First Observation of the Universal Periodic Corrections to Scaling: Magnetoresistance of Normal-Metal Self-Similar Networks

B. Doucot, W. Wang, J. Chaussy, B. Pannetier, and R. Rammal
Centre de Recherches sur les Très Basses Températures, Centre National de la Recherche Scientifique, 38042 Grenoble Cédex, France

and

A. Vareille and D. Henry
Centre National d'Etude des Télécommunications, 38240 Meylan, France

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In many physical problems where renormalization-group transformations are exact, periodic oscillating corrections to power-law behavior are usually expected. The magnetoresistance of a normal-metal self-similar network, which exhibits such behavior in the weak localization regime, is shown to provide the first experimental evidence for this phenomenon.

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In many physical problems where scaling properties play an important role, it is now well established that renormalization-group (RG) transformations are the natural tool for a detailed interpretation of the observed phenomena. In this context, self-similar structures appear as the ideal models for a simple and transparent illustration of scaling concepts. The most interesting feature of self-similar structures is probably the possibility to have exact RG transformations, which allow for a detailed study of the RG flow. The powerful RG method has been extended in recent years to various fields including a wide range of physical problems (condensed matter, statistical and quantum mechanics, fluid dynamics, disordered systems, etc.). In general, when a RG transformation holds exactly, there are two basic equations. The first one gives the transformation law of parameters under a scale change (length or time scale, for instance). The second one is simply the transformation equation of a given physical quantity (free energy, Green's function, etc.).

In order to illustrate the purpose of this Letter, let us consider the case of a one-parameter (x) RG and one physical observable $F(x)$. A simple example of a RG transformation is provided by the following functional equation for F:

$$\mu F(x)=F[\phi(x)]. \tag{1}$$

Here $F(x)$ is assumed to be a very well behaved function and μ denotes a positive real number. The function $\phi(x)$, which generates the RG flow, is usually used to extract the qualitative behavior as well as the stability of fixed points and critical exponents. For example if $x=0$ denotes a fixed point $[\phi(0)=0]$ and $\phi(x)=\lambda x+\ldots$ is the corresponding linearized transformation, then $\omega=\ln\mu/\ln\lambda$ is the critical exponent describing the power-law solution $F_0(x)=x^\omega$ of Eq. (1) near this fixed point.

Aside from this sort of local analysis of the RG flow near fixed points, one can be interested in the general solutions of Eq. (1). It appears that there is actually a large number of simple but nontrivial such solutions. Indeed, if we assume that $F_0(x)$ is a particular solution, then the general solution $F(x)$ is related to $F_0(x)$ in terms of a periodic function $p(x)$, with a period $\ln\mu$, through

$$F(x)=F_0(x)p(\ln F_0(x)). \tag{2}$$

Using the Fourier expansion of $p(x)$, and assuming the following expansion for the function $F_0(x)$,

$$F_0(x)=x^\omega\sum_{n=0}^{\infty}g_nx^n,$$

one ends up with the general solution (F real)

$$F(x)=\sum_{n,m=0}^{\infty}F_{m,n}x^{\omega+n}\cos\left[2\pi m\frac{\ln x}{\ln\mu}+\theta_{m,n}\right], \tag{3}$$

where $F_{m,n}$ and $\theta_{m,n}$ denote constant numbers.

The occurrence of these universal oscillations is actually a quite general phenomenon related to functional equations like Eq. (1), where $\phi(x)$ is an analytical function. In statistical mechanics,[1] this corresponds to critical "complex exponents" which are usually rejected for translationally invariant systems since they imply a length scale, or a finite size. However, these oscillations, although of a rather small amplitude, can appear in general when an exact RG transformation occurs, as is the case for self-similar structures, and this will be shown below. For a much more rigorous discussion of this point, we direct the reader to Bessis, Geronimo, and Moussa.[2]

In order to highlight these periodic oscillations, we have used a regular self-similar network made of submicronic Al wires: the Sierpinski gasket,[3,4] which is a 2D array of triangles (see Fig. 1 of Ref. 4) exhibiting a perfect dilation symmetry. The gasket structure has been realized by direct writing of the pattern on PMMA (polymethylmethacrylate) resist by use of an electron-beam microfabricator (Cambridge EBMF6). Then the Al was deposited in the open lines of the PMMA mask by thermal evaporation followed by a liftoff of the resist. The Al lines are 0.1 μm thick and ~ 0.3 μm wide. The elementary triangles, at the lowest length scale, are isosceles triangles of equal height and base: 3.2 μm. In order to optimize the signal-to-noise ratio in our resistance measurements, the structure was restricted to six stages of iterations: $0 \leqslant n \leqslant 5$, and this corresponds to the basic cell. The measured sample is a lattice of 1024 basic cells; the overall size of the network is 3.2×3.2 mm^2.

