Constraints on $\iota \to \gamma\gamma$ Provided by the Topological Susceptibility

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The chiral Ward identities for the pseudoscalar nonet including the glueball candidate $\iota(1450)$, together with the requirement that the topological susceptibility be positive, imply that $\Gamma(\iota\to\gamma\gamma)$ is typically small (< 1 keV), in agreement with recent experimental values. The topological susceptibility is, as expected, much smaller than pure-gauge-field estimates.

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The status of the state $\iota(1450)$ remains unclear.[1] There is a theoretical prejudice that it is a pseudoscalar glueball, but experimental confirmation of this hypothesis remains elusive.[2] It may be that this state is the same as the $E(1420)$,[3] and that the apparent absence of the mode $\iota\to\eta\pi\pi$ can be understood as an interference effect.[4] Measurements of decay modes, such as $\iota\to\gamma\gamma$, should constitute good evidence for or against the glueball hypothesis, but we must recognize that mixing with the pseudoscalar nonet is strong, so that the entire system must be dealt with as a whole before theoretical predictions of such important signatures can be extracted.

Let us consider the system consisting of the usual pseudoscalar nonet, π, K, η, and η', plus the presumed pseudoscalar glueball $\iota(1450)$, which in any case mixes strongly with the nonet. Some time ago we presented an incomplete analysis[5] based on the anomalous chiral Ward identities.[6-8] Given the uncertainties in the pseudoscalar decay constants, we could not achieve a definite solution, but rather a continuum of solutions for the various decay constants, depending on the parameters

$$\Delta = \tfrac{4}{3} m_K^2 F_K^2 - \tfrac{1}{3} m_\pi^2 F_\pi^2 - m_\eta^2 F_{8\eta}^2 \tag{1}$$

and

$$x = (F_{0\iota} - \tilde{F}_{0\iota})/(F_{0\eta'} - \tilde{F}_{0\eta'}), \tag{2}$$

where the decay constants are defined by (P = pseudoscalar)

$$m_P^2 F_{aP} = \langle 0|\partial^\mu A_\mu^a|P\rangle,$$

$$m_P^2 \tilde{F}_{0P} = \langle 0|\partial^\mu A_\mu^0 - (\tfrac{2}{3})^{1/2}\frac{3\alpha_s}{4\pi}\mathrm{Tr}G\tilde{G}|P\rangle. \tag{3}$$

For simplicity we had further used the U(3) values $F_{8\eta} = F_{0\eta'} = 1.00$. Crude typical constraints did emerge:

$$B(\iota\to K\bar{K}\pi) \leqslant 30\%, \tag{4}$$

$$\Gamma(\iota\to\gamma\gamma) \leqslant 5\ \mathrm{keV}. \tag{5}$$

The limiting values are consistent with pole-model predictions.[2]

This analysis did not explicitly refer to the topological susceptibility,

$$\chi_t = -i\tfrac{2}{3}\int (dx)\langle 0|T(3\alpha_s/4\pi)\mathrm{Tr}G\tilde{G}(x)(3\alpha_s/4\pi)\mathrm{Tr}G\tilde{G}(0)|0\rangle, \tag{6}$$

which is $6[d^2E/d\theta^2]_{\theta=0}$ in Witten's notation.[9] Although in Ref. 7 we recognized the important constraint that χ_t be positive, it was not imposed in Ref. 5, as pointed out by Williams.[10] Inclusion of this constraint leads to a much narrower space of solutions.

Here I wish to report on a systematic solution of the chiral Ward identities with the condition $\chi_t \geqslant 0$ imposed. We will see that this positivity requirement almost precludes any solution: χ_t naturally wants to be large and negative. We do find a small space of allowed solutions, most characterized by $\Gamma(\iota\to\gamma\gamma) < 1$ keV. This is quite consistent with the recently published value,[11,12] $\Gamma(\iota\to\gamma\gamma)B(\iota K\bar{K}\pi) < 1.6$ keV, but apparently inconsistent with the observed[13] $J/\psi \to \gamma(\rho\gamma)$ proceeding through the ι, since then a simple vector-dominance calculation gives $\Gamma(\iota\to\gamma\gamma) = (15\ \mathrm{keV})\times B(\iota\to K\bar{K}\pi)$.[14]

