

CP Nonconservation at the Z^0 Peak

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(Received 10 June 1986)

The measurement of a nonvanishing asymmetry $\alpha \equiv [\Gamma(s\bar{b}) - \Gamma(\bar{s}b)]/[\Gamma(s\bar{b}) + \Gamma(\bar{s}b)]$ would signal CP nonconservation in Z^0 decays. We study here this effect within the standard model. In the three-generation case, the α value comes out small because of the effective degeneracy of u and c quarks at these high energies. In the four-generation case, results are encouraging for the CERN e^+e^- collider LEP: One could have a branching ratio of this flavor-changing decay to the flavor-conserving one of $\sim 10^{-6}$ and reach α values near unity.

PACS numbers: 13.38.+c, 11.30.Er, 12.15.Ji, 13.65.+i

In this Letter we would like to discuss the prospects of CP -nonconserving effects at the e^+e^- colliders on top of the Z^0 peak. The observable proposed is the asymmetry in the flavor-changing decay of the Z^0 to $b\bar{s}$ and $s\bar{b}$,

$$\alpha = \frac{\Gamma(Z^0 \rightarrow s\bar{b}) - \Gamma(Z^0 \rightarrow b\bar{s})}{\Gamma(Z^0 \rightarrow s\bar{b}) + \Gamma(Z^0 \rightarrow b\bar{s})}, \quad (1)$$

as well as its trivial generalization to other $q\bar{q}$ pairs such as $d\bar{s}$, $b\bar{d}$. For reasons which will become clear below, the prospects are better for down quarks in the final state, at least in the standard theory. The considerations of up-quark final pairs such as $\bar{t}c$, $t\bar{c}$ could be appropriate in other models.

It is of crucial importance to look for CP -nonconserving effects in physical situations other than the K^0 - \bar{K}^0 system. In this context, experimental studies¹ in the B - \bar{B} or other heavy-flavor systems are interesting, although the theoretical expectation is rather gloomy (in the standard model). Those proposals keep the same profile as the K - \bar{K} ones insofar as they go either through the analysis of the mass matrix² or through exclusive or semi-inclusive decays³ to hadronic final states.

However, from now on it is imperative to find new CP -nonconservation guides appropriate to the colliders and supercolliders. In high-energy collisions, the natural asymptotic states are jets, not hadronic states. The question is to find out the optimal observables at every energy range. Furthermore, one should try to isolate observables in which the ingredients are intrinsic to the assumed basic theory responsible for CP nonconservation.

Let us start this program in the framework of the standard electroweak theory with three generations. There the CP -nonconserving effects stem⁴ from flavor

mixing of quarks nondegenerate in mass. One should be aware that the high statistics of future colliders does not necessarily imply good prospects for CP searches. In fact, with growing energy, although many channels open, the effective quark masses vanish and the CP -nonconserving effects are correspondingly small. The situation could change drastically in theories where CP nonconservation appears at high scales.

At the supercollider energies, the appropriate and reasonable observables should no longer explicitly depend on the different flavors. However, in the first generation of accelerators, it is perhaps still possible to identify the flavors at the jet level. Let us consider the CERN e^+e^- collider LEP I to start with. This Z^0 factory will provide several millions of Z^0 per year. It is worth our having a look at its flavor-changing decays and seeing whether there is already any CP effect at all in the standard theory.

The answer is affirmative. The interference of amplitudes with different weak phases as well as different absorptive parts provides a contribution at the one-loop level (Fig. 1) for these flavor-changing decays. The absorptive parts are available because the energy involved is $M_{Z^0} \geq 2m_q$, where m_q is the mass of the quark in the loop. These intermediate quarks may now be on the mass shell, a situation to be contrasted with typical low-energy analyses. As a consequence, different absorptive contributions appear in our case as a result of unitarity. Note that such a CP -nonconserving effect does not need an interference between topologically or chirally different diagrams; while the weak phase changes sign in going to the conjugate channel, the absorptive parts do not.⁵

Let us consider the flavor-changing decays of the Z^0 . To our knowledge, the corresponding rates have been estimated⁶ in view of the future colliders, but no

