Dynamical Selection Rules in $N\bar{N}$ Annihilation

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Observations of a strong dependence of nucleon-antinucleon annihilation modes, particularly $\pi\rho$ and πf, on orbital angular momentum and spin-isospin are interpreted in a quark-gluon description of the reaction dynamics.

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Annihilation of the nucleon-antinucleon ($N\bar{N}$) system into mesons offers a unique glimpse at the underlying dynamics of QCD (quantum chromodynamics). Here we apply a model based on QCD to the description of $N\bar{N}$ annihilation, and find dynamical selection rules (DSR) corresponding to a strong dependence of mesonic branching ratios on orbital angular momentum L. Recent experimental evidence for such phenomena places significant constraints on the topology of annihilation and the structure of the effective operator O which governs quark-antiquark ($Q\bar{Q}$) creation and annihilation at low momenta.

Evidence for DSR first appeared[1] in the channel $N\bar{N}(L=0)\to 3\pi$, where the quasi two-body (QTB) mode $p\bar{p}\to\pi\rho$ constitutes more than half of all $\pi^+\pi^-\pi^0$ events and is mainly produced from the $N\bar{N}$ channel $^{2I+1,2S+1}L_J={}^{13}S_1$ rather than $^{31}S_0$. The small $\pi\rho$ production from the $^{31}S_0$ state was confirmed by studies[2-4] of $\bar{p}n\to\pi^-\pi^-\pi^+$. Here, the $\pi^- f$ mode dominates $\pi^-\rho^0$: "At rest"[2] $N(^{31}S_0\to\pi^-\rho^0)$: $N(^{31}S_0\to\pi^- f)=1{:}5$, whereas "in flight,"[3] 1:8 is found, where N is the number of events.

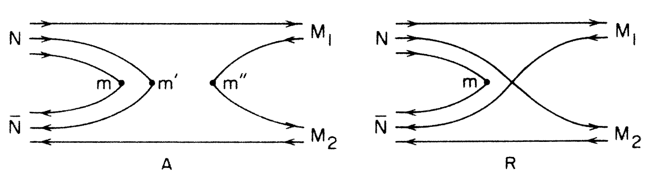

FIG. 1. Annihilation (A) and rearrangement (R) contributions to the $N\bar{N}\to M_1M_2$ reaction. Gluon exhanges are not shown explicitly. The effective operator O for quark-antiquark annihilation is symbolized by a dot. The indices m,m',m'' refer to the z components of the orbital angular momentum of each $Q\bar{Q}$ vertex.

Recent $p\bar{p}$ data[5] show that more than half of the 3π events come from $L=1$. This implies[6] that $N(L=1\to\pi\rho){:}N(L=1\to\pi^0 f)\approx 1{:}1$. For $L=1$, $\pi^\pm\rho^\mp$ comes mainly[5] from $^{11}P_1$ whereas $\pi^0 f$ comes from $^{33}P_1$.

To explain this selectivity, we consider the processes in Fig. 1, i.e., a transition from $N\bar{N}$ to a QTB final state M_1M_2 with a vertex for three (graph A) or one (graph R) $Q\bar{Q}$ pairs. We address the following two questions: (1) Which effective operator O acts at a $Q\bar{Q}$ vertex? (2) Given a choice of O, is graph A or R dominant? Our conclusion is the following[7]: *The data for $N\bar{N}\to 3\pi$ are consistent with O having vacuum quantum numbers $[0^{++}(0^+)$, $^{13}P_0$ in LS coupling], with graph A as the dominant contribution.* This choice of O, known as the "$^3P_0$ model," has been successfully applied to hadron decays.[8]

The QTB modes for $p\bar{p}\to\pi^+\pi^-\pi^0$ consistent with conservation of $\{J^{PC}(I^G)\}$ are listed in Table I. Independent of dynamical assumptions, initial $^{13}S_1$ and $^{11}P_1$ states produce only $\pi\rho$, by C conservation. For $^{31}S_0$ and $^{33}P_{1,2}$, on the other hand, the $\pi^0\epsilon$, $\pi^0 f$, and $\pi\rho$ modes compete. The small $\pi\rho$ production from $^{31}S_0$ and $^{33}P_{1,2}$ channels is the "$\pi\rho$ puzzle." We

TABLE I. Allowed transitions $p\bar{p}\to\pi^+\pi^-\pi^0$ for $L=0,1$. We omit channels like πg involving mesons with $J\geq 3$; l_f is the meson-meson relative orbital angular momentum.

