Improved Estimate of the Scalar-Glueball Mass

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We present results for the 0^{++} glueball mass (m_G) and the string tension (σ) in pure gauge lattice QCD with a four-parameter improved action. We suggest and confirm that previous estimates for $m_G/\sqrt{\sigma}$ have been too low because of the influence of the unphysical and nonuniversal phase structure in the fundamental-adjoint coupling plane. We find that $m_G \approx 1200$–1400 MeV using $\sqrt{\sigma} = 420$ MeV.

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Foremost among the aims of lattice quantum chromodynamics (LQCD) is to predict the spectrum of glueballs and other exotic states. A major step towards this goal would be a calculation of the glueball spectrum in lattice SU(3) without dynamical fermions. If the dominant effect of adding dynamical fermions is to shift the value of the bare coupling constant, then the spectrum deduced from the pure gauge theory could be directly confronted with experiment, except for the effects of decays and mixing.

Even this lesser goal of establishing the spectrum in the pure gauge theory is far from being attained. The only measurements of glueball correlators on a reasonably large lattice and with signals which unambiguously expose the lightest glueball state have been those of de Forcrand *et al.*,[1] who use the source method. Expressing their results as a ratio of the scalar-glueball mass (m_G) to the square root of the string tension ($\sqrt{\sigma}$) as determined by the source method in the same ensemble,[2] one finds $m_G/\sqrt{\sigma} = 1.96(7)$, 2,45(12), and 2.65(18) for couplings on the Wilson axis $6/g^2 = 5.5$, 5.7, and 5.9, respectively. Thus, there are significant scaling violations in $m_G/\sqrt{\sigma}$ for the range of couplings investigated along the Wilson axis. If we nevertheless take $m_G/\sqrt{\sigma} = 2.0$–2.65, a range suggested by earlier numerical work[3] and strong-coupling expansions along the Wilson axis,[4] as well as the above, then we obtain an estimate $m_G \approx 850$–1100 MeV, using the identification $\sqrt{\sigma} \approx 420$ MeV. This result may be hard to reconcile with the data on $\pi\pi$ phase shifts,[5] although a very recent analysis[6] of these and other data has suggested a possible extra state at about 980 MeV. Clearly it is of great phenomenological importance to firm up the lattice prediction before including dynamical fermions.

In recent years, a great effort has been devoted to Monte Carlo renormalization-group (MCRG) study of LQCD in order to determine the value of the Wilson-axis coupling beyond which there is scaling.[7,8] For K_F around 6.0, the string tension and deconfinement-transition temperature scale roughly in accord with the nonperturbative β function, but the glueball mass does not. In this Letter we suggest a reason why $m_G/\sqrt{\sigma}$ has not been exhibiting scaling, and why it may have been underestimated by calculations done along the Wilson axis. In confirmation, we present the results of a computation which removes at least part of the problem, suggesting that $m_G/\sqrt{\sigma} \approx 3.0(3)$. In physical units this corresponds to $m_G \approx 1200$–1400 MeV, a value certainly consistent with the experimental data.

We consider here pure gauge LQCD with the generic action

$$S[U] = \sum_\alpha K_\alpha \operatorname{Re}\operatorname{Tr}(U_\alpha), \tag{1}$$

where the U_α are various Wilson loops in different representations. Most simulations of this action have been performed in the plane defined by the fundamental (or "Wilson") and adjoint representations of the plaquette. In this plane it has been determined[9] that there is a line of first-order phase transitions that approach the Wilson axis from above and which terminate above the Wilson axis. This line of transitions, if continued beyond the end point, would intersect the Wilson axis at $K_F \approx 5.6$. Several numerical studies have measured the specific heat defined by

$$C_V K_F^{-2} \equiv \frac{\partial}{\partial K_F}\langle \mathrm{Re\,Tr}(U_\square)\rangle = \langle \textstyle\sum_{x_\mu} \mathrm{Re\,Tr}[U_\square(x_\mu)]\mathrm{Re\,Tr}(U_\square)\rangle|_{\text{connected}} \tag{2}$$

along the Wilson axis and in the fundamental-adjoint plane. In short, these studies have found a bump in the specific heat [see Fig. (6) of Bowler *et al.*[10]], which rapidly grows in magnitude as one moves closer to the end point. It is difficult to ascertain on the basis of simulations on small lattices whether C_V really diverges at the end point or not. Lacking any concrete evidence, we can only conjecture that the end point is a critical point and shall refer to it as the singular point. The string tension does not show pronounced effects due to the phase structure—in particular it remains finite even at the singular point.[10,11] The derivative $\partial\sigma/\partial K_F$ does become large, causing a dip in the β function. It should be remembered that these effects have nothing to do with the continuum limit of LQCD; rather they are lattice artifacts which could be removed by taking the continuum limit along a trajectory far away from the phase structure.