Let us now turn to the details. Pole saturation of (6) leads to

$$\chi_t = \chi_t^{\mathrm{YM}} - \sum_P m_P^2 (F_{0P} - \tilde{F}_{0P})^2, \tag{7}$$

where χ_t^{YM} is a contact term inserted to insure[15]

$$\chi_t \geqslant 0. \tag{8}$$

In I we disregarded (8) and, therefore, incorrectly set $\chi_t^{\mathrm{YM}} = 0$ in (7). Now χ_t can be evaluated by pole saturation provided we use the further Ward identity[10]

$$\int (dx)\langle 0|T\partial^\mu A_\mu^0\,\mathrm{Tr}G\tilde{G}|0\rangle = 0. \tag{9}$$

The result is

$$\chi_t = \sum_P m_P^2 \tilde{F}_{0P}(F_{0P} - \tilde{F}_{0P}). \tag{10}$$

When this is inserted into (3) of Ref. 7 (where

$S=-\chi_t$), we see that I(2.7) is correctly written as

$$3m_\pi^2F_\pi^2=\sum_P m_P^2[(F_{8P}+\sqrt{2}\tilde{F}_{0P})^2+2\tilde{F}_{0P}(F_{0P}-\tilde{F}_{0P})]. \tag{11}$$

The other equations of I are unchanged. By itself this modification does not have a large impact; it is the positivity requirement (8) which severely restricts the space of solutions. The reason for this is clear: The right side of (11) is small (in units where $F_\pi=1$, which I will henceforth use, it is 0.054). In order to achieve this either $\chi_t<0$ or

$$\chi_t\approx 0 \text{ and } (F_{8P}+\sqrt{2}\tilde{F}_{0P})^2\approx 0 \tag{12}$$

for all the relevant pseudoscalars P. SU(3) symmetry suggests that this is achieved as follows:

$$F_{8\eta}\approx 1.0,\quad \tilde{F}_{0\eta}\approx -0.7,$$
$$F_{8\eta'}\approx F_{8\iota}\approx \tilde{F}_{0\eta'}\approx \tilde{F}_{0\iota}\approx 0. \tag{13}$$

Supposing further that crudely $F_{0\eta'}\approx 1.0$, $F_{0\eta}\approx 0$, we can deduce $R=A(\iota\to\gamma\gamma)/A(\eta'\to\gamma\gamma)$ from I(2.15) to be roughly

$$R\sim 0.1/x \tag{14}$$

where $F_{0\iota}\approx x\sim 1$.

Armed with this expected qualitative behavior we can seek solutions of the equations in I, as modified [I(2.5), I(2.6), (11), I(2.8), the chiral Ward identities; I(2.10), I(2.12), based on $J/\psi\to P\gamma$ being mediated by the anomaly operator $\mathrm{Tr}G\tilde{G}$, and I(2.15), I(2.16), expressing mediation of $P\to\gamma\gamma$ through the electromagnetic anomaly], and as constrained by (8). I will scale decay constants by F_π, that is, set $F_\pi=1$, and express masses in gigaelectronvolts. The allowed region in the (Δ,x) plane is indicated in Fig. 1, for $F_{0\eta'}=1.00$. For various fixed values of Δ, the allowed regions in the $(F_{0\eta'},x)$ plane are shown in Fig. 2. The allowed regions are small, and easy to miss in a numerical search. $B(\iota\to K\bar{K}\pi)$ (which is a function of x only) and $\Gamma(\iota\to\gamma\gamma)=18.6R^2$ keV [we assume $\Gamma(\eta'\to\gamma\gamma)=5.5$ keV[16]] are shown as functions of x for the same Δ values in Fig. 3. It will be noted that, in each case, R [and $\Gamma(\iota\to\gamma\gamma)$] has a zero in the allowed region, but grows rapidly as x increases beyond that point. In Table I, I present some typical solutions