Initial state	Final state
$^{31}S_0$	$\pi^0\epsilon(l_f=0)$, $\pi^\pm\rho^\mp(l_f=1)$, $\pi^0 f(l_f=2)$
$^{13}S_1$	$\pi^0\rho^0$, $\pi^\pm\rho^\mp(l_f=1)$
$^{11}P_1$	$\pi^0\rho^0$, $\pi^\pm\rho^\mp(l_f=0,2)$
$^{33}P_1$	$\pi^0\epsilon(l_f=1)$, $\pi^\pm\rho^\mp(l_f=0,2)$, $\pi^0 f(l_f=1,3)$
$^{33}P_2$	$\pi^\pm\rho^\mp(l_f=2)$, $\pi^0 f(l_f=1,3)$

resolve it by showing that $\pi^0 f$ production dominates for these channels.

In the nonrelativistic limit, the $^3P_0$ vertex for O is

$$O = \lambda_p \chi_f \chi_c \chi_m (1m1-m|00) \mathcal{Y}_{1,-m}(\mathbf{k}_1-\mathbf{k}_2)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2). \quad (1)$$

χ_f, χ_c, and χ_m are flavor-singlet, color-singlet, and spin-1 wave functions. The Q and $\bar{Q}$ momenta are $\mathbf{k}_1$ and $\mathbf{k}_2$, and $(1m1-m|00)$ couples the $Q\bar{Q}$ p wave and unit spin to $J=0$. Equation (1) can be justified approximately in strong-coupling QCD on a lattice.[9] An effective one-gluon operator has been suggested[10] in place of (1), appropriate to the weak-coupling limit; this has some difficulties.[6,7] Other treatments[11] of the $^3P_0$ model give rise to quite different predictions than ours.

To evaluate the amplitudes $T^{\rm A}$ and $T^{\rm R}$ of annihilation (A) and rearrangement (R) (Fig. 1) we use harmonic oscillators for quarks in Q^3 or $Q\bar{Q}$ states. Momentum-space integrals were evaluated[12] by use of VEGAS. The numerical results are largely reproduced[7] by a "no-recoil" approximation, in which the finite meson size is neglected in certain angular factors. Here we quote results of this approximation. Omitting a momentum-conserving δ function, we find an amplitude

$$T_{i\alpha}^{\rm A,R}(L,l_f) = F_i^{\rm A,R}(L,l_f,k,q)\, g_{iLl_f}^{\rm A,R}(\alpha). \quad (2)$$

$i=\{ss\}$ or $\{sp\}$ labels the two-meson state ($s=\{\pi,\eta,\rho,\omega\}$, $p=\{\epsilon,\delta,A_1,A_2,D,f,B,H\}$), k (q) are initial (final) momenta, and $\alpha=\{m_L,m_{l_f},m,m',m''\}$ labels the z projections of L, l_f, and the three (A) or one (R) $Q\bar{Q}$ vertex, respectively. For given $\{L,l_f\}$, the decay widths Γ satisfy

$$\Gamma_i/\Gamma_j = w_i|F_i|^2/w_j|F_j|^2 \quad (3)$$

for A or R alone. The weights $w_i=\sum_\alpha g_i(\alpha)$, obtained by use of SU(6) wave functions for the mesons, are given in Table II for A.

