Generating Quantum Mechanical Superpositions of Macroscopically Distinguishable States via Amplitude Dispersion

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It is pointed out here that a coherent state propagating through an amplitude-dispersive medium will, under suitable conditions, evolve into a quantum superposition of two coherent states 180° out of phase with each other. The response of a homodyne detector to this superposition of macroscopically distinguishable states is calculated. Signatures which an experimentalist might look for in the homodyne detector's output in order to verify the generation of such states are described.

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Here the time evolution of an initial coherent state under the influence of an anharmonic-oscillator Hamiltonian is considered. The anharmonic term is taken to be proportional to $\hat{n}^k$, where $\hat{n}$ is the number operator and the integer k is greater than 1. This model is exactly soluble and the case when $k=2$ has been extensively studied recently by Milburn and Holmes.[1,2] We show that when k is even the initial coherent state evolves, after a suitable amount of time has elapsed, into a quantum superposition of two coherent states 180° out of phase with respect to each other. This behavior is hinted at by the behavior of the Q function studied by Milburn.[1]

The anharmonic-oscillator model can be regarded as describing the evolution of a coherent state injected into a transmission line with a nonlinear susceptibility, an optical fiber for example. Stated in electrical terms, it should thus be possible to engineer a nonlinear transmission line in which a sinusoidal signal rising 1 V positive will, after propagating sufficiently far along the transmission line, be converted into a quantum mechanical superposition of two sinusoidal signals in which one signal rises 1 V positive as the other drops 1 V negative. It is suggested that the process by which these Schrödinger's catlike states[3,4] (a quantum superposition of macroscopically distinguishable states) are generated from an initial (essentially classical) coherent state may be a fairly general property of nonlinear systems provided dissipation is kept sufficiently low.

A homodyne detector in which signal light is made to interfere with intense local oscillator light[5–8] of the same frequency on the surface of a photodetector provides a means by which an experimentalist could check to see if a quantum superposition of two coherent states has in fact been generated. In particular, it is shown that if the phase of the local-oscillator light is chosen properly an interference between the two coherent states of the superposition arises which can be detected as fringes in the probability distribution for the homodyne-detector's output current.

Choosing units such that $\hbar=1$, consider an anharmonic-oscillator Hamiltonian of the form

$$H=\omega\hat{n}+\Omega\hat{n}^k, \tag{1}$$

where ω is the energy-level splitting for the harmonic-oscillator part of the Hamiltonian and Ω is the strength of the anharmonic term. In the interaction picture where $\Omega\hat{n}^k$ is regarded as the interaction part of the Hamiltonian, and initial coherent state

$$|\alpha\rangle=\exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{2}$$

where $|n\rangle$ is the n-particle eigenstate, will evolve under the inflluence of the Hamiltonian according to

$$|\alpha,t\rangle=e^{-\Omega t\hat{n}^k}|\alpha\rangle. \tag{3}$$

Substituting Eq. (2) into Eq. (3) one can write

$$|\alpha,t\rangle=\exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\alpha^n\frac{\exp(-i\phi_n)}{\sqrt{n!}}|n\rangle, \tag{4}$$

where

$$\phi_n=\Omega t n^k. \tag{5}$$

By making the substitution $t=t'+2\pi/\Omega$ and noting that n^k is an integer, it is immediately apparent that

$$|\alpha,t+2\pi/\Omega\rangle=|\alpha,t\rangle; \tag{6}$$

that is, the state vector is periodic with a period $2\pi/\Omega$. Equation (4) is particularly easy to evaluate for special values of t. In particular when $t=\pi/\Omega$ then $e^{-i\phi_n}=(-1)^n$. Hence the state $|\alpha\rangle$ evolves into the state $|-\alpha\rangle$ at $t=\pi/\Omega$. Of more interest is what happens at the intermediate time $t=\pi/2\Omega$. At $t=\pi/2\Omega$ one has $e^{-i\phi_n}=1$ when n is even. When n is odd and k is even one has $e^{-i\phi_n}=-i$. When n and k are both odd

