Fusion Reactions of Polarized Deuterons

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Polarized and unpolarized $d+d \rightarrow n+{}^3\mathrm{He}$ fusion reaction cross sections in the center-of-mass energy region of 20–150 keV are calculated in a distorted-wave Born approximation. The calculated unpolarized cross sections and the anisotropy of the angular distributions are within 20% of the experimental data. The polarized cross sections are found to be $\sim 7.7\%$ of the unpolarized ones despite the inclusion of the *D*-state component in ^{3}He. This shows that the idea of a "neutron-lean" reactor may still be feasible.

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It has been suggested[1,2] that the spin degree of freedom in the plasma fusion reactions should be explored in light of the possibility that it could be utilized to enhance the desired fusion reactions rates and/or suppress the undesired reactions. Examples are given for the reactions $t(d,n)^4\mathrm{He}$ and $^3\mathrm{He}(d,p)^4\mathrm{He}$.[1,2] The cross sections of these reactions at low energies are largest at the $J^p=\frac{3}{2}^+$ resonances of the compound nuclei ^{5}He and ^{5}Li. At such low energies (i.e., 107 keV for the deuteron in the reaction $d+t$ and ~ 430 keV in the reaction $^3\mathrm{He}+d$), the reactions are dominated by the relative *S* wave in the entrance channel. Thus the combined spins of the $d+t$ and the $d+{}^3\mathrm{He}$ system are equal to $\frac{3}{2}$ in these resonance reactions. Since the combined spins of $d+t$ and $d+{}^3\mathrm{He}$ can be either $s=\frac{3}{2}$ or $s=\frac{1}{2}$, this shows that the $s=\frac{1}{2}$ channels do not take part in these resonance reactions essentially. Hence, if one polarizes the spins of the incoming nuclei in the parallel direction, i.e., $s=\frac{3}{2}$, these two reaction cross sections will be enhanced by $\sim 50\%$, a substantial gain. It was further demonstrated[1] that of the four mechanisms considered the depolarization rate is sufficiently slower than the fusion reaction rate so that fusion reactors with polarized nuclei are feasible as far as the depolarization is concerned.

An example for the simultaneous suppression of the undesired reaction is the polarized reaction $d(d,n)^3\mathrm{He}$ [and $d(d,p)t$] which could admit the possibility of a "neutron-lean" fusion reactor.[3] There is experimental evidence of such suppression shown by the partial-wave analysis of the tensor analyzing power in the reaction $d(d,p)t$.[4] The suppression is understood to occur when the deuterons are polarized in parallel (i.e., $s=2$) so that at low energies where the relative *S* wave dominates, the Pauli principle suppresses the two deuterons from approaching each other to initiate the reaction. In addition, since the total spin of the final state $n+{}^3\mathrm{He}$ can only be $s=1$ or 0, the conservation of angular momentum and parity dictates that the exit channel must be in the relative *D* wave which leads to additional suppression because this has to occur via the spin-orbit and tensor forces instead of the stronger central force.

The latter example of the suppression of the polarized reaction $d+d$ has raised a controversial issue. The argument for the suppression is based on the implicit assumption that the *D*-state probability in ^{3}He and the deuteron are small and they do not contribute in any significant way. However, it was pointed out recently by Hofmann and Fick[5] that the inclusion of the *D*-state probability in ^{3}He will allow the strong central force to contribute to the $s=2$ channel and could yield large cross sections in the polarized $d+d$ reaction. In their refined resonating-group calculation,[5] they found that the polarized reaction is not suppressed at all. In fact, at $E_{\mathrm{c.m.}} \simeq 20$ keV, the ratio of the polarized to unpolarized cross sections is around 1. This seems to concur with the findings of the *R*-matrix analysis.[6]