The crucial observation which follows from the numerical results is this: Since C_V is nothing but the connected plaquette-plaquette correlation function (with quantum numbers 0^{++}) at zero four-momentum, a singularity in the specific heat implies a zero-mass pole in the scalar-glueball channel. The ratio $m_G/\sqrt{\sigma}$ therefore vanishes at the singular point. A numerical calculation done very close to the singular point[12] did find the lattice value of m_G to be abnormally small. This then allows us to understand the numerical results for $m_G/\sqrt{\sigma}$ on the Wilson axis—they are depressed by the singular point. Finally, the anomalous scaling behavior of m_G along the Wilson axis can be explained as follows: The rapid increase in m_G as one moves towards stronger coupling (exponential if asymptotic scaling is valid) is suppressed, making the associated "β function" larger.[7]

If this explanation is correct then there are two options for extracting the continuum results. Move along the Wilson axis towards weaker coupling until $m_G/\sqrt{\sigma}$ reaches an asymptote; or use an action farther away from the singular point to decrease its effect.[13] We here report results from this second approach.

The improved action we have used consists of the plaquette in the fundamental, **8**, and **6** representations as well as the 1×2 rectangle in the proportion

$$\frac{K_8}{K_F} = -0.12, \quad \frac{K_6}{K_F} = -0.12,$$
$$\frac{K_{1\times 2}}{K_F} = -0.04, \tag{3}$$

when the traces are normalized to unity. It was determined by an MCRG calculation to lie close to the renormalized trajectory for the $\sqrt{3}$ renormalization-group transformation.[7] We expect the continuum limit to be smoother along this line of actions, i.e., with smaller corrections to the asymptotic mass ratios for a given (physical) lattice size. Whether or not this hope is realized, the action is farther away from the singular point and thus allows us to test our assertion made above. It has been pointed out that such actions may not have a positive-definite transfer matrix.[14] This would lead to an oscillatory component in the correlation functions for which we find no evidence (see Figs. 1 and 2).

By a finite-temperature numerical analysis,[15] we have determined that the deconfining transition for $N_t = 6$ occurs at $K_F = 10.15(5)$. Therefore, we have chosen to work at two values of the coupling constant: $K_F = 9.9$, which we have simulated on $6^3\times 21$ and $9^3\times 21$ lattices, and $K_F = 10.5$ on $9^3\times 21$ lattices, using a twenty-hit Metropolis algorithm. The data were taken every other sweep on 30 000 configurations for the $6^3\times 21$ lattice and on 18 000 configurations for the $9^3\times 21$ ones. The first 500 sweeps were discarded and a check for thermalization was made by reanalyzing without the first 2000 configurations. To make contact with calculations done on the Wilson axis, we compared large Wilson loops calculated with the improved action on $12^3\times 30$ lattices with those obtained on the Wilson axis. We find that our two couplings are roughly equivalent to $6/g^2 = 5.83$ and 5.96, respectively. The deconfinement transition for $N_t = 6$ occurs at $6/g^2 = 5.872(2)$ [15] on the Wilson axis. We thus have a simultaneous matching of the string tension and the deconfinement temperature between the improved-action and Wilson-action lattices. It is noteworthy that the ratio $T_c/\sqrt{\sigma}$ appears to be universal, while $m_G/\sqrt{\sigma}$ is not. This is a clear demonstration that to check scaling all mass ratios must be considered.

We have determined the string tension from the correlations between spatial Polyakov loops in presence of a cold source at $t = 0$. This method has less systematic uncertainties than the Wilson-loop method; for example there are no corner effects. We find a clean exponential falloff, with the signal being fitted well by a single exponential for $t = 3$ and beyond. The results are presented in Table I and a fit is shown in Fig. 1. We find the data at $K_F = 9.9$ consistent with the finite-size scaling form suggested by integration of the string fluctuation modes[16]:

$$\sigma(L) = \sigma(\infty) - \pi/3L^2 + O(L^{-3}). \tag{4}$$

TABLE I. Monte Carlo data for the scalar-glueball mass and the string tension.