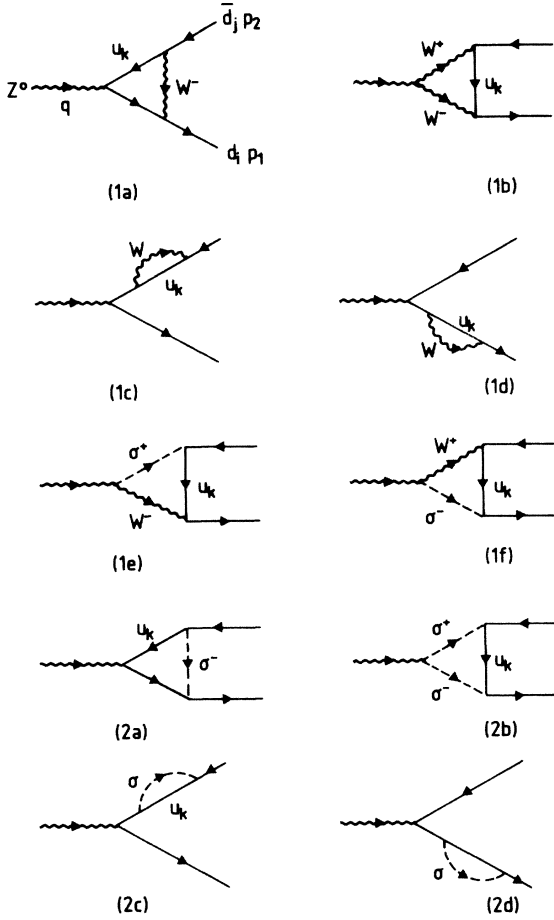


FIG. 1. Diagrams responsible for the flavor-changing decay $Z^0 \rightarrow d_i \bar{d}_j$ in the 't Hooft-Feynman gauge. External particles and momenta are indicated in diagram 1a.

attention has been paid to the CP -nonconserving effects in that context.

In the 't Hooft-Feynman gauge, the diagrams responsible for the flavor-changing vertex $Z^0 d_i \bar{d}_j$ are shown in Fig. 1. In a self-explanatory notation, the invariant T -matrix element is given by

$$T = \frac{g}{\cos\theta_w} \bar{u}_i(p_1/\lambda_1) \Gamma^\mu v_j(p_2, \lambda_2) \epsilon_\mu(q, \lambda) \quad (2)$$

where, in the limit of zero external quark masses, the

$$\Gamma(Z^0 \rightarrow d_i \bar{d}_j) = \frac{g^2}{\cos^2\theta_w} M_{Z^0} \frac{1}{8\pi} \left| \frac{g^2}{2^5 \pi^2} \left| \sum_k \xi_k I(r_k, s) \right|^2 \right. \quad (4)$$

With functions F_2 and F_3 defined by $F_2 \equiv I_2 - I_1$, $F_3 \equiv I_3 - I_1$, and by use of the unitarity of the Kobayashi-Maskawa (KM) matrix in three generations $\xi_1 + \xi_2 + \xi_3 = 0$, the rate ratio between flavor-changing and flavor-conserving decays can be written as

$$R = \frac{\Gamma(Z^0 \rightarrow d_i \bar{d}_j)}{\Gamma(Z^0 \rightarrow d_i \bar{d}_i)} = \frac{G_F^2 M_W^4}{\pi^4 \{1 + (1 - \frac{4}{3} \sin^2\theta_w)^2\}} |\xi_2 F_2 + \xi_3 F_3|^2 \simeq 4 \times 10^{-5} |\xi_2 F_2 + \xi_3 F_3|^2. \quad (5)$$

induced current at the one-loop level is of the $V-A$ type:

$$\Gamma^\mu = \frac{g^2}{2(4\pi)^2} \sum_k \xi_k I(r_k, s) \gamma^\mu L. \quad (3)$$

In Eq. (3), g is the $SU(2)$ gauge coupling, $\xi_k \equiv U_{ki}^* U_{kj}$ where U_{ki} is the flavor matrix element in generation space and k labels the quark running in the loop, $s \equiv (M_Z/M_W)^2$, $r_k \equiv (m_k/M_W)^2$, and L is the left-handed projector $L \equiv (1 - \gamma_5)/2$. The limit of the zero quark mass for external legs is well defined in each diagram by itself, except for the self-energy ones in which one has to sum diagrams (1c) + (1d) and (2c) + (2d) of Fig. 1.

The calculation of the flavor-changing form factor $I(r_k, s)$ contains interesting features and it will be presented elsewhere for real and virtual Z^0 . We present here the points relevant to our discussion for Z^0 decays. The diagrams (2) in Fig. 1 are at least of order r_k , whereas this is not so *a priori* for the diagrams (1). However, because of the unitarity of the flavor-mixing matrix, the pieces independent of r_k are irrelevant in the effective vertex Γ^μ [Glashow-Iliopoulos-Maiani (GIM) cancellation]. The final result for the flavor-changing form factor is finite. In the 't Hooft-Feynman gauge the cancellations operate in the following form. Diagrams (1e) and (1f) of Fig. 1 are finite. In diagrams (1a), (1b), and (1c) + (1d), the pole term in the dimensionally regularized amplitude is quark-mass independent and so it is GIM-cancelled. The divergences in diagrams (2) cancel among themselves for a given quark intermediate state (fixed k).

