Comment on "Anomalously Broad Spin Distributions in Sub-barrier Fusion Reactions"

In a recent Letter, Vandenbosch *et al.*[1] reported on large anisotropies in fission-fragment angular distributions following low-energy heavy-ion fusion reactions. These authors presented their results as evidence for anomalously large values of $\langle l^2\rangle$, the average of the square of the compound nuclear spin. In this Comment, we argue that this is not a very likely explanation. We also suggest further measurements which could clarify this problem.

A simple estimate of the ratio of fission fragments emitted at 180° and 90° gives

$$\frac{W(180)}{W(90)} \simeq 1+\frac{\langle l^2\rangle}{4K_0^2},$$

where K_0^2 characterizes the distribution of the spin projection along the symmetry axis at the saddle point for fission. Vandenbosch *et al.*[1] argue that K_0^2 is known from fission resulting from α-capture reactions forming the same compound nucleus, so that the observed asymmetry in the heavy-ion cases determine $\langle l^2\rangle$. These values, however, are at least twice as large as those estimated from an analysis of the measured total fusion cross sections.[1] An alternative explanation could be that the heavy-ion collisions at low energy produce an extended, isomeric state which mimics the fission yield of the compound nucleus but has about half the effective moment of inertia.

It should be possible to resolve this issue by extension of the fission measurements to even lower bombarding energies. This is because the quantity $\langle l^2\rangle$ (and all moments of the compound-nucleus spin distribution) has a constant, model-independent value in the limit where the fusion cross section depends exponentially on energy.

The relevant expression for the angular distribution of fission fragments following fusion is given by

$$W(\theta)=\frac{\pi\hbar^2}{2\mu E}\sum_{l=0}^{\infty}(2l+1)T_l\sum_{K=-l}^{l}|Y_{lK}(\theta,0)|^2\exp\left(-\frac{K^2}{2K_0^2}\right)\left[\sum_{K=-l}^{l}\exp\left(-\frac{K^2}{2K_0^2}\right)\right]^{-1}.$$

Here μ is the reduced mass of the reaction partners, E is the bombarding energy in the center-of-mass frame, and T_l is the fusion probability for partial wave l. This assumes that the compound system decays only by fission.[1] Thus integrating over all angles gives the total fusion cross section

$$\sigma_f(E)=\int W(\theta)\,d\Omega=\frac{\pi\hbar^2}{2\mu E}\sum_l(2l+1)T_l\equiv\sum_l\sigma_l,$$

which, of course, is independent of K_0^2. The moment $\langle l^2\rangle$ is defined with respect to the distribution σ_l/σ_f.

In standard fusion calculations based on barrier penetration, the dependence of T_l on E and l has the form

$$T_l=T\left\{\left(V_b+\frac{\hbar^2 l(l+1)}{2\mu r_b^2}-E\right)\epsilon^{-1}\right\},$$

where V_b is the barrier height, r_b is its radius, and ϵ is related to its curvature. This basic form can also be used as the starting point when one allows for a distribution of barriers due to dynamic coupling effects. The main point to be emphasized here is that E and l^2 are essentially interchangeable variables, as one would expect from the form of the radial Schrödinger equation. Also ϵ can be defined empirically from the logarithmic derivative of $E\sigma_f(E)$ at low energy.

Following this line of reasoning, one can obtain the relation

$$\langle l^2\rangle=\frac{1}{\sigma_f E}\frac{2\mu r_b^2}{\hbar^2}\int_{-\infty}^{E}dE'\,E'\sigma_f(E'),$$

and similar relations for all moments of the spin distribution. This expresses in a concrete way what one would intuitively expect; namely, the energy dependence of σ_f determines the spin content of σ_l. For this reason it is difficult to imagine how any model, or parametrization, which fits $\sigma_f(E)$ over a range of energies could fail to give reasonable values of $\langle l^2\rangle$. Thus, it seems more plausible that the anisotropy should be taken as evidence for a small value of K_0^2.

Moreover, well below the barrier where $E\sigma_f(E)$ depends exponentially on E, the spin distributions become independent of energy. In particular, the relation above yields the limiting value of $\langle l^2\rangle=(2\mu r_b^2/\hbar^2)\epsilon$. The values of $\langle l^2\rangle$ presented in Ref. 1 are up to 5 times larger than this limit. It would therefore be of great interest to extend the measurements to even lower energies.

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C. H. Dasso,[a] H. Esbensen, and S. Landowne
Argonne National Laboratory
Argonne, Illinois 60439

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[a]Permanent address: The Niels Bohr Institute, Copenhagen, Denmark.

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