Transport Processes in Heavy-Fermion Superconductors

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We show that in anisotropic superconductors in which impurity scattering is the dominant relaxation process at low temperatures, the viscosity and ultrasonic attenuation in the hydrodynamic regime, when calculated in the Born approximation, tend to constant values while the thermal conductivity, κ, varies as T. These results are in conflict with experiment. We show that if multiple-scattering effects are taken into account, the ultrasonic attenuation and κ/T fall with decreasing temperature.

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In this Letter we describe calculations, in the hydrodynamic limit, of transport coefficients for anisotropic superconducting states; in this limit, the calculation of the ultrasonic attenuation reduces to a calculation of the viscosity. We assume that impurities are the dominant scatterers of electrons. We first show that, if one treats the scattering in the Born approximation, as is commonly done in studies of metals, the viscosity at low temperatures, and hence the ultrasonic attenuation for essentially all orientations of the wave vector and the polarization of the ultrasonic wave, are independent of temperature, while the thermal conductivity is proportional to T in essentially all directions. Both of these results are in conflict with experiment. We then show that if we take multiple-scattering effects into account, we can account for some features of the experimental data on the ultrasonic attenuation of UPt_3 and UBe_{13}.[1] Finally we discuss how the failure of the Born approximation can come about physically and consider the application of our theoretical results to the thermal conductivity of UPt_3 and UBe_{13}.[2]

First of all let us estimate the mean free path of quasiparticles in anisotropic superfluids. For simplicity we shall assume that the effective scattering potential in the normal state is a constant, and that the impurity concentration is such that one is in the clean limit, so that depairing effects can be neglected. For triplet superconductors we use the usual representation of the gap matrix, $\Delta_{\mathbf{p}}$ in the form $i\sigma_2\boldsymbol{\sigma}\cdot\boldsymbol{\Delta}_{\mathbf{p}}$,[3] and for singlet superconductors we shall denote the gap as Δ. In this Letter we confine ourselves to perturbations which are independent of the spin variable; the collision integral for quasiparticles in the superfluid for this case is

$$\frac{\partial n_{\mathbf{p}}}{\partial t} = -\frac{2\pi}{\hbar} n_i \sum_{\mathbf{p}'} |v|^2 \mathcal{C} \delta(E_{\mathbf{p}} - E_{\mathbf{p}'}) \times (\delta\bar{n}_{\mathbf{p}} - \delta\bar{n}_{\mathbf{p}'}) \quad (1)$$

in the Born approximation. Here $E_{\mathbf{p}}$ is the quasiparticle energy in the superconductor in equilibrium, given by $E_{\mathbf{p}} = (\xi_{\mathbf{p}}^2 + \Delta^2)^{1/2}$ in the singlet case, and $E_{\mathbf{p}} = (\xi_{\mathbf{p}}^2 + \boldsymbol{\Delta}_{\mathbf{p}}\cdot\boldsymbol{\Delta}_{\mathbf{p}}^*)^{1/2}$ for the triplet case, where $\xi_{\mathbf{p}}$ is the quasiparticle energy in the normal state, measured from the Fermi energy. (We assume time-reversal invariance is valid, so that the nonunitary contributions to $\boldsymbol{\Delta}_{\mathbf{p}}$ vanish.) $\delta\bar{n}_{\mathbf{p}}$ is the deviation of the distribution function from the Fermi function evaluated using the actual quasiparticle energies in the nonequilibrium state, and n_i is the density of impurities. The factor $\mathcal{C}$ contains the coherence factors coming from the Bogoliubov transformation between quasiparticles in the normal and superconducting states, and is given by $\frac{1}{2}[1 + (\xi\xi' - \Delta^2)/(EE')]$ for the singlet case and by $\frac{1}{2}\{1 + [\xi\xi' - \mathrm{Re}(\boldsymbol{\Delta}\cdot\boldsymbol{\Delta}'^*)]/(EE')\}$ for the triplet case. The quasiparticle relaxation time is given by

$$\tau_{\mathbf{p}}^{-1} = (2\pi/\hbar)\, n_i \sum_{\mathbf{p}'} |v|^2 \mathcal{C} \delta(E - E').$$