Graphs R and A give dramatically different results. We find

$$T_{i\alpha}^{\rm R}(L,l_f)=0, \quad l_f \geq 1, \quad (4)$$

independent of the "no-recoil" approximation. Thus the transitions $L=0 \rightarrow \pi\rho(l_f=1)$, $\pi A_2(l_f=2)$, and $\pi f(l_f=2)$ are forbidden in rearrangement. In addition,

$$\Gamma(^{33}P_1 \rightarrow \pi^\pm\rho^\mp(l_f=0)) = 18\Gamma(^{11}P_1 \rightarrow \pi^\pm\rho^\mp(l_f=0))$$

from R alone. These predictions are in complete disagreement with data (πA_2 is sizable[13] from $L=0$ and $^{33}P_1 \rightarrow \pi\rho$ is not seen[5]).

For the graph A in the $^3P_0$ model, we find

$$F_i^{\rm A}(L,l_f,k,q) \approx N_i^{\rm A}(L,l_f)(kR)^L(qR)^{l_f}\exp[-R^2(k^2/9+q^2/4)]. \quad (5)$$

The usual penetrabilities $(kR)^L$ and $(qR)^{l_f}$ are modified by a Gaussian from wave-function overlap. For fixed $k\rightarrow 0$ and L, $F_i^{\rm A}(l_f,q)$ has a maximum when $l_f=\frac{1}{2}R^2q^2$. For $R=0.8$ fm, Eq. (5) gives

$$|F_{\pi\rho}^{\rm A}(L=0,l_f=1)/F_{\pi f}^{\rm A}(L=0,l_f=2)|^2 \approx 0.03,$$
$$|F_{\pi\rho}^{\rm A}(L=1,l_f=0)/F_{\pi f}^{\rm A}(L=1,l_f=1)|^2 \approx 0.08. \quad (6)$$

Thus, $L=0\rightarrow sp(l_f=2)$ and $L=1\rightarrow sp(l_f=1)$ transitions are *kinematically* favored. (Precise values are sensitive[7] to R.) This kinematical matching of q and l_f is crucial and, together with the w_i of Table I, explains the absence of $\pi^\pm\rho^\mp$ from $^{31}S_0$ and $^{33}P_1$. In particular,

$$\Gamma(^{31}S_0 \rightarrow \pi^\pm\rho^\mp, \pi^0 f, \pi^0\epsilon) \approx 1{:}15{:}0.3.$$

Note that $^{31}S_0 \rightarrow \pi^0\epsilon$ is small because of the kinematic suppression of $L=0\rightarrow sp(l_f=0)$ transitions. (We have used a mass[9] of 1090 MeV/c^2 for the very broad ϵ.) Thus

$$\frac{N(^{31}S_0\rightarrow\pi\rho)}{N(^{13}S_1\rightarrow\pi\rho)} \approx \frac{1}{3}\frac{\Gamma(^{31}S_0\rightarrow\pi\rho)}{\Gamma(^{31}S_0\rightarrow\pi\rho+\pi f+\pi\epsilon)} \approx \frac{1}{50}. \quad (7)$$

This explains the dominance of the transition $^{13}S_1\rightarrow\pi\rho$ in the $p\bar{p}$ atom. This conclusion does not depend qualitatively on R. For $L=1$, we obtain

$$\frac{N(^{33}P_1\rightarrow\pi^+\rho^-+\pi^-\rho^+)}{N(^{11}P_1\rightarrow\pi^+\rho^-+\pi^-\rho^++\pi^0\rho^0)} \approx \frac{1}{10}, \quad \frac{N(L=1\rightarrow\pi\rho)}{N(L=1\rightarrow\pi f)} \approx 1, \quad (8)$$

TABLE II. Relative spin-flavor weights w_l for $p\bar{p} \to \pi^+\pi^-\pi^0$ in quasi two-body states. We have ignored $n\bar{n}$ and other admixtures in the atomic wave function, and defined $w_l = 1$ for $\pi^+\rho^- + \pi^-\rho^+$ by convention. The w_l values correspond to graph A of Fig. 1 with $^3P_0$ $Q\bar{Q}$ vertices.

