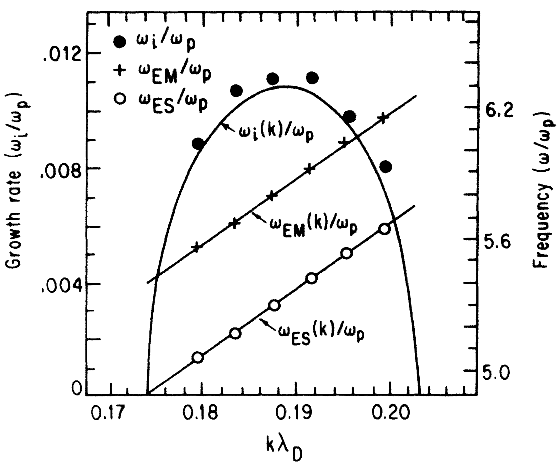

FIG. 1. Theoretical (solid curves) and simulation (discrete points) growth rate and frequency vs wave number for the case with $\gamma_0=3.2$, $\epsilon_0=0.06$, $\omega_0=0.5\omega_p$.

The maximum growth rate would be (for $\gamma_0 >> 1$)

$$\omega_i^{\max} \cong (\epsilon_0/2\sqrt{2})\omega_p\gamma_0^{-3/4}(\omega_p/\omega_0)^{1/2}. \qquad (4)$$

A numerical study for the instability spectrum and the corresponding growth rate given by Eqs. (2) and (3), along with computer simulation results, is given in Fig. 1. The excellent agreement between the theory and the simulation ensures the accuracy of the modeling as well as the approximations of the theory. The spectrum relation $\omega_{\rm EM}=\omega_{\rm ES}+\omega_0$ from computer simulations confirms that the instability is due to the coupling of the negative-energy electrostatic beam modes with the high-frequency electromagnetic modes by the pump. Figure 2 shows the relative power spectrum for electrostatic and electromagnetic waves from the simulation. The electromagnetic power is much larger than the electrostatic power. This means that the reduced electron kinetic energy is mainly transferred to electromagnetic energy. This can also be clearly seen in Fig. 3, where the time evolutions of the electrostatic, electromagnetic, and electron kinetic energy are shown.

Of considerable importance is the nonlinear saturation level since it determines the efficiency of radiation production. We can define the efficiency as $\eta=(\gamma_0-\langle\gamma_s\rangle)/(\gamma_0-1)$, where $\langle\gamma_s\rangle$ is the average electron relativistic factor at the saturation level. From Fig. 3 we would get a value $\eta=10\%$. For some other parameters, efficiencies as high as 20% were obtained.[8] Since the saturation mechanism is due to electron trapping in the electrostatic wave, we can roughly derive a formula to predict the efficiency. At saturation the electron beam speed is, on the average, slowed down to the phase velocity of the generated electrostatic beam mode

$$v_p \cong v_0-\gamma_0^{-3/2}(1+3k_n^2\lambda_{\rm D}^2)^{1/2}\omega_p/k_n.$$

Neglecting the electrostatic wave energy and electron thermal energy, we obtain the efficiency

$$\bar{\eta}=(\gamma_0-\gamma_p)/(\gamma_0-1), \qquad (5)$$

where $\gamma_p=(1-v_p^2/c^2)^{-1/2}$. A comparison of the computer simulations with the estimated values given by Eq. (5) is shown in Fig. 4.

The above fluid analysis, of course, is only valid in the Raman regime. In the Compton regime ($k\lambda_{\rm D} \gtrsim 1$), the plasma waves are heavily Landau damped. We have also investigated this regime by increasing the electron thermal-velocity spread in our computer simulations.[8] The instability seems to be weakened, and the growth rate reduced. However, for $k\lambda_{\rm D}\sim 1$, efficiencies of greater than 5% are still obtainable.

Let us use an astron beam as an example of a feasible electron beam to determine what we might expect

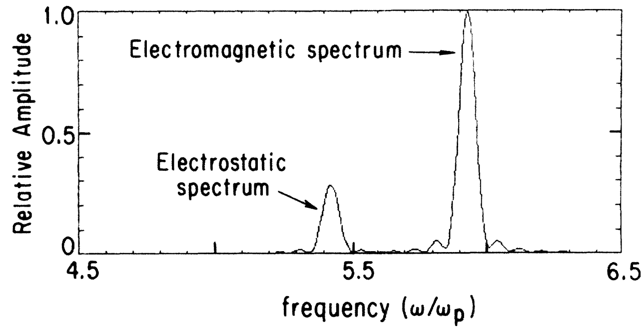

FIG. 2. Unstable electromagnetic and electrostatic power spectrum for the case with $\gamma_0=3.2$, $\epsilon_0=0.06$, $\omega_0=0.5\omega_p$. The electrostatic power is enlarged by a factor of 20.