The resistance measurements were performed between the top node of the lattice and the two other nodes. The resistivity ratio of the Al film between room temperature and 4.2 K was found to be 6.4. The resistance of the sample above the superconducting critical temperature ($T_c = 1.23$ K) was 2.47 Ω. Measurements of the magnetoresistance $R(H)$ were performed at $T = 1.30$ K (above T_c) with a maximum current of 1 μA, with use of a four-probe ac bridge.[5]

In the temperature range of interest, weak localization effects have been studied recently on 1D and 2D clean Al films by different groups.[6] Above T_c, the low-field magnetoresistance is governed both by localization effects and by Maki-Thomson superconducting fluctuations. The contribution of the Maki-Thomson term can be understood by means of a simple temperature-dependent parameter $\beta(T/T_c)$, which is independent of the localization dimension.[7] The factor $\beta(T/T_c)$, which is field independent in the low-field regime, has no effect on the fine structure of the magnetoresistance (MR). However, because of its divergence as T_c is approached, $\beta(T/T_c)$ provides an enhancement factor to the weak localization effect which allows a large improvement of the experimental accuracy.

Typical magnetic fields range between 1 mOe and 10 Oe. Within this range, the critical temperature $T_c(H)$ of the network has been found to exhibit a very rich structure, as shown in the inset of Fig. 1.[8] This curve, which will be described elsewhere,[9] shows five levels of self-similarity and provides an accurate calibration of the flux quantization[10] at the different hierarchy stages of the Sierpinski gasket. In our sample, the magnetic field which corresponds to one superconducting flux quantum $\phi_0 = hc/2e$ in the elementary triangles is 4.4 Oe, and the fine structure was observed

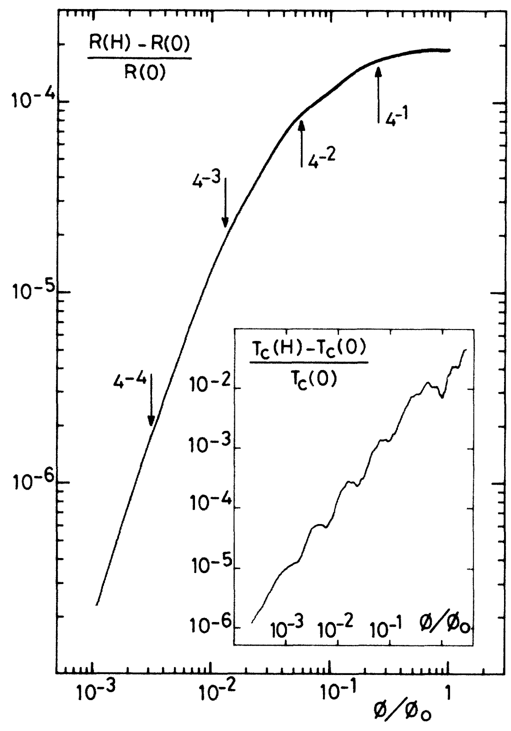

FIG. 1. MR of a Sierpinski gasket of Al submicronic wires. The inset shows the normal-superconducting phase boundary $T_c(H)$ of the same network. The magnetic field scale corresponds to the range of one superconducting quantum flux ϕ_0 through the elementary triangular cell. Arrows indicate the positions of reduced fluxes $\phi/\phi_0 = 4^{-n}$, $n = 1, 2, \ldots$.

down to $\phi_0/512$.

The MR, shown in Fig. 1, was measured at $T = 1.30$ K: $R(H) - R(0)$ vs H in a log-log plot. We see that the fine structure, which was observed on the $T_c(H)$ line, is no longer present. There only remain peaks at integer values of ϕ/ϕ_0 (not shown here), where ϕ is the magnetic flux through an elementary triangle. This is a manifestation of the Al'tshuler-Aronov-Spivak effect, observed also on regular Euclidean networks.[5] The limiting value of the slope of this curve is 2 in the very low-field regime (Fig. 2), and this agrees with the expected value (see below). However, close inspection of the MR curve reveals kinks at precise values of the magnetic flux. This behavior contrasts with the MR behavior on regular Euclidean networks,[5] where no structure occurs at $\phi < \phi_0$. Figure 2 shows the variation of the slope, i.e., the logarithmic derivative $\partial \ln \Delta R(H) / \partial \ln H$. As the field is lowered, the slope increases up to 2 but exhibits periodic oscillations in this logarithmic plot; the associated period of these oscillations is exactly ln4.