K_F	Lattice	$\sigma(L)L$	$\sqrt{\sigma(\infty)}$	$m_G(L)$	$\frac{m_G(\infty)}{\sqrt{\sigma(\infty)}}$
9.9	$6^3\times 21$	0.32(1)	0.287(3)	0.79(11)	...
9.9	$9^3\times 21$	0.63(2)	0.288(4)	0.89(8)	3.1(3)
10.5	$9^3\times 21$	0.38(1)	0.235(2)	0.67(6)	3.0(3)

The extrapolated values quoted in Table I are obtained by use of this relation. It is worth mentioning that the value of $\sqrt{\sigma}$ extracted from Wilson loops is systematically larger. This is a feature common to all calculations. In this regard our result for $m_G/\sqrt{\sigma}$ is a comparative study.

Finally, we have measured the 0^{++} glueball mass on the same cold-source configurations, using 1×1, 1×2, and 2×2 Wilson loops. On the $9^3\times 21$ lattices we find that the data from $t=3$ to 9 can be well fitted with a single exponential plus a constant. A variational calculation[17] shows that the glueball wave function is dominated by the 2×2 loops. The best fits are also obtained with 2×2 loops; we quote these results in Table I, and Fig. 2 shows the fit at $K_F=10.5$. The errors quoted on these numbers are larger than the spread among the values obtained by use of different size loops. Despite larger statistics, the glueball signal on the $6^3\times 21$ lattice is not as good. As a result of the large errors in $m_G(L)$, we can only obtain a weak estimate of the three-scalar-glueball coupling constant. Using the finite-size scaling form[18]

$$m(L)=m(\infty)\left[1-\frac{3}{16\pi}\left(\frac{\lambda}{m(\infty)}\right)^2\frac{1}{m(\infty)L}\exp\left(-\frac{\sqrt{3}}{2}m(\infty)L\right)[1+O(L^{-1})]\right], \tag{5}$$

we find $\alpha_{GGG}\equiv(3/16\pi)[\lambda/m(\infty)]^2=70(70)$. Then, the infinite-volume results using Eq. (5) are $m_G=0.90(8)$ and 0.70(6) at $K_F=9.9$ and 10.5, respectively.

To summarize, Table I gives our values for $m_G/\sqrt{\sigma}$. As argued earlier, this result is larger than the value on the Wilson axis obtained with the same method. We are repeating the calculations at $K_F=9.2$ and 11.2 to check for improved scaling and to see whether $m_G/\sqrt{\sigma}$ has reached its continuum limit.

We also attempted to calculate the tensor-glueball mass using a source appropriately constructed from the elements of Z_3. Unfortunately, like others before,[1] we do not see a signal beyond a distance of three lattice spacings, and thus do not quote a result. We are thus unable to verify the claim[19] that the 2^{++} glueball may be nearly degenerate with or even lighter than the 0^{++} one. We also looked for a scalar-glueball signal in the correlations of spatial Polyakov loops in the adjoint representation.[19] Only on the $6^3\times 21$ lattice were we able to see a reasonable signal.[20] It extended up to $t=5$, and gave a value of m_G consistent with the previous one.

In conclusion, we have suggested an explanation for the pattern of numerical results for the glueball masses, and have presented results with an improved action supporting that explanation. Our results suggest that the scalar glueball lies in the experimentally murky region between 1200 and 1400 MeV.

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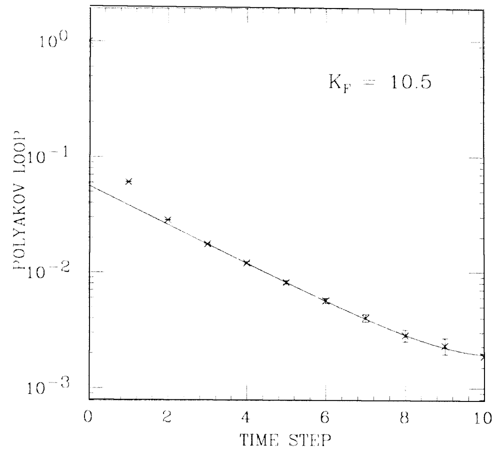

FIG. 1. Fit to the string-tension data.

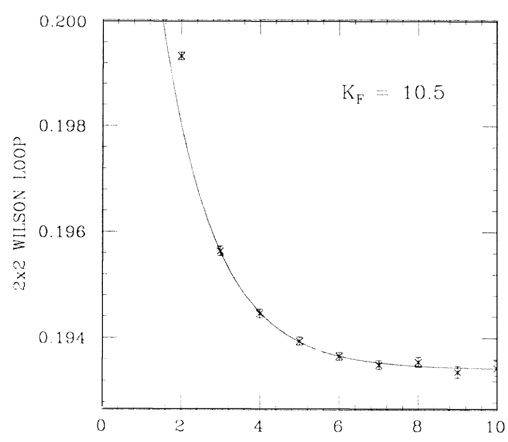

FIG. 2. Fit to the 0^{++} glueball data.

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