Despite these results from the *R*-matrix analysis and the resonating-group calculation, we think that the problem deserves further critical scrutiny[7] because these results contradict the partial-wave analysis of the tensor analyzing power for $d(d,p)t$ at $E_d \simeq 290$ keV.[4] Furthermore, the central force contribution through the *D*-state probability in ^{3}He that Hofmann and Fick argued for is certainly allowed, but the magnitude of the matrix element in their calculation is inscrutably large. The reason that the matrix element $\langle {}^3D_2, {}^1D_2 | V_c | {}^5S_2 \rangle$ for the central force V_c between the $^5S_2$ *d*-*d* channel and the $^3D_2, {}^1D_2$ $n{}^3$He channel is smaller than the matrix element $\langle {}^1S_0 | V_c | {}^1S_0 \rangle$ is related to the smaller internucleon *D*-wave component in the incoming *d*-*d* system in the relative *S* wave,[7] a smaller *D*-wave projection[8] for the central potential onto the relative coordinate between *n* and ^{3}He, and a number of cancellations upon antisymmetrization of the total wave function. The cancellation arises from the permutation symmetry upon antisymmetrization of the spin, isospin, and spatial coordinates of the nucleons. This is quite transparent in the distorted-wave Born-approximation (DWBA) formalism where the antisymmetrization of the total wave function is treated analytically. On top of the above-mentioned reduction factors, the D-state probability in ^{3}He is merely $\sim 4\%$. Therefore, based on the above reasonings, we estimated[7] that the ratio of the polarized to the unpolarized cross section due to the central force should be 10^{-3}–10^{-4}. This is 3 to 4 orders of magnitude smaller than that obtained in the recent resonating-group calculation.[5]

In order to verify our point quantitatively, we carried out a DWBA calculation. Since the $d+d$ reaction cross sections are of the order of ~ 10 mb,[9] they are more than 2 orders of magnitude smaller than the elastic d-d and n-^{3}He cross sections.[10,11] This shows that the off-diagonal matrix elements in the coupled d-d and n-^{3}He channels are at least an order of magnitude smaller than the diagonal matrix elements. Therefore, the perturbative calculation such as DWBA should be reasonably good.

The DWBA formalism has been adopted by Boersma[12] to study the dd reactions at these low energies. Contrary to what is implied in the Reply of Fick and Hofmann,[13] there is no criticism of the suitability of DWBA for these reactions in Ref. 12. Instead, the success on the energy dependence of the cross sections is stressed and a more realistic dd wave function (which was not available at that time) beyond the Coulomb wave function used is called for improvement. The present work represents an attempt to answer this call for improvement.

Effective potentials (including the folded Coulomb potential in the entrance channel) are used to generate the distorted waves between the d and d in the entrance channel and between the n and ^{3}He in the exit channel. They are fitted to the d-d elastic-scattering phase shifts given by Thompson[14] and the n-^{3}He elastic cross sections.[11] The internal deuteron wave function ($2G$) is taken from the work of Chwieroth, Tang, and Thompson[15] and the ^{3}He wave function from the work of Bransden, Robertson, and Swan.[16] The same nucleon-nucleon interaction[17] as used in the recent resonating-group calculation[5] (called S in Ref. 5) is employed to calculate the matrix elements in the Born approximation so that the results can be compared with those in Ref. 5.

The unpolarized reaction cross section for the DWBA is written as

$$\sigma = \frac{4\pi^2}{k^2}\frac{1}{(2S_d+1)}\{[\langle {}^1S_0|V_c|{}^1S_0\rangle^2 + \langle {}^3P|V_c|{}^3P\rangle^2](1-P_D) + (2s+1)\sigma_{\text{pol}}\} \tag{1}$$

with the polarized cross section σ_{pol} written as

$$\sigma_{\text{pol}} = \frac{4\pi^2}{k^2}\frac{1}{(2s+1)}\{\langle {}^3D_2,{}^1D_2|V_T+V_{LS}|{}^5S_2\rangle^2(1-P_D) + {}_D\langle {}^3D_2,{}^1D_2|V_c+V_{LS}+V_T|{}^5S_2\rangle^2 P_D\}, \tag{2}$$

where k is the relative wave number in the entrance channel in the c.m. system. P_D is the D-state probability in ^{3}He which we take to be 4%. V_c, V_{LS}, and V_T denote the central, the spin-orbit, and the tensor interactions, respectively. The subscript D in Eq. (2) denotes the matrix elements for the D-state component in ^{3}He. The normalization of the relative wave functions in the entrance and the exit channels include the factors

$$(\mu k/4\pi^2\hbar^2)^{1/2}, \quad (\mu' k'/4\pi^2\hbar^2)^{1/2},$$

respectively [μ (μ') and k (k') are the reduced mass and the relative wave number in the entrance (exit) channel], so that the matrix elements are dimensionless. These matrix elements are calculated for the relative S and P waves in the entrance channel. The relative D wave is very small[5] at these energies and is therefore neglected.