The oscillations of the slope of the MR curve are actually a clear signature of the phenomena discussed in the introduction. Furthermore, the experimental data can be analyzed up to a high level of accuracy by use of the weak-localization theory for normal-metal networks.[11] For a regular network of identical strands of length a, with a coordination number z, the integrated correction to the resistance is given by

$$\frac{\Delta R}{R}=\frac{\kappa}{2}\left[\left(1-\frac{2}{z}\right)\frac{\eta\cosh\eta-\sinh\eta}{\eta\sinh\eta}+\frac{2}{N}\sinh\eta\sum_{\alpha=1}^{N}\lambda_\alpha^{-1}\right]. \tag{4}$$

In Eq. (4), $\eta=a/L_\phi$, where $L_\phi\equiv(D\tau_\phi)^{1/2}$ is the length over which dephasing of the electron wave function results from inelastic processes (D is the diffusion constant). The dimensionless factor is given by $\kappa=2e^2L_\phi/\pi\hbar\sigma_0 S$, where σ_0 is the bulk conductivity of wires and S is their transverse section area. The sum in Eq. (4) is taken over the N eigenvalues λ_α of the Hermitian matrix Q, defined as follows: $Q_{\alpha\alpha}=z\cosh\eta$ and $Q_{\alpha\beta}=-e^{-i\gamma_{\alpha\beta}}$ for nearest-neighboring nodes α,β on the network, where $\gamma_{\alpha\beta}=(2\pi/\phi_0)\int_\alpha^\beta \mathbf{A}\cdot d\mathbf{l}$ is the magnetic phase factor induced by the vector potential $\mathbf{A}$.

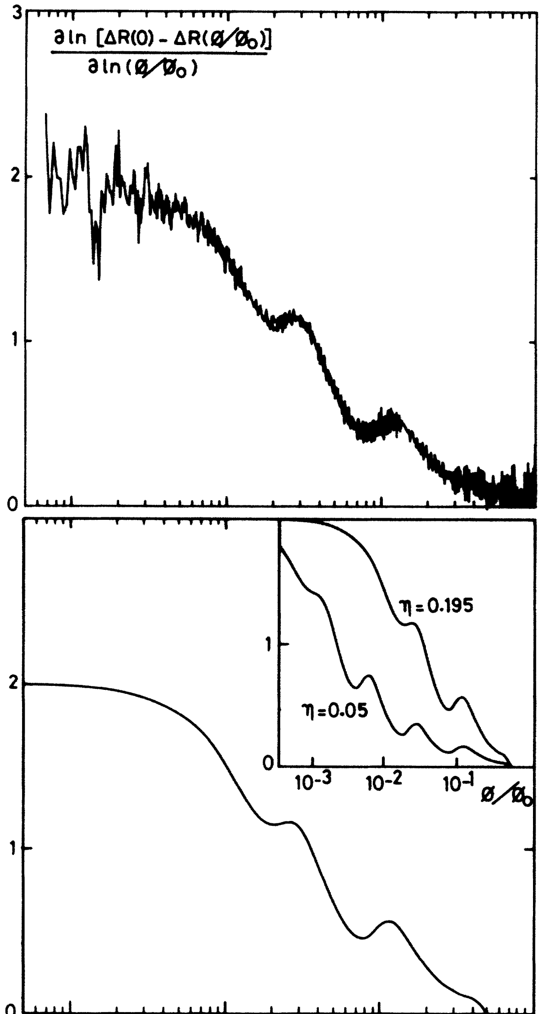

FIG. 2. Logarithmic derivative $\partial\ln[\Delta R(0)-\Delta R(H)]/\partial\ln(\phi/\phi_0)$ of the magnetoresistance. The upper part corresponds to the experimental data. The lower part is the result of the RG calculation of the same quantity ($\eta=0.195$). On the same plot, these two parts cannot be distinguished and they are separated for the sake of clarity. In the inset, the same curve is reproduced, on reduced scales, with another one corresponding to a smaller value of $\eta=a/L_\phi$ ($\eta=0.05$) which leads to more and more periodic oscillations. Here ϕ/ϕ_0 denotes the reduced flux through the elementary triangular cell.

In the case of the Sierpinski gasket ($z=4$), it is not easy to find a closed expression for the sum involved in Eq. (4). However, it is possible to write down an exact RG transformation (scaling factor $b=2$), which can be used to perform a numerical calculation of $\Delta R/R$, up to any desirable precision. Rather cumbersome algebra is involved in this decimation procedure and the details can be found in Ref. 11. The results for the logarithmic derivative of the MR are shown in Fig. 2. The only adjustable parameter in these calculations is the ratio $\eta=a/L_\phi$, and a perfect agreement is obtained for $\eta=0.195\pm0.005$. Here we have neglected minor corrections due to unequal lengths of the sides of the elementary triangle. We note that the number of maxima on Fig. 2 is very sensitive to the value of η and this is illustrated in the inset. The period of oscillation is equal to ln4, as expected, and corresponds to the ratio of the enclosed fluxes by elementary cells at two successive stages. Note that the value of $\mathcal{L}_\phi$ so obtained is consistent with the estimates[11] extracted from the results of Ref. 5.