$e^{-i\phi_n} = -i(-1)^{(n-1)/2}$. The resulting state Eq. (4) can then be recognized as a generalized coherent state introduced by Titulaer and Glauber[9] and discussed by Stoler[10] and by Bialynicka-Birula.[11] Generalized coherent states for which ϕ_n is periodic in n can be expressed as a superposition of a finite number of coherent states. In particular, when k is even

$$|\alpha, \pi/2\Omega\rangle = (1/\sqrt{2})[e^{-i\pi/4}|\alpha\rangle + e^{i\pi/4}|-\alpha\rangle] \quad (7)$$

and when k is odd

$$|\alpha, \pi/2\Omega\rangle = \tfrac{1}{2}[|\alpha\rangle - |i\alpha\rangle + |-\alpha\rangle + |-i\alpha\rangle]. \quad (8)$$

It has now been shown that under the evolution of the Hamiltonian Eq. (1) an initial coherent state $|\alpha\rangle$ will evolve into a coherent superposition of a finite number of coherent states which are macroscopically distinguishable when $|\alpha|$ is large.

For simplicity the discussion will now be restricted to the case when k in Eq. (1) is even. Then, as can be seen from Eq. (7), at $t = \pi/2\Omega$ the initial coherent state has evolved into a coherent superposition of the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ which are 180° out of phase with respect to each other.

Before evaluating the response of a homodyne detector to this state it is instructive to evaluate what happens to the state upon passing through a beam splitter, particularly since the beam splitter can be used to model medium or detector losses.[5,8] To this end consider a beam splitter in which light enters the port a_1. A fraction η of the light passes through the beam splitter and exits port b_1, the rest of the light $1-\eta$ exits port b_2. A beam splitter must also have a second input port a_2 and it will be assumed that no light enters this port. The mode transformation performed by the beam splitter may be taken to be

$$b_1 = \eta^{1/2}a_1 + (1-\eta)^{1/2}a_2,$$
$$b_2 = -(1-\eta)^{1/2}a_1 + \eta^{1/2}a_2. \quad (9)$$

Consider first the case when the input state $|\text{in}\rangle$ has the form $|\text{in}\rangle = |\alpha\rangle_1|0\rangle_2$; that is a coherent state $|\alpha\rangle$ enters port a_1 of the beam splitter and the vacuum state enters through port a_2. Expressed in terms of the creation operator $a_1^\dagger$ for the mode a_1, the input state is

$$|\text{in}\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{n!}(a_1^\dagger)^n|0\rangle_1|0\rangle_2. \quad (10)$$

Solving Eq. (9) for a_1 and substituting this into Eq. (10) one can show that the output state is given by[12]

$$|\text{out}\rangle = |\eta^{1/2}\alpha\rangle_1|-(1-\eta)^{1/2}\alpha\rangle_2,$$

where $|\eta^{1/2}\alpha\rangle_1$ is the coherent state leaving port b_1 and $|-(1-\eta)^{1/2}\alpha\rangle_2$ is the coherent state leaving port b_2. Applying these same techniques, one can show that the state Eq. (7), upon passing through a beam splitter, becomes

$$|\text{out}\rangle = (1/\sqrt{2})[e^{-i\pi/4}|\eta^{1/2}\alpha\rangle_1|-(1-\eta)^{1/2}\alpha\rangle_2 + e^{i\pi/4}|-\eta^{1/2}\alpha\rangle_1|(1-\eta)^{1/2}\alpha\rangle_2]. \quad (11)$$

A homodyne detector[5-8] observing the light leaving the port b_1 of the beam splitter measures the operator

$$x = (1/\sqrt{2})[e^{i\theta}b_1 + e^{-i\theta}b_1^\dagger], \quad (12)$$

where the local oscillator phase θ is controlled by the experimenter.