The unpolarized cross sections and the anisotropy C_2/C_0 in the angular distribution $[d\alpha/d\Omega = C_0 + C_2P_2(\cos\theta) + \ldots]$ at 25 to 150 keV c.m. energies are plotted in Figs. 1 and 2, respectively, together with the experimental data and the results from Hofmann and Fick.[5] It is seen that our calculated unpolarized cross section and the angular anisotropy are generally

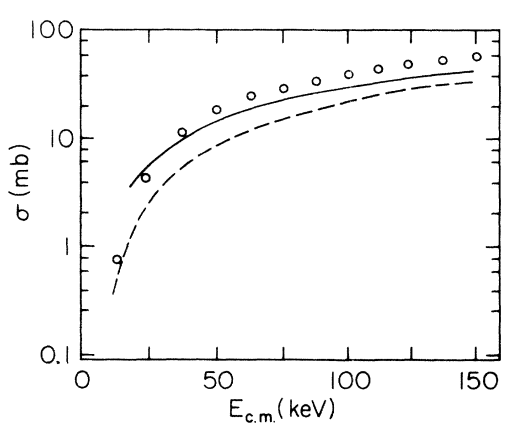

FIG. 1. The $d(d,n)^3$He reaction cross section as a function of the center-of-mass energy in the entrance channel. The dots are the experimental data. The solid line is from our DWBA calculation and the dashed line is from the resonating-group calculation in Ref. 3.

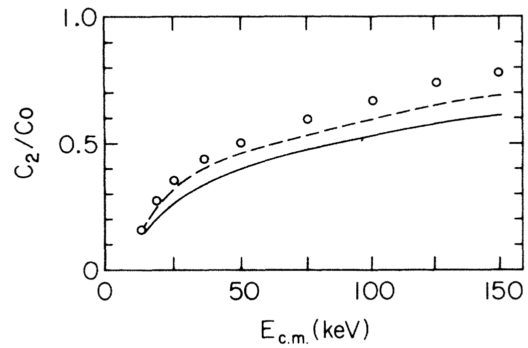

FIG. 2. The anisotropy C_2/C_0 vs the c.m. energy of the deuterons. The dots represent the experimental data. The solid line is our prediction and the dashed line is from Ref. 3.

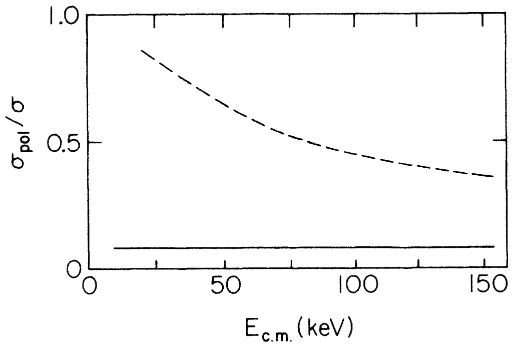

FIG. 3. The ratio of the polarized to unpolarized cross sections. The solid line is our prediction and the dashed line is from Ref. 3.

within 20% of the experimental data in this energy region which justifies our approximation based on the DWBA calculation. It is worthwhile to point out that our results on the unpolarized cross sections represent an improvement over those in the resonating-group calculation[5] which are roughly a factor of 2 smaller than the measured cross sections.

The results of the matrix elements (dimensionless) in different channels due to the various parts of the *NN* interaction are tabulated in Table I for the center-of-mass energies of the deuterons at 50, 100, and 150 keV. From Table I we have learned that the *P*-wave contribution constitutes $\sim 4\%$ of the total cross sections. Thus, it verifies that in this energy range the cross sections are indeed dominated by the *S* wave in the entrance channel.

It is interesting to note that the matrix elements in Table I scale with energy. This is the result of the low-energy limit. As a consequence, the cross section also scales with energy, according to Eq. (1). This agrees well with the experimental result in this energy region, i.e., 50–100 keV (see Fig. 1).

The presence of the *D*-state component in ^{3}He allows an $s=\frac{3}{2}$ wave function in ^{3}He which gives rise to an $s=2$ matrix element by the central force. According to Table I, its magnitude turns out to be an order of magnitude smaller than that of the *S*-wave, $s=0$ matrix element due to the central force. This is attributed to a number of cancellations due to the permutation symmetry among the interacting nucleons upon antisymmetrization which involves large numbers and reflects the fact that the *D*-wave composition between nucleons in different incoming deuterons is smaller than the corresponding *S*-wave composition at these low energies. Multiplied by $\sim 4\%$ for the *D*-state component in ^{3}He, we predict that the central force yields polarized cross sections which are $\sim 1.3\times10^{-4}$ times the unpolarized cross sections, in agreement with our earlier estimate. This is contradictory to the findings of the resonating-group calculation which purports that the central force is responsible for their $\sim 1{:}1$ ratio of the polarized to the unpolarized cross sections.