$L=0$ transitions	w_l	$L=1$ transitions	w_l
$^{31}S_0 \to \pi^\pm\rho^\mp(l_f=1)$	1	$^{11}P_1 \to \pi^\pm\rho^\mp(l_f=0)$	1
$^{13}S_1 \to \pi^\pm\rho^\mp(l_f=1)$	1	$^{33}P_1 \to \pi^\pm\rho^\mp(l_f=0)$	$(\frac{5}{3})^2$
$^{31}S_0 \to \pi^0 f(l_f=2)$	$\frac{1}{2}$	$^{33}P_1 \to \pi^0 f(l_f=1)$	$\frac{500}{27}$
$^{31}S_0 \to \pi^0\epsilon(l_f=0)$	$\frac{32}{27}$	$^{33}P_1 \to \pi^0\epsilon(l_f=1)$	$(\frac{5}{3})^2/6$

consistent with the data.[5] In Eqs. (7) and (8), we have taken a pure $p\bar{p}$ initial state and also assumed that the 5π final states do not drastically change the statistical factor of $2J+1$ for the 3π mode. Note that, in our interpretation, the absence of events corresponding to $^{31}S_0 \to \pi^\pm\rho^\mp(l_f=1)$ and $^{33}P_1 \to \pi^\pm\rho^\mp(l_f=0)$ is *not* due to a selection rule which forbids these transitions in Born approximation, but rather a strong competition from the $\pi^0 f$ channel; see Table II. However, we do obtain a selection rule $L=1 \to ss(l_f=2)$ in the "no-recoil" limit, as well as the absence of $l_f \geqslant 3$ amplitudes for *all* transitions from $L=0,1$ states. Thus, the transitions $^{11}P_1, ^{33}P_{1,2} \nrightarrow \pi\rho(l_f=2)$ and $^{33}P_{1,2} \to \pi^0 f(l_f=3)$ listed in Table I are forbidden in this limit. The smallness[5] of the transition $^{33}P_2 \to \pi^0 f(l_f=1)$ has been explained[6] as an effect of isospin mixing. Note that intrinsic spin S is not conserved in our model, i.e., $\Delta S=0$ and 1 transitions are of comparable strengths [for example, $^{13}S_1 \to \pi\rho(\Delta S=0)$ and $^{11}P_1 \to \pi\rho(\Delta S=1)$]. This is also the case for ordinary meson decays.[8,9]

The $^3P_0$ model predicts other approximate DSR's and strong L dependence. For A, we expect $\bar{K}K^*$ from the $^3S_1$ channel to dominate that from $^1S_0$, as data suggest.[13] Note that $\bar{K}K^*$ cannot come from R. Another case is $p\bar{p} \to \pi^+\pi^-\eta, 2\pi^0\eta$ reactions,[14] where $^{11}S_0$ dominates $^{33}S_1$. This appears to support the $\Delta S=0$ rule of the conventional three-body rearrangement model.[15] However, it also comes from *any* QTB model, since the only modes leading to $\pi\pi\eta$ are

$$^{11}S_0 \to \epsilon\eta(l_f=0), f\eta(l_f=2), \pi^\pm\delta^\mp(l_f=0), \pi^\pm A_2^\mp(l_f=2), \quad ^{33}S_1 \to \pi^\pm A_2^\mp(l_f=2), \tag{9}$$

and $A_2 \to \pi\eta$ is a minor decay mode compared to $\pi\rho$. Our model gives testable predictions for the relative rates in Eq. (9). As another example, we find that $^{13}S_1 \to \pi^\pm\rho^\mp(l_f=1), \pi^\pm B^\mp(l_f=2)$, and $^{11}S_0 \to \pi^\pm A_2^\mp(l_f=2)$ rates are comparable. Because the q values are similar, the comparison of sp modes with the same l_f is not sensitive to the radius R, and hence a good test of the w_l.