Introducing the operator

$$y = (1/\sqrt{2})[e^{i\theta}b_2 + e^{-i\theta}b_2^\dagger] \quad (13)$$

the x,y representation $\psi_{\text{out}}(x,y) = \langle x,y|\text{out}\rangle$ of the state Eq. (11) can be constructed using standard techniques.[13] One finds

$$\psi_{\text{out}}(x,y) = (1/\sqrt{2})[e^{-i\pi/4}\psi_\gamma(x)\psi_{-\delta}(y) + e^{i\pi/4}\psi_{-\gamma}(x)\psi_\delta(y)], \quad (14)$$

where $\gamma = \eta^{1/2}\alpha$ and $\delta = (1-\eta)^{1/2}\alpha$ and the wave functions on the right-hand side have the form

$$\psi_\beta(x) = \frac{1}{\pi^{1/4}}\exp\left\{-\frac{x^2}{2} + \frac{2x\beta e^{i\theta}}{\sqrt{2}} - \left(\frac{\beta e^{i\theta}}{\sqrt{2}}\right)^2 - \frac{|\beta|^2}{2}\right\}. \quad (15)$$

The probability distribution

$$P(x) = \int_{-\infty}^{\infty} dy\, \psi_{\text{out}}^*(x,y)\psi_{\text{out}}(x,y) \quad (16)$$

for the output current x delivered by the homodyne detector independent of what has left port b_2 of the beam splitter can now be evaluated. Setting $\alpha = |\alpha|e^{i\phi}$ one finds

$$P(x) = (1/2\sqrt{\pi})\Big(\exp[-\{x - \sqrt{2\eta}|\alpha|\cos(\theta+\phi)\}^2] + \exp[-\{x + \sqrt{2\eta}|\alpha|\cos(\theta+\phi)\}^2]$$
$$+ 2\exp[-2(1-\eta)|\alpha|^2]\exp[-x^2 - 2\eta|\alpha|^2\cos^2(\theta+\phi)]\sin[2\sqrt{2\eta}|\alpha|\sin(\theta+\phi)x]\Big). \quad (17)$$

Probability distributions of this form have been obtained by Caldeira and Leggett[14] and by Walls and Milburn[15] in their investigations of the rate of loss of coherence in a damped harmonic oscillator. The first two terms of this probability distribution represent two Gaussian hills centered at $x=\sqrt{2\eta}|\alpha|\cos(\theta+\phi)$ and $x=-\sqrt{2\eta}|\alpha|\cos(\theta+\phi)$, respectively. If the local oscillator phase angle θ is adjusted such that $\cos(\theta+\phi)=\pm 1$ and $|\alpha|$ is sufficiently large, the probability distribution will consist essentially of two well-separated Gaussian hills. Already, for $|\alpha|=2$ and $\eta=1$ the Gaussian hills are well separated as can be seen in Fig. 1(a). The last term of Eq. (17) represents an interference which arises because the state Eq. (7) is in a coherent superposition of the states $|\alpha\rangle$ and $|-\alpha\rangle$. The observability of this interference term is enhanced by adjusting the local-oscillator phase such that $\cos(\theta+\phi)=0$. Figure 1(b) depicts the resulting interference fringes for the case when $|\alpha|=2$ and $\eta=1$. Hence by adjusting the local-oscillator phase θ such that $\cos^2(\theta+\phi)=1$ and then to $\cos^2(\theta+\phi)=0$ an experimentalist can verify first that the state has macroscopically distinguishable components (particularly when $|\alpha|$ is large) and then that the state is a coherent superposition rather than a statistical mixture. As $|\alpha|$ becomes larger the number of interference fringes increases, as can be seen by comparing Fig. 1(b) with Fig. 2(a) where $|\alpha|=5$.

From the exponential $\exp[-2(1-\eta)|\alpha|^2]$ appearing in the interference term one can see that the interference fringes rapidly fade as the loss $1-\eta$ becomes larger than $1/2|\alpha|^2$. This is depicted in Figs. 2(a)–2(c) as the loss is increased from 0 to 0.02 and then to 0.05. Hence for large $|\alpha|$ even a very small amount of loss will wash out the interference fringes and make the resulting probability distributions indistinguishable from that of a statistical mixture of the states $|\alpha\rangle$ and $|-\alpha\rangle$. The severity with which losses tend to destroy coherence between macroscopically distinguishable states has been noted by a number of authors and we refer the reader to Milburn and Holmes[1,2] for references. From a practical point of view, in order not to be limited by medium losses or detector inefficiencies, the experiment described here could only be realistically performed for small $|\alpha|$.