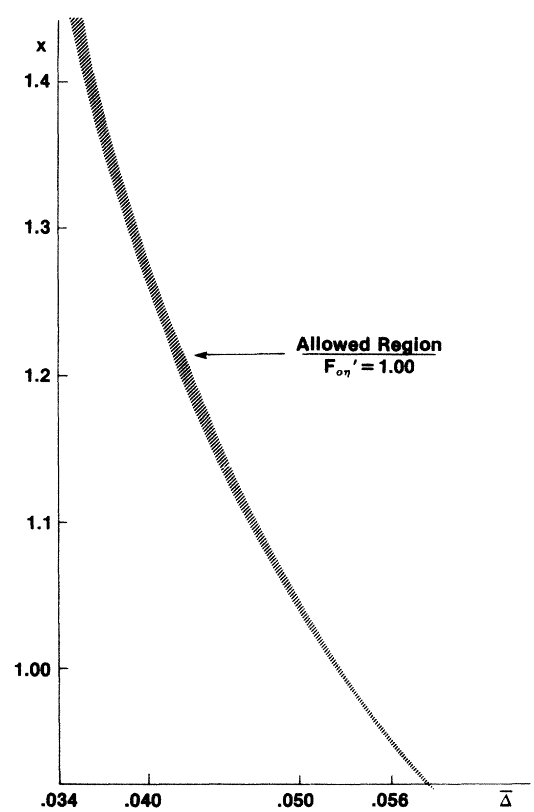

FIG. 1. Allowed region in the (Δ,x) plane for $F_{0\eta'}=1.00$. Here $\bar{\Delta}=\Delta/m_{\eta'}^2$.

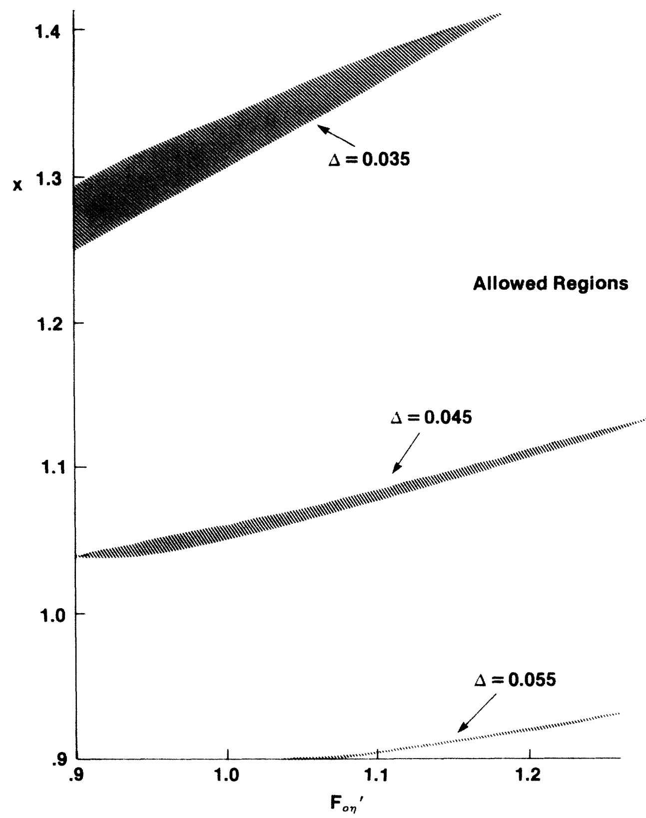

FIG. 2. Allowed regions in the $(F_{0\eta'},x)$ plane for various values of Δ.

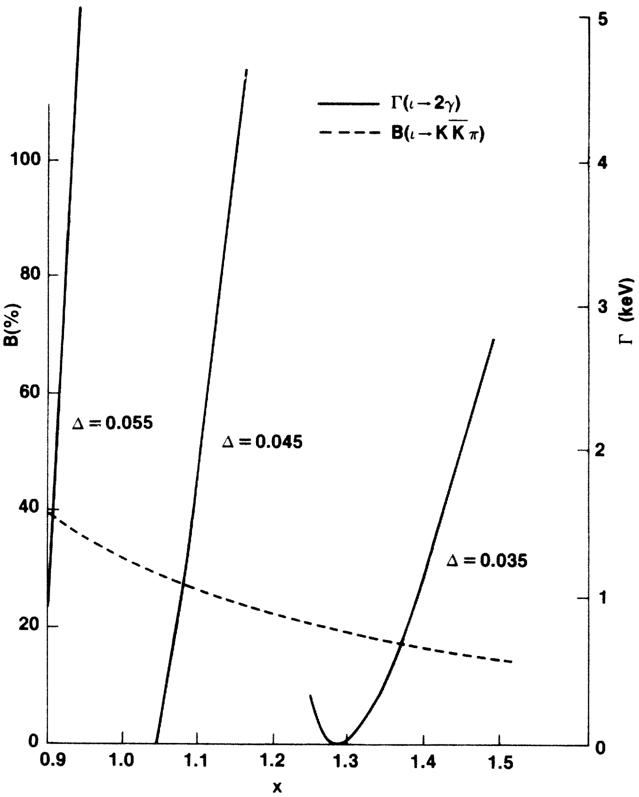

FIG. 3. Branching ratio for $\iota \to K\bar{K}\pi$, and the two-photon decay rate of ι, as functions of x and Δ.

