Spectrum Supersymmetry of Regge Trajectories

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An infinite-dimensional representation of SU(2,2/1) can combine meson and baryon trajectories with daughters into an infinite supermultiplet.

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The equality of the slopes for the meson and baryon Regge trajectories is one of the most interesting problems in hadron spectroscopy. It has been suggested that this problem can be studied by use of supersymmetries.[1] This is based on the belief that inside the excited baryon QCD leads to the formation of diquarks which are well separated from the remaining quark.[2]

In nuclear physics a similar picture led to the introduction of spectrum supersymmetries.[3] The microscopic interpretation is that the additional nucleon of an even-odd nucleus which is added to the core of an even-even nucleus couples only weakly. This results in similar level structure and level spacing for even-odd and even-even nuclei. The collective models for these phenomena are dynamical supersymmetries based on $U(6/N)$.[3]

The similarity in the microscopic pictures of hadrons and nuclei suggests a similarity for the collective models. A collective model for hadrons should, therefore, have a relativistic spectrum supersymmetry which combines baryons and mesons into supermultiplets. To describe the—in principle—infinite number of resonances on a Regge trajectory the supermultiplet must be infinite.

We want to start with the superconformal group SU(2,2/1), which was also suggested in Ref. 1 and whose infinite-dimensional representations are fairly well known.[4] It has $\mathrm{Osp}(1,4)\supset \mathrm{SO}(3,2)\supset \mathrm{SO}(3)_{S_{ij}} \otimes \mathrm{SO}(2)_{\Gamma_0}$ and $\mathrm{SU}(1,1/1)\supset \mathrm{Osp}(1,2)\supset \mathrm{SO}(2,1) \supset \mathrm{SO}(2)_{\Gamma_0}$ as different subalgebras. The generators of SU(2,2/1) are denoted[5] by S_{ab} $(a,b=0,1,2,3,5,6)$, R, and T_α, S_α, $\alpha=1,2$. $S_{\mu\nu}$, $\Gamma_\mu = S_{\mu 6}$, $\mu=0,1,2,3$, and $Q_\alpha=\frac{1}{2}(T_\alpha+S_\alpha)$ are the generators of the supergroup Osp(1,4); S_{56}, S_{05}, and $S_{06}=\Gamma_0$ are the generators of SO(2,1). The Osp(1,2) chain has been suggested as spectrum supergroup for the nonrelativistic harmonic oscillator and other nonrelativistic models with spin-orbit coupling.[6] The SO(3,2) chain has been suggested as spectrum generating group for the relativistic rotating vibrator[7] (a mutation of the relativistic string with noncanonical intrinsic position and momentum). SU(2,2/1) is the smallest supergroup that can combine both.

The properties of the representations of a noncompact group or supergroup which are most important for physical applications are the reduction with respect to the (maximal) compact subgroup and the matrix elements of the group generators. The former describes the spectrum and the latter transition amplitudes. Here we will discuss the spectrum, which is best illustrated by the weight diagram (or K type). A weight diagram displays which irreducible representations (irreps) of the (maximal) compact subgroup $\mathrm{SO}(4)_{S_{ij}S_{i5}} \otimes \mathrm{SO}(2)_{\Gamma_0} \supset \mathrm{SO}(3)_{S_{ij}} \otimes \mathrm{SO}(2)_\Gamma$ occur in an irrep of $\mathrm{SU}(2,2/1)\supset \mathrm{Osp}(1,4)$. We are here only interested in a special class of irreps of SU(2,2/1), the so-called positive-energy, massless representations, which are conventionally denoted[4] by $D_S(s+1;s,0;s+1)$ where s is a number which can take any integer or half-integer value: $s=0,\frac{1}{2},1,\frac{3}{2},\ldots$. The only important aspect in this notation is that these representations are characterized by the value of s (the subscript S stands for supergroup). The physical meaning of this value is obtained from the reduction of this irrep with respect to the sub(super)group chain $\mathrm{SU}(2,2/1)\supset \mathrm{Osp}(1,4) \supset \mathrm{SO}(3,2)$, which is given by