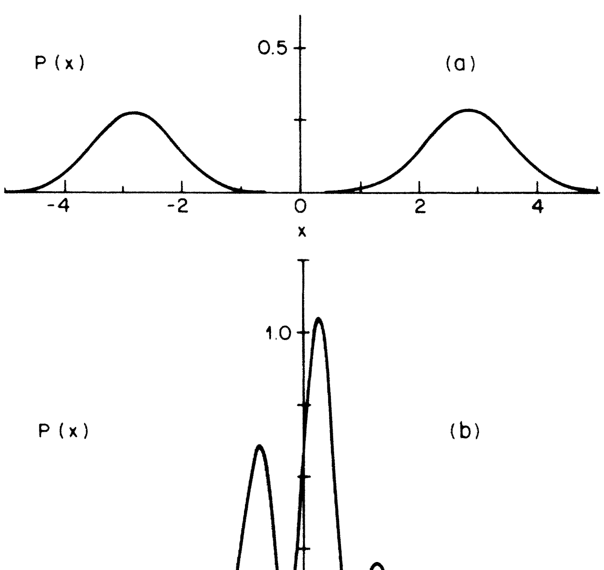

FIG. 1. The probability distribution $P(x)$ for the homodyne-detector's output current x for the case when $\eta=1$ and $|\alpha|=2$. In (a) the homodyne detector's phase is adjusted so that the Gaussian hills are maximally separated, i.e., $\cos(\theta+\phi)=1$. In (b) the homodyne detector's phase is adjusted such that $\sin(\theta+\phi)=1$. Here the probability distribution exhibits interference fringes.

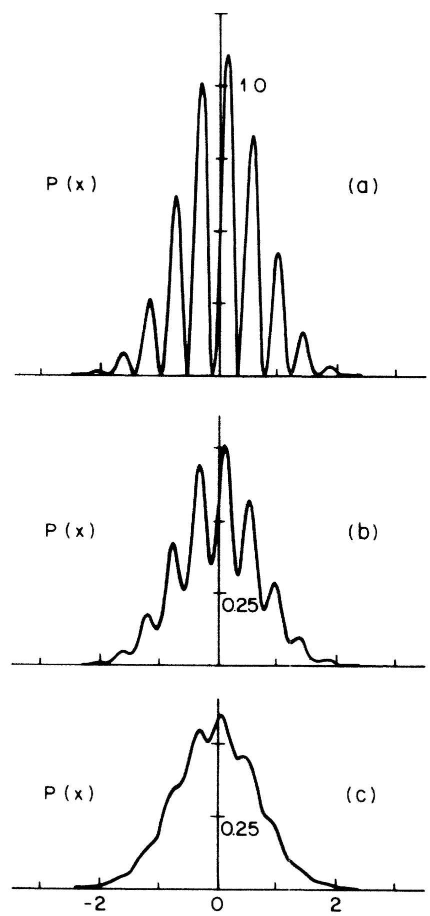

FIG. 2. The probability distribution $P(x)$ for the homodyne detector's output current x for the case when $|\alpha|=5$ and $\sin(\theta+\phi)=1$. In (a), (b), and (c) the detector efficiency is respectively 1, 0.98, and 0.95. As the detector efficiency is decreased the fringe visibility rapidly degrades.

Fortunately as already noted, even for $|\alpha|=2$ (the mean number of photons in this case is 4) the two coherent states of the superposition are well separated.

The Hamiltonian, Eq. (1), is rather special and may be difficult to realize in practice. We have also performed accurate numerical simulations[16] of strongly pump-depleted four-wave mixers governed by the Hamiltonian

$$H=\Omega\, aab^{\dagger}b^{\dagger}+\text{H.c.},$$

where a is the signal mode and b is the pump mode. If the system is started out initially with the signal in a vacuum state and the pump in a coherent state then when the system has evolved to the point where nearly all the pump energy has been transferred to the signal mode, the signal consists of two well-separated hills when viewed with a homodyne detector whose local-oscillator phase has been adjusted appropriately. When the local-oscillator phase is adjusted $\pi/2$ rad from the setting which maximizes the distance between the hills, interference fringes are clearly visible. This suggests that the process by which a coherent state is converted into a coherent superposition of two macroscopically distinguishable states may be a fairly general property of nonlinear systems with sufficiently low dissipation.

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