For a triplet superconductor $\boldsymbol{\Delta}_{\mathbf{p}} = -\boldsymbol{\Delta}_{-\mathbf{p}}$, and therefore the coherence factors average to unity; we find that the scattering rate is determined by the density of quasiparticle states, $N_s(E_{\mathbf{p}})$ for a single spin state and for one branch of the spectrum in the superconductor:

$$1/\tau_{\mathbf{p}} = (1/\tau_N)[N_s(E_{\mathbf{p}})/N(0)]. \quad (2)$$

Here τ_N is the relaxation time in the normal state, and $N(0)$ is the normal-state density of states. On the other hand, for a singlet superconductor, one finds

$$\tau_p^{-1} = \tau_N^{-1}[N_s(E_p)/N(0)](\xi_p/E_p)^2 = \tau_N^{-1}|\xi_p|/E_p$$

which, since the quasiparticle velocity is $v_F\xi_p/E_p$, leads to the well-known result that the quasiparticle mean free path, l, in the superconductor is equal to its value, l_N, in the normal state. Here v_F is the Fermi velocity. For triplet states which have nodes of the gap at points, $N_s(E)$ is proportional to E^2 at low densities, and therefore the mean free path of thermally excited quasiparticles varies as T^{-2} at low temperatures, since their velocities are of order v_F. For states with lines of nodes, $N_s(E)$ varies as E, and the mean free path varies as T^{-1}. Similar results have been independently obtained by Rice, Coffey, and Ueda.[4]

According to elementary kinetic theory, the viscosity is given by an expression of the form $\sum_{\mathbf{p}}(-\partial n_{\mathbf{p}}/\partial E_{\mathbf{p}})p_F^2 v_p^2 \tau_p$ which is easily shown to be of order $N_s(T)p_F^2 v_F^2 \tau(T)$. On making use of Eq. (2), we see that this is of order $N(0)p_F^2 v_F^2 \tau_N$, the temperature-independent normal-state viscosity. For the Anderson-Brinkman-Morel (ABM) state, detailed calculation yields for the *zz,zz* component of the viscosity, with z being the polar axis, a low-temperature limit which is $\frac{5}{2}$ times the normal-state viscosity. Thus the viscosity tends to a temperature-independent value for triplet states with nodes either at points or on lines. The corresponding expression for the thermal conductivity, κ, takes the form $T^{-1}\sum_{\mathbf{p}}(-\partial n_{\mathbf{p}}/\partial E_{\mathbf{p}})(E_{\mathbf{p}}v_{\mathbf{p}})^2\tau_p$ which is of order

$$N_s(T)Tv_F^2\tau(T) = N(0)v_F^2\tau_N T \sim \kappa_N(T_c)(T/T_c),$$

where κ_N is the normal-state thermal conductivity. These conclusions are confirmed by solving the Boltzmann equation exactly.

To go beyond the Born approximation we follow the calculations of Salomaa, Pethick, and Baym,[5] who considered scattering of quasiparticles from negative ions in ^{3}He-A. The present problem is simpler. First, since we are interested only in states close to the Fermi surface, we may eliminate those far away, and write the equation for the scattering matrix $\mathcal{T}$ in terms of the normal-state K matrix, which we take to be independent of initial and final momenta. $\mathcal{T}$ then depends only on energy, and is given in Nambu space by $\mathcal{T} = \mathcal{K} + \mathcal{K}(\sum_{\mathbf{p}}\mathcal{G})\mathcal{T}$, where the propagator is

$$\mathcal{G}(\mathbf{p},E) = \frac{1}{E^2 - E_{\mathbf{p}}^2}\begin{pmatrix} E+\xi_{\mathbf{p}} & -\Delta_{\mathbf{p}} \\ -\Delta_{\mathbf{p}}^{\dagger} & E-\xi_{\mathbf{p}} \end{pmatrix}, \tag{3}$$

and the sum over $\mathbf{p}$ is to be taken only over states close to the Fermi surface. For states with odd-parity gaps the off-diagonal matrix elements of $\sum_{\mathbf{p}}\mathcal{G}$ vanish, and therefore we find

$$N(0)\mathcal{T} = \frac{N(0)k_N}{\tau_3 + ig\pi N(0)k_N} \equiv \frac{-\tan\delta_N/\pi}{\tau_3 = ig\tan\delta_N} \equiv \frac{-1/\pi}{\tau_3\cot\delta_N - ig(E)}, \tag{4}$$

where τ_3 is the Pauli matrix in Nambu space. Here $k_N = -\tan\delta_N/\pi N(0)$ is the scalar normal-state K matrix, δ_N is the corresponding phase shift and

$$g(E) = \frac{i}{\pi}\int\frac{d\Omega_{\hat{\mathbf{p}}}}{4\pi}\,d\xi_{\mathbf{p}}\frac{E}{E^2 - E_{\mathbf{p}}^2}, \tag{5}$$

the diagonal component of $\sum_{\mathbf{p}}\mathcal{G}$ divided by $-i\pi N(0)$, describes the influence of the superconducting state on the scattering amplitude. We have chosen our normalization such that in the normal state g is unity, in which case the matrix elements of $\mathcal{T}$ are given by the usual result $\mp e^{\pm i\delta_N}\sin\delta_N/\pi N(0)$. In the superconducting state g generally has both real and imaginary parts for energies less than the maximum value of the gap, Δ.