$$D_S(s+1;s,0;s+1) \underset{\mathrm{Osp}(1,4)}{\longrightarrow} D_S(s+1,s) \longrightarrow D(s+1,s) \oplus D(s+3/2,s+1/2) \tag{1}$$

where $D_S(s+1,s)$ denotes a positive-energy representation of Osp(1,4)[8] and $D(j_0+1,j_0)$ denote the irreps of SO(3,2) that have been used for the quantal relativistic oscillator in Ref. 7. There j_0 has been interpreted as the total constituent spin; i.e., if the extended relativistic object described by $D_S(s+1;s,0;s+1)$ is considered to consist of two and three quarks then $j_0=s+\frac{1}{2}$ and $j_0=s$ are the sums of the spins of the quarks. We, therefore, choose $s=\frac{1}{2}$ in (1) and obtain a representation which describes baryon resonances $(j_0=\frac{1}{2})$ and positive-CP, normal-j^P meson resonances ($j_0=1$) by one irrep:

$$D_S(\tfrac{3}{2};\tfrac{1}{2},0;\tfrac{3}{2}) \underset{\mathrm{SO}(3,2)}{\rightarrow} D(\tfrac{3}{2},\tfrac{1}{2}) \oplus D(2,1). \qquad (2)$$

Figure 1 is the weight diagram of the irrep $D_S(\frac{3}{2};\frac{1}{2},0;\frac{3}{2})$, except that it has been distorted in order to illustrate the results that will be discussed below. Each level represents a weight (μ,j) where μ is the eigenvalue of Γ_0 and $j(j+1)$ is the eigenvalue of $\mathbf{S}^2$. Each level, therefore, belongs to a state with angular momentum j and principal quantum number μ, which according to Ref. 7 will be interpreted as vibrational

