Dissipative Flow of Liquid ^{4}He in the Limit of Absolute Zero

K. W. Schwarz
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
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The problem of dissipative superflow in the absence of normal-fluid friction is considered within the hydrodynamic approximation. It is found that microscopic surface roughness gives rise to new mechanisms of dissipative vortex motion and vortex regeneration, accounting not only for the critical velocities at which self-sustaining dissipation becomes possible, but also for the occurrence of discrete dissipative events in oscillating flow.

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Superfluid ^{4}He behaves as an irrotational ideal fluid which may contain quantized vortex filaments. In addition, it supports a gas of elementary excitations (the normal fluid) which can exert an external frictional force on the fluid near the core of a quantized vortex. This essentially hydrodynamical description of superfluid dynamics is valid down to a scale comparable to the healing length of the superfluid wave function, a distance which both theory and experiment suggest to be on the order of a few angstroms.

Superflow characteristically becomes dissipative above some critical velocity, apparently contradicting the concept of ideal-fluid behavior. This is observed even at low temperatures where the role of the normal fluid becomes insignificant. The question addressed in this paper is whether such a phenomenon can be understood strictly within the hydrodynamical context, or whether it requires us to invoke some new mechanism, such as the direct quantum-mechanical nucleation of quantized vortices. Although this question has been widely discussed since the early article of Feynman,[1] the issue has remained unresolved, and in fact no credible models have emerged on either side. Recent developments once again compel us to face this issue. On the one hand, a successful description of dissipative superflow (superfluid turbulence) above 1 K has been developed within the hydrodynamical approximation.[2,3] The normal-fluid frictional force plays an important role in this model, and the question arises whether the theory can sensibly be extended to low temperatures where such forces disappear. On the other hand, a recent remarkable paper[4] reports the observation of dissipation in oscillating flow through a small orifice at very low temperatures. While dissipation in ^{4}He at low temperatures has been observed previously, these authors offer persuasive evidence that in their experiment it occurs in the form of discrete, more or less identical dissipative events, such as might arise from the repetitive (but not necessarily periodic) nucleation and growth of quantized vortices. Since the dimensions of their orifice are still several orders of magnitude larger than the healing length, it would seem natural to look first for a hydrodynamic explanation. In what follows, I shall concentrate on the problem of dissipation in steady flow through an infinitely long channel, which presumably corresponds to the state of superfluid turbulence. The problem of oscillating flow through an orifice is more complicated, and at present only a qualitative discussion is possible.

As originally pointed out by Anderson,[5] any mechanism which causes vortices to move across the flow direction is intrinsically dissipative. To understand that this is a hydrodynamic effect, let us recall that the equation of motion of a frictionless, incompressible, irrotational fluid has a first integral (the Bernoulli equation)

$$\rho\, \partial\Phi/\partial t + p + \tfrac{1}{2}\rho v^2 = f(t), \tag{1}$$

where ρ is the density, p is the pressure, and the velocity $\mathbf{v}$ is related to the velocity potential Φ by $\mathbf{v} = \nabla\Phi$. The function $f(t)$ has no dynamical consequences and can be set equal to zero. The energy flux in the fluid has the form $\mathbf{v}(p + \frac{1}{2}\rho v^2)$, so that Eq. (1) relates changes in Φ to the energy flow. Consider a vortex filament changing its configuration [Figs. 1(a) and 1(b)] while subjected to a fixed applied flow field $\mathbf{v}_a$. As shown in Fig. 1(c), a surface S^* can be drawn to define a singly connected volume V^* in which Eq. (1) can be applied. Note that S^* excludes the vortex core and that it generates a local barrier B which makes Φ single valued, and across which Φ is discontinuous by the quantum of circulation $\kappa = h/m_4$. It then follows directly[6] from Eq. (1) that if a vortex changes its configuration by any process whatsoever, thus changing the local barrier, a well-defined amount of energy

$$\Delta E = -\rho\kappa \mathbf{v}_a \cdot \Delta \int d\mathbf{B} \tag{2}$$

flows into V^* through S_1 and S_2, and out of V^* into that part of S^* surrounding the barrier. Here the normal of $d\mathbf{B}$ is oriented toward the direction in which the vortex carries fluid through the barrier. The energy dissipated according to Eq. (2) must be provided by the mechanism[7] which is keeping the fluid moving. I emphasize that there is nothing in the argument lead-