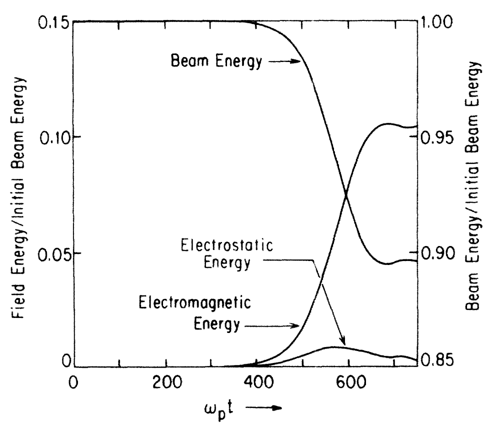

FIG. 3. Time evolution of electrostatic, electromagnetic, and electron kinetic energy for the case with $\gamma_0=5.0$, $\epsilon_0=0.1$, $\omega_0=0.5\omega_p$. The electrostatic energy is enlarged by a factor of 100.

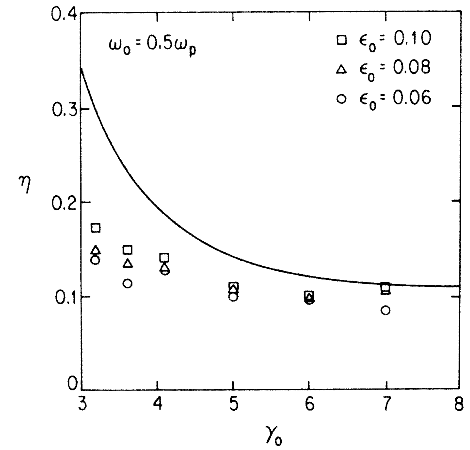

FIG. 4. Theoretical efficiency vs initial beam energy and the results (discrete point) from simulations.

from the ac FEL. Typical astron-beam parameters are density $n_0 = 3\times10^{12}$ cm^{-3}, energy spread $\Delta\gamma/\gamma_0 \cong 10^{-3}$, and $\gamma_0 = 10$. Thus the Debye length $\lambda_D \sim 1$ μm and plasma frequency f_p $(=\omega_p/2\pi)$ ~ 15.6 GHz. Using an ac electric field of frequency $f_0 = 5$ GHz with amplitude $E_0 = 20$ MV/m ($\epsilon_0 \cong 0.12$) would produce radiation at frequency $f \sim 1000$ GHz ($\lambda \cong 30$ μm) so that $k\lambda_D = 2\pi\lambda_D/\lambda \cong 0.2$, and thus Eq. (4) holds and can be used to predict a growth rate $\omega_i \cong 0.014\omega_p$. Amplifying the signal by ten e foldings would require an interaction time $\tau = 10/\omega_i \cong 7.5\times10^{-9}$ sec and thus the interaction length would be $L = c\tau \cong 2.2$ m. These parameters are quite comparable to those of the conventional-wiggler FEL and can be easily realized in the laboratory.

As for a more specific design, we would suggest the propagation of an electron beam along the symmetric axis (here labeled the x direction) of a rectangular wave guide of length L, width W (along the z direction), and height h (along the y direction), operating near cutoff so that an ac electric field $E_0\hat{e}_y\cos(\omega_0 t)$ exists on axis. The design criteria would be the following: (C-1) $0.01 < (\epsilon_0/\gamma_0)(\omega_p/\omega_0) < 0.1$, (C-2) $\lambda << d_y$ and $\lambda << d_z$, (C-3) $d_y < h - 2\epsilon_0 c\omega_p/\gamma_0\omega_0^2$, (C-4) $d_z << w$, (C-5) $w << L$ and $d_z << \gamma_0 c/\omega_0$; where λ $(\sim \pi c/\gamma_0\omega_0)$ is the radiated wave length, and d_y and d_z are respectively the electron beam width along the y direction and the z direction. Holding to criterion C-1 ensures that the imposed ac electric field is strong enough to make meaningful gain in a reasonable distance, but not so strong as to excite higher harmonic modes. Holding to criterion C-2 ensures that the plane-wave approximation used in this Letter holds. Holding to criterion C-3 prevents the electrons from hitting the walls in the course of their ac motion. Holding to criterion C-4 implies that the ac wave form approximates very closely a temporally oscillating but spatially uniform electric field in the interaction region. Also, as long as $w << L$ in criterion C-5 is satisfied, the induced magnetic field (due to Ampère's law) in the interaction region is essentially parallel to the symmetric axis (x axis) with magnitude $B_{\rm ind}(z,t) \cong (\omega_0 z/c)E_0\sin(\omega_0 t)$, where z $(0 \le z \le d_z/2)$ is the distance away from the symmetric plane $z = 0$. The maximum of the amplitude of $B_{\rm ind}$ has a value $(\omega_0 d_z/2c)E_0$ and is invariant under Lorentz transformation to the beam frame. Therefore, holding to $d_z << \gamma_0 c/\omega_0$ ensures that this induced field $B_{\rm ind}$ is very small (compared to the ac electric field which, in the beam frame, has electric amplitude $\gamma_0 E_0$ and magnetic amplitude $\gamma_0\beta_0 E_0$) and is thus negligible. In practice, it is easy to satisfy the above five criteria simultaneously. For the above astron-beam example, choosing $d_y \sim d_z \sim 2$ mm, $h \sim 1$ cm, $w \sim 3$ cm, and $L \sim 4$ m would be appropriate.