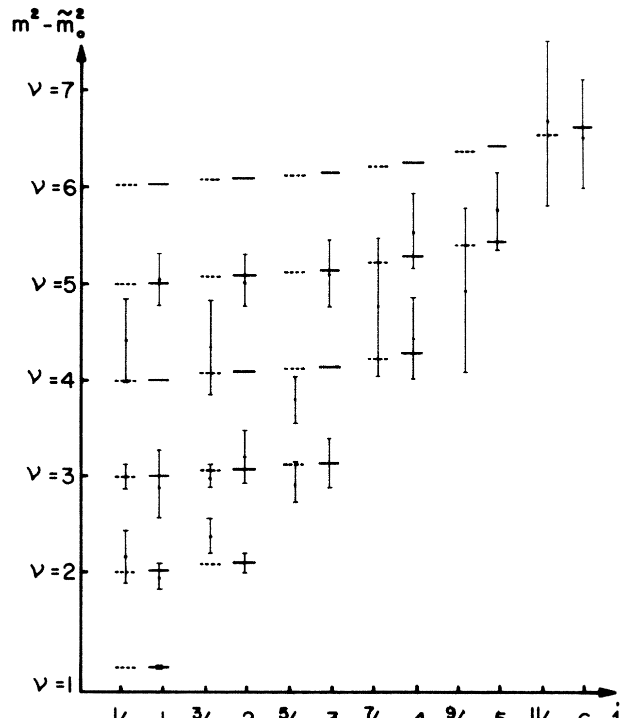

FIG. 1. Weight diagram of an Osp(1,4) representation which has been modified into a mass-level diagram. On the horizontal axis we have plotted j, whose physical interpretation is the spin of the resonances. If the levels for a fixed value of ν had been plotted on a horizontal parallel to the j axis then this would be the weight diagram of the representation $D(\frac{3}{2},\frac{1}{2}) \oplus D(2,1)$. Instead we have plotted the level with a vertical coordinate of $m^2-\tilde{m}_0^2$, where the value of $\tilde{m}_0^2$ (baryons) $-\tilde{m}_0^2$ (meson) has been fixed such that the ground-state levels coincide, and m has been determined from a fit of the nucleon resonances and of the $Y=0$, $CP=+1$, $j^P=$normal meson resonances by the mass formula (3). ν is the new vibrational quantum number which has been assigned to the resonance: ν is the eigenvalue $(\Gamma_0-\frac{1}{2})$ for baryons; ν is the eigenvalue (Γ_0-1) for mesons. Not shown in this figure is the prediction of $m=2758$ MeV for $j=7^-$ [experimental $M(2750)$] and of $m=2791$ MeV for $j=\frac{13}{2}^+$ [experimental $N(2700)$].

quantum number $\nu=\mu-1$ for mesons and $\nu=\mu-\frac{1}{2}$ for baryons. Figure 1 would be exactly the weight diagram of $D_S(\frac{3}{2};\frac{1}{2},0;\frac{3}{2})$ if we had plotted μ and not ν versus j and if all levels with the same value of μ were plotted on the same horizontal line.

The representations (1) have the remarkable property that the Casimir operators $\mathscr{C}(\mathrm{SO}(3))=j(j+1)$ and $C(\mathrm{SO}(2,1))=q(q-1)$ have the same value, so that $q=j+1$; therefore j also labels an irrep of SO(2,1). Thus the column of levels with a fixed value for j in Fig. 1 belongs to an irrep $D^+(q)$ of SO(2,1). As two irreps, $D^+(q)\oplus D^+(q+\frac{1}{2})$, combine into an irrep of Osp(2,2)$\supset$Osp(1,2), any two adjacent columns in Fig. 1 form a weight diagram of Osp(2,2). Figure 1 is thus an extension of Fig. 2 of Fubini and Rabinovici[9]—the weight diagram of Osp(2,2)—which includes all integer and half-integer values of j, not just $j=l$ and $j=l+\frac{1}{2}$ for a fixed value of l. These weight diagrams of Osp(2,2) have been used[6] to describe the spectrum of a radial supersymmetric oscillator. The representation (2) thus provides a relativistic generalization of the supersymmetric oscillator of Ref. 6 and a supersymmetric extension of the relativistic oscillator of Ref. 7.

The physical interpretation of the SU(2,2/1) as relativistic spectrum supersymmetry is derived from the physical interpretation of the SO(3,2) subalgebra of Ref. 7. This interpretation is related to but not identical with the conventional interpretation; in particular neither Γ_0+S_{05} nor Γ_0 is the energy. We use constraint Hamiltonian quantum mechanics. In addition to the SU(2,2/1) generators the Poincaré group generators P_μ and $J_{\mu\nu}=M_{\mu\nu}+S_{\mu\nu}$ are defined[10] and only through the constraint relation is the energy (at rest) related to Γ_0:

$$P_0 \overset{c}{=} \alpha'^{-1}\Gamma_0+\tilde{m}_0^2=\textstyle\sum_\alpha \frac{1}{2}\{Q_\alpha,Q_\alpha^\dagger\}+\tilde{m}_0^2,$$

where the symbol $\overset{c}{=}$ means equal by constraint. This follows from the relativistic Hamiltonian, which for the simplest case ("relativistic harmonic oscillator") is postulated as

$$H=v(P_\mu P^\mu-\alpha'^{-1}\textstyle\sum\frac{1}{2}\{\hat{Q}_\alpha,\hat{Q}_\alpha^\dagger\}-\tilde{m}_0^2) \qquad (3)$$