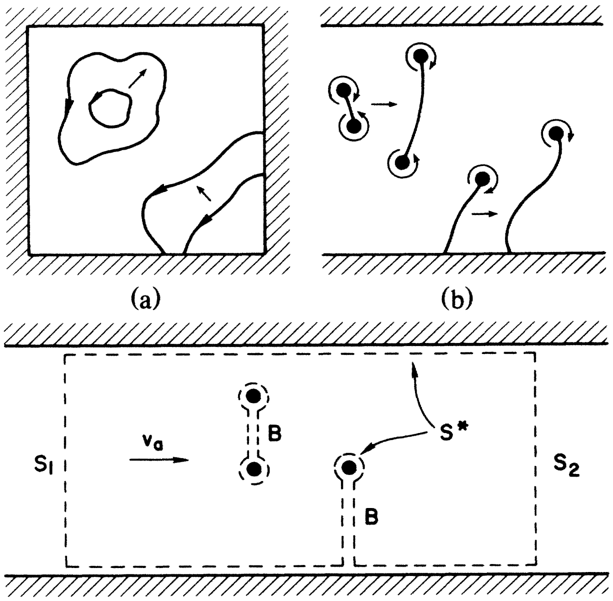

FIG. 1. Dissipative vortex motion: (a) head-on view, flow into figure; (b) side view, flow from left to right; (c) construction for calculating energy flows.

ing to Eq. (2) to explain why dissipative vortex motion actually occurs. This requires the addition of physical processes going beyond the simple ideal-fluid picture.

The language of ^{4}He physics differs slightly from that of fluid mechanics. Instead of Φ one uses the phase $\phi = m_4\Phi/\hbar$. In addition, since the excitation gas behaves as a separate hydrodynamic entity, only a part of the fluid will act as an ideal irrotational fluid. This results in the replacement of ρ by the superfluid density ρ_s and of $p+\frac{1}{2}\rho_s v_s^2$ by μ/m_4, where μ is the chemical potential per superfluid atom. Evaluated between 2 and 1, Bernoulli's equation in the rest frame of the channel then takes the form

$$\hbar\,\partial(\phi_2-\phi_1)/\partial t = -(\mu_2-\mu_1), \tag{3}$$

which has the appearance of the Josephson frequency relation. Considerable confusion has arisen from the notion that, because it looks quantum mechanical, Eq. (3) has a dynamical content beyond that contained in Eqs. (1) or (2). Certain disclaimers should therefore be kept in mind. First, the validity of this equation as applied to ^{4}He is not in need of experimental verification, as claimed for instance in Ref. 4. It follows directly from the well established dynamical properties of the superfluid. Second, this "phase slippage" equation merely describes the energy flow which occurs when vortices appear and grow. It does not provide any information about how and why they do so, and therefore again it does not explain why dissipative vortex motion (i.e., phase slippage) actually takes place. In particular, Eq. (3) does not imply that vortex-mediated dissipation should be "quantized," or that it should involve some characteristic "Josephson frequency." Such effects would require an orifice on the order of angstroms, several orders of magnitude smaller than any which have yet been studied, and would presumably involve a different kind of physics from that which is relevant here.

The preceding discussion is meant to make it clear that the central issue in understanding dissipative superflow is to identify the mechanisms which cause quantized vortices to appear and to move so as to dissipate energy in the sense of Eq. (2). Until now, this has been achieved only for situations in which the normal-fluid friction plays an important role. The motion of a quantized vortex filament is accurately approximated by[8]

$$\dot{\mathbf{s}} = \beta\mathbf{s}'\times\mathbf{s}'' + \mathbf{v}_s + \alpha\mathbf{s}'\times(\mathbf{v}_n - \mathbf{v}_s + \beta\mathbf{s}'\times\mathbf{s}''), \tag{4}$$

where $\mathbf{s}(\xi,t)$ describes the configuration of the filament, the primes denote the derivative with respect to the arc length ξ, α is a known temperature-dependent friction constant, $\mathbf{v}_n$ and $\mathbf{v}_s$ are the local average normal and superfluid velocity fields, and $\beta = (\kappa/4\pi)\times\ln(c_1/s''a_0)$, with c_1 a constant of order 1 and $a_0 \approx 10^{-8}$ cm the effective core radius of the filament. Nonlocal vortex interactions are described by the statement that at sufficiently close distances vortices will reconnect to each other or to boundaries. Application of these ideas to a vortex tangle in the situation where α and $\mathbf{v}_n - \mathbf{v}_s$ are large[3] leads to a self-sustaining dissipative state in which the friction makes loops grow outward so as to give $\Delta E > 0$ in Eq. (2), the loops eventually annihilating at the boundaries, while the supply of such loops is constantly regenerated through the process of line-line reconnection.[9] Although very successful in the regime to which it applies, this hydrodynamic model of dissipative superflow obviously fails when α or $\mathbf{v}_n - \mathbf{v}_s$ become small, since the mechanism causing nonconservative loop growth no longer operates.