Because of technical reasons, the wiggler period of the conventional-magnet FEL is currently limited to $\lambda_w \gtrsim 2$ cm, thus limiting the frequency of the laser to $f \sim 30\gamma_0^2$ (GHz). On the other hand, by use of a superconductor cavity, an ac electric field as strong as $E_0 > 20$ MV/m with frequency $f_0 \gtrsim 10$ GHz has been achieved.[5] The technology of such superconducting cavities is developing rapidly; we expect that strong ac field with $f_0 \sim 150$ GHz and good Q should be possible and believe that this is a fruitful area for experimental research. This could lead to a laser frequency of $f \sim 300\gamma_0^2$ (GHz). As for an electromagnetic-pumped free-elecron laser (EM FEL), the lack of a coherent, tunable intense radiation source for operation of the laser in the attractive subcentimeter regime prevents simple, independent implementation of such an idea. The two-stage approach[9,10] which combines the conventional-wiggler FEL and EM FEL can also be realized in combining the ac FEL and EM FEL. Since the ac field is spatially homogeneous in the laboratory frame, technical accuracy is probably easier to achieve than for the wiggler magnetostatic field (i.e., precision frequency is probably easier to maintain than precision spatial dependence). In addition, the ac field can be turned on and off when the ac wiggler is desired, as for example in a storage ring. Also, if we taper the entrance of the wave guide such that the ac electric field is gradually cut off toward the entrance, an adiabatic transition to the ac wiggling field for the electrons can be achieved. Furthermore, we should also mention that the ac electric field could be produced in a plasma; for example, fields of 10^7 V/cm have been produced by the beat wave mechanism at a frequency of 3×10^{12} c/sec.[11] Propagating a modest-energy electron beam (10 MeV) parallel to the wave front would produce radiation at a frequency $f \sim 6000\gamma_0^2$ (GHz) $= 9.6\times10^{15}$ Hz ($\lambda \simeq 312$ Å). These calculations suggest that this mechanism may be involved in explaining the experimental results of Kato, Benford, and Tzach[12] a beam-plasma experiment that produced intense radiation with frequency up to $45\omega_p$.

In summary, we list some of the main advantages of the ac FEL: (1) The pumping field is temporally oscillating but spatially uniform, and thus technical accuracy is easier to maintain. (2) An adiabatic transition to the ac wiggling field for the beam electrons can be achieved. (3) The ac field can be turned on and off at will. (4) The generated laser frequency is $\sim 2\gamma_0^2 f_0$, where f_0 can be 10–150 GHz for superconducting cavities, or > 3000 GHz if a plasma electrostatic wave is considered.

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Note added.—Since this paper was submitted, it has come to our attention that there are some related activities.[13]

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[3]In a conventional free-electron laser, a relativistic electron beam propagates along a periodic transverse magnetostatic structure (a wiggler) and produces radiation at frequency $\sim 2\gamma_0^2 c/\lambda_w$ (for $\gamma_0 >> 1$), where λ_w is the period of the wiggler. See for example, J.M.J. Madey, J. Appl. Phys. **42**, 1906 (1971); L. R. Elias *et al.*, Phys. Rev. Lett. **36**, 717 (1976); D. A. G. Deacon *et al.*, Phys. Rev. Lett. **38**, 892 (1977); D. B. McDermott *et al.*, Phys. Rev. Lett. **41**, 1368 (1978); R. K. Parker *et al.*, Phys. Rev. Lett. **48**, 238 (1982); A. N. Didenko *et al.*, IEEE Trans. Nucl. Sci. **28**, 3169 (1981); C. W. Roberson *et al.*, in *Millimeter Components and Techniques, Part II,* Infrared and Millimeter Waves, Vol. 10, edited by Kenneth J. Button (Academic Press, New York, 1983), R. W. Warren *et al.*, IEEE J. Quantum electron **19**, 391 (1983), T. J. Orzechowski *et al.*, Phys. Rev. Lett. **54**, 889 (1985); G. Bekefi, R. E. Shefer, and W. W. Destler, Appl. Phys. Lett. **44**, 280 (1983); F. A. Hopf *et al.*, Phys. Rev. A **21**, 302 (1976); W. B. Colson, Phys. Lett. **59A**, 187 (1976); W. H. Louisell *et al.*, Phys. Rev. A **19**, 288 (1979); N. M. Kroll, P. L. Morton, and M. N. Rosenbluth, IEEE J. Quantum Electron. **17**, 1436 (1981); P. Sprang, C. M. Tang, and W. M. Manheimer, Phys. Rev. A **21**, 302 (1980); W. A. McMullin and G. Bekefi, Phys. Rev. A **25**, 1826 (1982); R. C. Davidson, J. S. Wurtele, IEEE Trans. Plasma Sci. **13**, 464 (1985); R. C. Davidson and H. S. Uhm, J. Appl. Phys. **53**, 2910 (1982); *Free Electron Generation of Extreme Ultraviolet Coherent Radiation—1983,* edited by J. Madey and C. Pelligrini, AIP Conference Proceedings No. 118 (American Institute of Physics, New York, 1984).

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