(v is the Lagrange multiplier; $1/\alpha'$ and $\tilde{m}_0$ are system parameters). In (3) $\hat{Q}_\alpha=D_{\alpha\beta}(\hat{P}_\mu)Q_\beta$ where $D(\hat{P}_\mu)$ is an operator matrix and the operator $\hat{P}$ is defined $\hat{P}_\mu=P_\mu M^{-1}$, $M^2=P_\mu P^\mu$; $D(\hat{p})$ is the two-dimensional $(\frac{1}{2},0)$ representation of the inverse Lorentz boost $L(\hat{p}):p_\mu\rightarrow(m,0,0,0)$. In the full representation μ is the eigenvalue of $\hat{P}_\mu\Gamma^\mu$ and $j(j+1)$ is the eigenvalue of $(-\hat{W}_\mu\hat{W}^\mu)$ with $\hat{W}_\mu=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\hat{P}^\nu J^{\rho\sigma}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\hat{P}^\nu S^{\rho\sigma}$, and to each level (μ,j) corresponds an irreducible representation of the Poincaré group with spin j and additional quantum number μ.[10] The SU(2,2/1) operators describe only the intrinsic dynamics of an extended relativistic object.

The simple Hamiltonian (3) will lead—through its constraint—to the mass formula

$$m^2(\mu,j) = \alpha'^{-1}\mu + \tilde{m}_0^2;$$

other Hamiltonians will lead to other mass formulas. We will test the mass formula

$$m^2 = \tilde{m}_0^2 + \alpha'^{-1}\mu + \lambda^2 j(j+1), \tag{4}$$

where m^2 is the mass square of the hadron that has been assigned to the level (μ,j).

The empirical parameters $1/\alpha'$, λ^2, and $\tilde{m}_0^2$ have been determined by fits to the meson and nucleon spectrum separately. We have done this for many hadron towers and will report here only the result for the ρ, ω, and nucleon towers. The hadron resonances usually assigned to the ρ, ω, and nucleon trajectories have been assigned in Fig. 1 to the levels (μ,j) with $\mu = j+1$ (corresponding to the yrast states of nuclear physics). The levels with $\mu > j+1$ (higher vibrational excitations) represent the daughters. Note that there is no $j=0$ level in our representation (2) which, as usual, would have caused trouble. The mass values with error bars drawn in Fig. 1 are the resonances which have been included in the fit. In addition other resonances have been reported for which the evidence is weak and which have therefore not been used in the fit.[11]

It turned out that the values for $1/\alpha'$ and λ^2 for the baryon and meson towers agreed within error. Therefore a joint fit to the meson and nucleon resonances was performed which gave the values

$$1/\alpha' = 1.03 \pm 0.036 \text{ GeV}^2,$$

$$\lambda^2 = 0.015 \pm 0.008 \text{ GeV}^2,$$

with $\chi^2/n_D = 9.9/28$.

The result shows that the rotator contributions (proportional to λ^2) are small and that (3) provides a good approximation. And it shows that the slope is indeed the same for the meson and nucleon trajectories and their daughters. There is no representation of SU(2,2/1) whose weight diagram contains only the trajectory without daughters. But there is one representation of Osp(1,4) which could do this. It also contains, however, a $j=0$ state.

In this Letter we have proposed a relativistic generalization of methods that have previously been used in molecular physics. An extension of these mathematical methods into the relativistic domain and their application to hadron physics naturally suggests a transfer of the underlying physical ideas: When one analyzes molecular and nuclear spectra and structure in terms of rotators and oscillators, the notion of constituents (electrons, nucleons) becomes unimportant; the "parts" are the collective motions. The Regge recurrences and the daughters are—according to our model—vibrational excitations [with weak angular momentum dependence (rotational bands)]. The only time we had to refer to quarks was when we chose the representations (value of s), similar to the situation in nuclear physics. Though molecules, nuclei, and hadrons are made of completely different constituents, similar features (vibrational excitations with rotational bands) appear again and again, because not the constituents but the motions determine these features. The method which best displays this unity in physics is the one that uses groups. Supergroups occur when states of integer and half-integer spin are to be compared. The equality of the slopes for the meson and baryon trajectories can be explained as a manifestation of the spectrum supersymmetry given by SU(2,2/1).

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