for $\Delta = 0.035$, 0.045, and 0.055; the variation in x of the decay constants across the allowed region is rather slight. We see that the qualitative features indicated by our crude solution (13) and (14) are borne out, except that $F_{0\eta} \sim 0.5$. Although this might appear to represent an unacceptably large SU(3) violation, it is probable that glueball mixing with the η is significant and that as a consequence $F_{0\eta}$ and $\tilde{F}_{0\eta}$ are large. The latter is indeed unavoidably large [see (12)] and Williams[10] finds $\tilde{F}_{0\eta} \sim -0.7$ also, although he presents solutions with $F_{0\eta} \sim 0.2$. It would appear, then, that we cannot use this substantial SU(3) violation to exclude our solution. We should, of course, tread cautiously: We may be omitting crucial physics, such as further 0^- glueball states or continuum contributions. More specifically, we may be so far from the large-N limit that the saturation of the chiral Ward identities is seriously in error. We should also note that in this analysis we have made an exact fit to the $\gamma\gamma$ widths—not because we are unaware of the considerable extrapolation involved, but because we need to limit the parameter space at this primitive stage of our understanding of the phenomena.

To conclude, I again emphasize that the freedom in decay-constant space is severely restricted by the requirement that the topological susceptibility be positive. Correspondingly, very small values of the $\iota \to \gamma\gamma$ branching ratio are the rule, although values (now presumably excluded[11,12]) as large as 5 keV are possible, consistent with the earlier presumed measurements[13] of $\iota \to \rho\gamma$. Our typical value of $B(\iota \to K\bar{K}\pi)$ is 30%, which is probably too low by perhaps a factor of 2.[11] (Williams[10] had earlier found similar solutions, with small $\gamma\gamma$ branching ratios, but values for the $K\bar{K}\pi$ branching ratio near unity.) However, given the incompleteness of our treatment of the physics and the sensitivity to poorly known parameters, it seems we have achieved a qualitatively valid description of the phenomena.[17]

TABLE I. Representative solutions to the chiral Ward identities. All decay constants are measured in units of $F_\pi = 93$ MeV, and masses in units of 1 GeV.

	A	B	C
Δ	0.035	0.045	0.055
x	1.33	1.05	0.92
$F_{8\eta}$	1.00	1.00	1.00
$F_{8\eta'}$	−0.14	−0.14	−0.18
$F_{8\iota}$	−0.09	−0.11	−0.11
$F_{0\eta}$	0.51	0.36	0.70
$F_{0\eta'}$	1.00	1.00	1.20
$F_{0\iota}$	1.34	0.90	1.10
$\tilde{F}_{0\eta}$	−0.67	−0.66	−0.66
$\tilde{F}_{0\eta'}$	0.05	0.19	0.11
$\tilde{F}_{0\iota}$	0.08	0.04	0.09
R	−0.11	−0.08	−0.41
$\Gamma(\iota \to \gamma\gamma)$	0.23 keV	0.12 keV	3.1 keV
$B(\iota \to K\bar{K}\pi)$	18%	29%	38%
χ_t	0.023	0.016	0.025

Finally, let me comment on the values of χ_t that we have found. From Table I we see that typically $\chi_t \approx 0.02$, or, when we restore units, $\chi_t = (115\ \text{MeV})^4$. This is much smaller than the pure-gauge-field estimates: For example $\chi_t = (280\ \text{MeV})^4$, $(380\ \text{MeV})^4$, and $(240\ \text{MeV})^4$, from Ref. 9, Fox *et al.*,[18] and Cornwall and Soni,[19] respectively.[20] It remains a challenge for QCD to show how quark loops can accomplish this dramatic reduction in the topological susceptibility.

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