The main result to be reported here is that when the hydrodynamic formalism is extended to account for the inevitable presence of microscopic surface irregularities on the walls of the flow channel, a second set of processes leading to nonconservative vortex motion and vortex regeneration can be identified. These processes do not involve the normal-fluid friction α and therefore can give rise to self-sustaining dissipation in the zero-temperature limit, entirely within the fluid-dynamical context. Recall that a vortex filament terminating on a perfectly smooth surface will move along the surface without hindrance. When the end of the vortex encounters a bump, however, it will remain pinned there until it bows over to within some critical angle of the surface, at which point it will jump off and resume its motion.[8] Quantized vortices in ^{4}He will pin on bumps of only a few angstroms, so that in practice

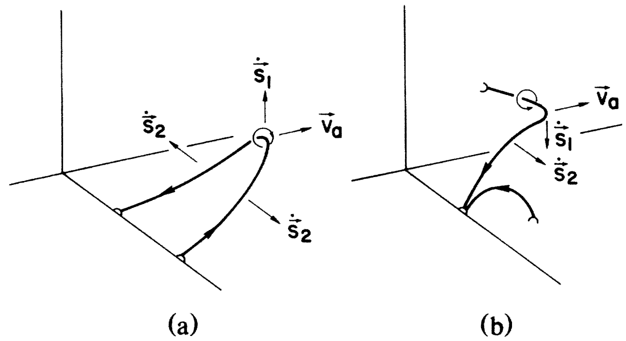

FIG. 2. Zero-temperature mechanisms for (a) dissipation, and (b) vortex regeneration.

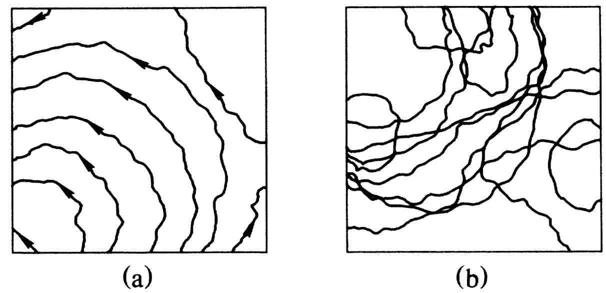

FIG. 3. (a) Quantitative calculation of the dissipative growth of a vortex across the channel. The vortex starts at the lower left corner, and the driving velocity is into the figure. The pinning and release of the vortex as it moves down the channel gives rise to the kinks propagating along the vortex. (b) Snapshot of the self-sustaining dissipative state close to v_c^*, showing that it consists primarily of isolated loops growing across. The vortices in unit length of the channel are projected onto a plane perpendicular to the flow direction.

this process is always expected to occur. Consider now a vortex loop pinned as shown in Fig. 2(a). As the applied velocity $\mathbf{v}_a$ (equal to $\mathbf{v}_s$ in the present instance) sweeps the loop downstream, the vortex spooling off the pinning site, the tip will develop an increasingly pronounced self-induced velocity $\dot{\mathbf{s}}_1$ as shown. The resulting upward distortion then generates the self-induced velocities $\dot{\mathbf{s}}_2$, which cause the loop to grow across the flow field in a dissipative manner. Quantitative numerical calculations [Fig. 3(a)] show that this is not a particularly delicate process, efficient dissipative motion developing whenever a vortex hangs up on a pinning site.

It is not difficult to see that the flow energy is dissipated by being fed into the growing vortex lines which then annihilate at the walls, presumably degrading into thermal excitations by some as yet not clearly understood process. It is simpler and more illuminating however to think about the momentum exchange between the superfluid and its environment. External forces such as mutual friction or pinning forces act on the vortices which in turn move so as to pass this momentum flux into the superfluid as a reaction force. Thus, the presence of quantized vortices can act to couple the superfluid to its environment in such a way as to decelerate it. For example, it is clear in Fig. 2(a) that the vortex is exerting a downstream force on the boundary via its interaction with the pinning sites. Conversely, the boundary must be exerting a retarding force on the superfluid via its interaction with the vortex flow field. The cross-stream motion developed by the vortex is the mechanism through which this momentum flux is passed out into the superfluid, thus decelerating it. From a general point of view, the broken translational symmetry allows the surface to couple to the vortices, causing it to exert a retarding force on the superfluid.