We have checked from our result that the limit $s \rightarrow 0$ reproduces the results already known in the literature.⁷ In this low-energy limit, the amplitudes do not provide any absorptive parts. For our purposes, we have to analyze the form factor for $s = (M_E/M_W)^2$ and its behavior changes drastically for on-shell Z^0 bosons. Diagrams (1a) and (2a) of Fig. 1 have then an absorptive part for quark masses such that $r < s/4$, with a value which is r dependent. Diagrams (1b), (1e), (1f), and (2b) are real as long as $s < 4$, as corresponds to the actual physical situation. Diagrams (1c) + (1d) and (2c) + (2d) are real.

The result for the decay widths reads

If we take, for instance, $m_u \simeq 0$, $m_c \simeq 1.5$ GeV, $m_t \simeq 45$ GeV, the function F gets values $F_2 = (-6 + i7) \times 10^{-4}$, $F_3 = -0.26 + i0.73$. The corresponding results for R are shown in Table I. Only channel $s\bar{b}$ offers some prospects of having a branching ratio close to the experimental possibilities of future e^+e^- colliders.

With the same parametrization, the asymmetry

$$\alpha \equiv \frac{\Gamma(Z^0 \rightarrow d_i \bar{d}_j) - \Gamma(Z^0 \rightarrow d_j \bar{d}_i)}{\Gamma(Z^0 \rightarrow d_i \bar{d}_j) + \Gamma(Z^0 \rightarrow d_j \bar{d}_i)}$$

is given by

$$\begin{aligned} \alpha &= \frac{-4 \operatorname{Im}(\xi_2 \xi_3^*) \operatorname{Im}(F_2 F_3^*)}{|\xi_2 F_2 + \xi_3 F_3|^2 + |\xi_2^* F_2 + \xi_3^* F_3|^2} \\ &\simeq -2 \frac{\operatorname{Im}(\xi_2 \xi_3^*) \operatorname{Im}(F_2 F_3^*)}{|\xi_2 F_2 + \xi_3 F_3|^2}, \end{aligned} \quad (6)$$

where the last expression is obtained by considering ξ_2 and ξ_3 approximately real in the denominator. Finally,

$$\alpha \simeq 10^{-12} \sin \delta / R \quad (7)$$

for present accepted values of the Kobayashi-Maskawa mixing angles.

From the table and Eq. (7) it is observed that the rate for a given flavor-changing channel and the CP -nonconserving effect are correlated: When the rate grows, the asymmetry diminishes by the same amount. This is a feature of the Kobayashi-Maskawa model with three generations, because in it the number of invariant parameters associated with CP nonconservation

TABLE I. Estimates of the branching ratio R of the flavor-changing decays $Z^0 \rightarrow d_i \bar{d}_j$ to the flavor-conserving decay in the standard theory with three and with four generations, as well as the corresponding CP asymmetries α . s_δ is the sine of the KM phase; s_ϕ is the sine of a combination of the phases for four generations.

	$R^{(3)}$	$R^{(4)}$	$ \alpha^{(3)} $	$ \alpha $	$ \alpha^{(4)} $
$Z^0 \rightarrow d\bar{s}$	10^{-11}	10^{-8}	$10^{-1} s_\delta$		s_ϕ
$Z^0 \rightarrow d\bar{b}$	10^{-9}	10^{-8}	$10^{-3} s_\delta$		s_ϕ
$Z^0 \rightarrow s\bar{b}$	10^{-7}	10^{-6}	$10^{-5} s_\delta$		s_ϕ

is a unique universal one. Furthermore, the small value of F_2 (compare with $F_3!$) is understood as a consequence of the near degeneracy, at these energies, of the u and c quarks. Remark that the CP -nonconserving effect is always proportional to the combination of form factors $\operatorname{Im}(F_2 F_3^*)$. Writing

$$\operatorname{Im}(F_2 F_3^*) = \operatorname{Im}(I_1 I_2^* + I_2 I_3^* + I_3 I_1^*), \quad (8)$$

one sees that the result is antisymmetric under the exchange of any pair of intermediate quarks. As long as the different I_k are the same functions of m_k [$I_k = I(m_k)$], Eq. (8) gives zero in the limit when any two of the three quarks are degenerate. This is the reason why the measurable asymmetry comes out small in the standard model: At the Z^0 energies, m_c and m_u behave as if almost degenerate. Furthermore, in the limit when $m_k \ll M_W$, we can extract the explicit dependence of the observable by performing an expansion of $I(r, s)$ in r , leading to

$$\operatorname{Im}(F_2 F_3^*) \simeq \frac{1}{2} \operatorname{Im}\{I'(0, s) I''(0, s)\} (r_1 - r_2)(r_2 - r_3)(r_3 - r_1), \quad (9)$$

where the derivatives I' and I'' are independent of flavor.