The main features of the localization corrections to resistance can be summarized as follows. At zero magnetic field, one has $\Delta R/R\sim\tau_\phi^{(2-\tilde{d})/2}$ ($\tilde{d}<2$) in the scaling regime, where τ_ϕ is the phase-coherence breaking time and $\tilde{d}$ is the spectral dimensionality of the structure.[11] However, because of anomalous diffusion, this result can be cast in a more familiar form, $\Delta R/R\sim\mathcal{L}_\phi^{-\beta}$, where $\mathcal{L}_\phi/a=(L_\phi/a)^{\tilde{d}/\bar{d}}$ is the true phase-coherence length on the gasket and $\bar{d}$ is the fractal dimensionality. Here $\beta=\bar{d}(\tilde{d}-2)/\tilde{d}$ denotes the localization exponent, as defined by Rammal and Toulouse.[12] In finite magnetic fields, the MR exhibits a crossover between a quadratic dependence $(\Delta R/R)(H)-(\Delta R/R)(0)\sim H^2$ at low fields ($H\mathcal{L}_\phi^2 << \phi_0$) and a power-law behavior $(\Delta R/R)(H)\sim H^{\beta/2}$ at larger fields ($H\mathcal{L}_\phi^2 >> \phi_0$). We note that for Euclidean structure ($\bar{d}=\tilde{d}=d$) the known results are recovered,[13] in particular the logarithmic behavior for 2D films where $\beta = 0$.

The power-law behavior $\Delta R/R \sim H\beta/2$ is actually the manifestation of the singular shape of the spectrum edge $\epsilon(H)$ of the operator $\underline{Q}$, at low magnetic field[3,11]: $\epsilon(0) - \epsilon(H) \sim H^{\bar{d}/\tilde{d}}$ $[\epsilon(0) = z = 4]$. Such a behavior is reflected on the line $T_c(H)$ shown in Fig. 1 (see also Ref. 4). On the other hand, the RG calculation reproduces[11] this overall behavior, with $\bar{d} = \ln 3/\ln 2$, $\tilde{d} = 2\ln 3/\ln 5$, as it should be. Periodic modulations are superposed on the power law, with a period ln4 in a log-log plot.

The physical origin of the oscillation can be traced back to the fact that the considered structure exhibits a dilation symmetry only under a discrete subgroup of the dilation group. Indeed, at low fields, the magnetic flux renormalizes as[11]

$$\phi_n/\phi_{n=0} \simeq 4^n[4 + 13(\tfrac{3}{20})^n]/17, \tag{5}$$

at stage n of decimation and $\mathcal{L}_\phi = \infty$ (scaling regime). The maximal modulations are obtained for reduced fluxes ϕ/ϕ_0 corresponding to integer values of the renormalized flux. According to Eq. (5), ϕ_n is the same for (ϕ, n) and $(\phi/4, n+1)$, and this leads to the observed period of ln4. Another approach to this result involves a careful study of the RG flow, which leads to the following equation for $\Delta R/R$:

$$(\Delta R/R)(\phi/\phi_0) = \tfrac{5}{3}(\Delta R/R)(4\phi/\phi_0). \tag{6}$$

Equation (6) has the same form as Eq. (1) and the general solution can be written as

$$\frac{\Delta R}{R}\left(\frac{\phi}{\phi_0}\right) = \left(\frac{\phi}{\phi_0}\right)^{\beta/2} p\left(2\pi\frac{\ln(\phi/\phi_0)}{\ln 4}\right), \tag{7}$$

where p denotes a periodic function, with period 1. Equation (7) shows that $\eta \equiv a/L_\phi$ governs just the low-field cutoff and does not play any role in the self-similar regime[11] where the predicted oscillations are observed. It is important to notice that in Fig. 2, we have plotted $(\Delta R/R)(0) - (\Delta R/R)(H)$ instead of $(\Delta R/R)(H)$. We note that this behavior is due again to the singular self-similar edge of the spectrum of the operator Q, which reflects the self-similarity of the structure. Furthermore, there is no such oscillation at zero magnetic field, as a function of temperature for instance. Of course, the lack of exact self-similarity can lead to the disappearance of the obtained periodic oscillations.

To summarize, we believe that the periodic corrections to scaling reported in this Letter are the manifestation of a very general phenomenon observed here for the first time. Such corrections are associated in a natural way with an exact RG transformation, which generates a singular measure. In the present case, this can also be seen on the spectral measure as well as the spectrum (Cantor-type set) of the operator Q. The existence of such a RG transformation depends of course on the validity of the weak localization theory. Thus the observation of both the predicted power law and the periodic corrections should be viewed as a strong evidence for weak localization theory.

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