The spin-orbit and tensor forces, on the other hand, yield larger contributions to the polarized cross sections via the *D*-state component in ^{3}He (Table I). Especially, the tensor force gives the largest contribution since it could connect a *D* wave to an *S* wave. However, according to our calculation, the combined central and these spin-dependent forces yield polarized cross sections which are $\sim 7.7\%$ of the unpolarized ones in this low-energy region. This is certainly consistent with Ad'yasevich and Fomenko's analysis[4] which predicted the polarized cross section to be about 5% of the unpolarized one at $E_d \simeq 290$ keV. However, this is much smaller than those predicted by the reso-

TABLE I. The DWBA matrix elements (dimensionless) in different channels at three different energies due to the various parts of the *NN* interaction. V_c stands for the central force, V_{LS} the spin-orbit force, and V_T the tensor force. Matrix elements with subscript *D* are those with the *D*-state component in ^{3}He (the 4% *D*-state probability is not multiplied in these matrix elements).

	$\langle ^1S_0\lvert V_c\rvert ^1S_0\rangle$	$\langle ^3P\lvert V_c\rvert ^3P\rangle$	$\langle ^3D_2\lvert V_{LS}\rvert ^5S_2\rangle$	$\langle ^3D_2,{}^1D_2\lvert V_T\rvert ^5S_2\rangle$	$_D\langle ^3D_2,{}^1D_2\lvert V_c\rvert ^5S_2\rangle$	$_D\langle ^3D_2,{}^1D_2\lvert V_{LS}\rvert ^5S_2\rangle$	$_D\langle ^3D_2,{}^1D_2\lvert V_T\rvert ^5S_2\rangle$
50	0.29×10^{-1}	0.31×10^{-2}	0.25×10^{-2}	0.33×10^{-2}	-0.12×10^{-2}	0.61×10^{-2}	-0.15×10^{-1}
100	0.58×10^{-1}	0.63×10^{-2}	0.49×10^{-2}	0.66×10^{-2}	-0.24×10^{-2}	0.12×10^{-1}	-0.30×10^{-1}
150	0.86×10^{-1}	0.94×10^{-2}	0.74×10^{-2}	0.99×10^{-2}	-0.36×10^{-2}	0.18×10^{-1}	-0.45×10^{-1}

nating-group calculation[5] and the R-matrix analysis.[6] The ratios of the polarized to unpolarized cross sections at low energies are plotted in Fig. 3 together with those given by Hofmann and Fick.[5]

To unravel the situation further, we turn to the tensor analyzing powers which have certain bearings on the issue of the $s=2$ channel. It was pointed out in Ref. 5 that, without the $s=2$ channel contributing, the angular distributions of T_{20} and T_{22} (A_{zz} and A_{xx-yy}) should be symmetric with respect to 90° while those of T_{21} (A_{xz}) antisymmetric with respect to 90°. The experimental results[18] on the angular distributions of the tensor analyzing powers for the reaction $d(d_{\mathrm{pol}},n)^3\mathrm{He}$ at $E_d=320$ keV (160 keV in the center-of-mass system) reveals that the angular distributions of T_{20} and T_{22} are rather symmetric with respect to 90° and those of T_{21} are quite antisymmetric with respect to 90°. These are certainly contradictory to the implications of Hofmann and Fick in their article.[5] Results plotted on the contour diagrams at lower energies ($E_d=60$, 80, 105, and 205 keV)[18] show that the asymmetries from the expected symmetries without the $s=2$ channel contribution are even smaller. This suggests that the $s=2$ channel is relatively unimportant at such low energies.

Combined with the experimental results on the tensor analyzing powers and our DWBA calculation, we feel that the polarized ($s=2$) cross sections may well be strongly suppressed (at the 8%–15% level) despite the presence of the D-state component in ^{3}He. Thus a "neutron-lean" fusion reactor based on the suppression of the polarized d-d reactions may be attainable. However, more experimental data on the tensor analyzing powers and theoretical works involving the D-state component in the deuteron are still desired.

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