It should be noted that for a given driving velocity there is a minimum radius of curvature or loop size which can be involved in this growth mechanism. Furthermore, when a given vortex grows across, as in Fig. 3(a), it will eventually reconnect across the corners, forming loops which apparently are oriented in the wrong way to cause further dissipation. Hence once again one must face the question of how new macroscopic loops of the correct orientation and size to serve as growth centers are created. The surprising answer, discovered by direct numerical simulations, is that a vortex driven into a corner as shown in Fig. 3(a) will, under the combined effects of pinning and $\mathbf{v}_a$, undergo a complicated further development which results in its flipping over to form a small loop of the correct orientation to act as a new growth loop, provided that $\mathbf{v}_a$ is large enough. As illustrated in Fig. 2(b), the applied velocity first distorts the line so as to generate a self-induced motion $\dot{\mathbf{s}}_1$ which brings the line towards the bottom plane. The secondary velocity $\dot{\mathbf{s}}_2$ which then develops rotates the plane of the vortex. When it finally reconnects to the bottom plane, it forms a loop oriented in the right direction to serve as a growth center. Since a given loop will grow to impact several corners, this unexpected regeneration effect can cause the vortex population to increase from a single initial pinned vortex. Thus, above a critical driving velocity a steady dissipative state can be maintained in which vortex loops grow by pinning and release, and regenerate by reflecting back out of the corners that they are driven into. The mechanisms which characterize this zero-temperature dissipative flow state are entirely different from those which are active in normal-fluid–driven turbulence, but they too are a straightforward consequence of classical vortex dynamics.

Previously developed vortex-tangle codes,[8] modified to include vortex pinning and release, have been used to calculate the properties of the zero-temperature dissipative state. In the computations, a rough surface is parametrized by the angle θ (measured from the horizontal) at which the vortex depins and the distance λ it then moves before reattachment. A typical snapshot of the resulting state [Fig. 3(b)] shows that it differs considerably from homogeneous, normal-fluid–driven turbulence in its appearance.[3] In an infinitely long channel, this state cannot sustain itself below a critical velocity given approximately by

$$v_c = v_c^*(\theta, \lambda/D)(\kappa/4\pi D)\ln(c_1 D/a_0), \tag{5}$$

where D is the channel size, and the dimensionless velocity v_c^* is a function of θ and λ, as well as the channel geometry. The dependence of v_c^* on θ and λ was found to be rather weak, the calculated values varying from $v_c^* \approx 25$ when $\theta = 25°$ and $\lambda/D = 0.13$ to $v_c^* \approx 15$ when $\theta = 5°$ and $\lambda/D = 0.03$. These results may be compared to the temperature-independent, pure-superflow critical velocity measured by Baehr, Opatowsky, and Tough[10] in 1.34×10^{-2}-cm-i.d. glass channels. Their value of 1.5 cm/sec translates into $v_c^* = 18$, in excellent agreement with the values calculated here from first principles. One may conclude that the new fluid-dynamical mechanisms reported here are not only of conceptual interest, but in fact allow us to calculate the properties of the zero-temperature dissipative state to a reasonable approximation.

The zero-temperature dissipative mechanisms supplement rather than replace those driven by the normal-fluid friction. Thus, they are also expected to be important at high temperatures, particularly when $\mathbf{v}_n - \mathbf{v}_s$ is small while $\mathbf{v}_n$ and $\mathbf{v}_s$ are large, a situation for which the frictional term in Eq. (4) becomes relatively unimportant. Preliminary calculations indicate that at least some of the great complexities[11] exhibited by superfluid turbulence above 1 K arise precisely because there exist these two simultaneously operating and interacting sets of processes. A great deal of further work will clearly be required to sort out these complications.

Finally, I note that the present findings provide a natural explanation of the observations of Avenel and Varoquaux,[4] entirely within the hydrodynamic context. To see this, it is important to realize that the critical velocity $(\kappa/2\pi D)\ln(b/a_0)$ for vortex depinning[12,13] is about an order of magnitude smaller than the critical velocity [Eq. (5)] for self-sustaining dissipation. Below the depinning velocity, vortices pinned in the corners cannot move and there is no dissipation. Just above the depinning velocity, they can break free and move across the orifice as in Fig. 3(a), but they will now repin in the opposite corners rather than regenerate to maintain a steady state. In an oscillating flow, vortices will therefore move back and forth across the orifice in discrete stages, dissipating approximately the same amount of energy $\Delta E = \rho_s \kappa v_a A$ each time, where A is the cross-sectional area of the orifice. This is exactly the result reported in Ref. 4. While the explanation given here is somewhat idealized, its plausibility raises the exciting prospect that dissipative dynamics at the individual-vortex level can be explored by comparing fluid-dynamical calculations with experiments of the Avenel-Varoquaux type. On the other hand, there appears to be no obvious justification for thinking that some new quantum nucleation process is at work here.

In conclusion, a consideration of microscopic surface roughness has led to the discovery of new mechanisms of dissipative vortex motion and vortex regeneration in superfluid ^{4}He. These mechanisms do not depend on normal-fluid friction and therefore operate in the zero-temperature limit. In addition to explaining the existence of self-sustaining dissipation at low temperatures, they seem to account for the occurrence of discrete dissipative events in oscillating flows, and for some of the complicated features of superfluid turbulence at higher temperature.

Brewer[14] has proposed an interesting alternative explanation of the Avenel-Varoquaux observations, based on the idea that their experiment contains an additional flow path.

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