We urge consistent treatment of the data on the basis of the QTB hypothesis, since numerous ss and sp modes involving two broad mesons (having in some cases very sizable predicted branching ratios) have been omitted in existing analyses. The usual rearrangement picture[15] allows $L=0 \to sss$ and $L=1 \to ssp$, but $L=1 \to sss$ is forbidden. However, for QTB modes the sequence $L=1 \to sp(l_f=0)$, $p \to ss(l_f=0)$ is allowed. Experiments involving "tagged" formation of $p\bar{p}$ atoms with $L=1$ and detection of annihilation involving *neutral* mesons[16] are very important. They can test new DSR, such as $\rho^+\rho^-$ vs $\rho^0\rho^0$. We anticipate an enhancement of the $\pi^+\pi^-\pi^0\pi^0$ mode, due to strong $^{33}S_1 \to \rho^+\rho^-(l_f=1)$. Other sensitive tests of the mechanism of annihilation include $\{\rho^0,\omega\}$ interference studies (i.e., relative phases) at low energies, ϕ production (e.g., $\pi\phi$), and, if feasible, polarization of vector and tensor mesons arising from interference of amplitudes with differing l_f. The latter test the validity of the DSR $L=1 \nrightarrow ss(l_f=2), sp(l_f=3)$.

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[1]C. Baltay *et al.*, Phys. Rev. **140**, B1039 (1965); M. Foster *et al.*, Nucl. Phys. **B6**, 107 (1968).

[2]P. Anninos *et al.*, Phys. Rev. Lett. **20**, 402 (1968).

[3]S. N. Tovey *et al.*, Phys. Rev. D **17**, 2206 (1978).

[4]T. E. Kalogeropoulos *et al.*, Phys. Rev. D **24**, 1759 (1981); L. Gray *et al.*, Phys. Rev. D **27**, 307 (1983); D. Bridges *et al.*, Phys. Rev. Lett. **56**, 211, 215 (1986).

[5]S. Ahmad *et al.*, in *Proceedings of the Seventh European Symposium on Antiproton Interactions, Durham, England, 1984*, edited by M. R. Pennington, IOP Conference Series No. 73 (Hilger, London, 1985), p. 283.

[6]C. B. Dover, in *Medium Energy Nucleon and Antinucleon Scattering*, edited by H. V. von Geramb, Lecture Notes in Physics Vol. 243 (Springer-Verlag, Berlin, 1985), p. 80.

[7]Earlier discussion is contained in C. B. Dover and P. M. Fishbane, Nucl. Phys. **B244**, 349 (1985); details will be presented in a forthcoming manuscript.

[8]A. LeYaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D **8**, 2223 (1973).

[9]N. Isgur and J. Paton, Phys. Rev. D **31**, 2910 (1985); R. Kokoski and N. Isgur, University of Toronto Report No. UTPT-85-05, 1985 (to be published).

[10]J. A. Niskanen and F. Myhrer, Phys. Lett. **157B**, 247 (1985); M. Kohno and W. Weise, Nucl. Phys. **A454**, 429

(1986); H. R. Rubinstein and H. Snellman, Phys. Lett. **165B**, 187 (1985); E. M. Henley, T. Oka, and J. Vergados, Phys. Lett. **166B**, 274 (1986).

[11]A. M. Green, V. Kuikka, and J. A. Niskanen, Nucl. Phys. **A446**, 543 (1985); A. Faessler, S. Furui, and S. B. Khadkikar, Nucl. Phys. **A424**, 495 (1984); S. Furui, to be published.

[12]G. P. Lepage, J. Comput. Phys. **27**, 192 (1978).

[13]R. Armenteros and B. French, in *High Energy Physics,* edited by E. H. S. Burhop (Academic, New York, 1969), p. 284.

[14]S. Devons *et al.,* Phys. Lett. **47B**, 271 (1973).

[15]M. Maruyama and T. Ueda, Nucl. Phys. **A364**, 297 (1981).

[16]E. Aker *et al.* (Crystal Barrel Collaboration), CERN Report No. CERN/PSCC/85-56, 1985 (unpublished).