In view of the above remarks, it is relevant to consider what the situation would be in the standard electroweak theory, but with four generations of quarks. When we neglect F_2 for the above reasons, the CP -nonconserving observable gives

$$\alpha \simeq -2 \frac{\operatorname{Im}(\xi_3 \xi_4^*) \operatorname{Im}(F_3 F_4^*)}{|\xi_3 F_3 + \xi_4 F_4|^2}, \quad (10)$$

where $\operatorname{Im}(\xi_3 \xi_4^*)$ is not a unique universal quantity: It now depends on the channel considered.

In order to illustrate the situation, let us still consider $s\bar{b}$ as the final state. For $m_{t'} \simeq M_W$, we have $F_4 = -0.50 + i0.75$, so that $\operatorname{Im}(F_3 F_4^*) \simeq -0.2$, which is a typical value increasing somewhat with the mass of t' , t' being the $\frac{2}{3}$ -charged quark of the fourth generation. We have analyzed the four-generation mixing matrix using the parametrization of Botella and Chau.⁸

Two of the new angles are constrained, whereas the other one, s_u , remains free. One gets the following behavior:

$$R(Z^0 \rightarrow s\bar{b}) \simeq 4 \times 10^{-5} s_u^2 c_u^2 \lambda^2 |F|^2, \quad (11)$$

where $F \sim F_3, F_4$ and λ is the parameter which, in the Wolfenstein parametrization⁹ of the KM matrix, takes the value 0.22. This expression is to be compared with the corresponding one for three generations where

$$R(Z^0 \rightarrow s\bar{b}) \simeq 4 \times 10^{-5} \lambda^4 |F_3|^2. \quad (12)$$

In this channel $s\bar{b}$ one gets an "enhancement factor" when going to four generations, which is $(s_u c_u / \lambda)^2$. In the most favorable case, the flavor-changing rate to $s\bar{b}$ would increase by an order of magnitude. A similar situation occurs in the $d\bar{b}$ channel; however, the channel $d\bar{s}$ now has a factor $(c_u / \lambda)^4$, which could mean an enhancement of three orders of magnitude.

It is the CP asymmetry itself that increases remark-

ably in the four-generation case. In fact, one sees from Eq. (10) that in this case α could reach values near unity because $|\xi_3| \sim |\xi_4|$ and $|F_3| \sim |F_4|$ and one has appropriate relative phases between ξ_3 and ξ_4 as well as $\text{Im}(F_3 F_4^*) = -0.2$. As a consequence, in the limit where both the u and c quark masses are effectively zero ($F_2 \sim 0$), the CP -nonconserving effect does not disappear and α could reach values near unity. The relevant product $\alpha R \approx 10^{-6}$ for four generations in the standard model is probably a result not far from the capabilities of the next generation of e^+e^- colliders.

We have focused our analysis of flavor-changing Z^0 decays on final states made out of down-quark pairs. The reason is that in order to obtain a sizable asymmetry in the standard model with (three) four generations, one needs at least two quarks with masses comparable to $M_{W,Z}$. This does not need to be the case in other models. Any other CP -nonconserving model which is independent of the quark-mass difference $m_c^2 - m_u^2$ should also give predictions not outside the realm of the first generations of colliders. This could be the case, for instance,¹⁰ for left-right-symmetric models, Higgs models of CP nonconservation, and supersymmetric models with phases in the gauge-fermion and scalar-quark mass matrices. The corresponding detailed analysis should be done in order to determine if and when the effect is phenomenologically interesting.

We have benefitted from many discussions on this topic with F. J. Botella, and we acknowledge useful comments by A. De Rújula and J. Ellis. One author (J.B.) is indebted to the CERN Theoretical Physics Division for hospitality, and another (A.S.) is grateful for a Fellowship from the Plan de Formación del Personal Investigador (MEC-Spain). This work has been partly supported by Comisión Asesora de Investigación Científica y Técnica, Spain, under Contract No. 1740-82.

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