In the limit of strong multiple scattering ($\tan\delta_N >> |g|^{-1}$), $\mathcal{T}$ is simply $[i\pi N(0)g(E)]^{-1} = t_N/g$ and

$$\frac{1}{\tau_{\mathbf{p}}} = \frac{1}{\tau_N}\frac{1}{|g(E_{\mathbf{p}})|^2}\frac{N_s(E_{\mathbf{p}})}{N(0)}, \tag{6}$$

which differs from the Born approximation result, Eq. (2), by the factor $|g|^{-2}$. As a result, the viscosity will no longer be temperature independent, but will be of order $|g(T)|^2\eta_N$. As the energy tends to zero, $g(E)$ likewise tends to zero for any pairing state, and consequently the result for the viscosity holds only down to temperatures $\sim T_1$ at which $|g(T_1)| = \cot\delta_N$, and the strong-multiple-scattering coupling approximation fails. For the ABM state[6] ($|\Delta_{\mathbf{p}}| = \Delta\sin\theta$) a detailed calculation shows that

$$|g|^2 = (\tfrac{1}{2}\pi x)^2 + \{\tfrac{1}{2}x\ln[(1+x)/(1-x)]\}^2$$

for $x<1$ and $\{\frac{1}{2}x\ln[(x+1)/(x-1)]\}^2$ for $x>1$, where $x = E/\Delta$. Thus, since $|g|\propto x$ for small x, at temperatures $T_1 << T << \Delta$ the viscosity and ultrasonic attenuation will be of order T^2,[7] and for $T<T_1$ they will tend to constant values of order $\cos^2\delta_N$ times their normal-state values, and hence far below those in the normal state. An analogous calculation for the thermal conductivity, κ, shows that in the strong-multiple-scattering limit for temperatures $T_1 << T << \Delta$, κ behaves as T^3, and for $T<T_1$ it is of order $\cos^2\delta_N\kappa_N(T_c)(T/T_c)$, since in this latter limit the mean free path varies as $1/T^2$.

We next consider the polar state, which has $\Delta_{\mathbf{p}} = \Delta\cos\theta$ and is representative of states with nodes on lines. Here a detailed calculation shows that

$$|g|^2 = (\tfrac{1}{2}\pi x)^2 + (x\ln\{[1+(1-x^2)^{1/2}]/x\})^2$$

for $x<1$ and $[x\arcsin(1/x)]^2$ for $x>1$. Thus since $|g|^2\propto x^2\ln^2 x$ for small x, the viscosity and ultrasonic attenuation vary as $T^2 \ln^2 T$ while the thermal conductivity varies as $T^3 \ln^2 T$ for $T_1 << T << \Delta$. For $T << T_1$, the viscosity and ultrasonic attenuation are of order $\cos^2\delta_N$ times their normal-state values, and the thermal conductivity is of order $\cos^2\delta_N \kappa_N(T_c)(T/T_c)$.

At first sight it might appear that the transport properties of heavy-fermion metals in the normal state would be altered significantly from those of ordinary metals. The following simple argument demonstrates that this is not the case. From the kinetic-theory expressions above one can see that η_N is of order $np_F l_N$, and that κ_N is of order $nl_N T/p_F$. Quite generally τ_N^{-1} takes the form

$$2\pi n_i |t_N|^2 N(0) = (2/\pi) n_i \sin^2\delta_N / N(0)$$

and $l_N = v_F \tau_N = p_F^2/4\pi n_i \sin^2\delta_N$, from which one sees that η_N and κ_N depend only on the impurity density, the phase shift, and the Fermi momentum, *but not on* m^*. τ_N, on the other hand, is proportional to m^*, so that for comparable impurity densities, effects of impurity scattering on quasiparticle lifetimes in either the normal or superconducting states are far less important in heavy-fermion systems than in normal superconductors. The above calculations of the viscosity are easily extended to allow for anisotropic-deformation-potential contributions[8] which can change the magnitude of the viscosities without altering the asymptotic forms of the temperature dependence found in their absence. Since the magnitude of the ultrasonic attenuation in heavy-fermion systems is comparable to that found in normal metals,[1] then, as Varma[9] has emphasized, the deformation-potential contributions to the viscosity cannot be much larger than the terms we have considered here.

We expect that other states with nodes of the gap at points or lines will have transport properties similar to those calculated above for the ABM and polar states. In the above considerations we have not taken into account the anisotropy of the transport coefficients. This can easily be done, and exact results for the various components of the transport coefficients may be found at arbitrary temperatures. The estimates of the transport coefficients we have made apply to the largest tensor components of the viscosity and thermal conductivity, and are therefore appropriate for all but those particular geometries in which the largest components give no contribution.

Finally we discuss why phase shifts may be close to $\pi/2$ in heavy-fermion systems. A dimensionless measure of the strength of the scattering is $N(0)V$, where V is a typical electron-impurity scattering matrix element. In ordinary metals $N(0)V$ is of order unity or somewhat larger, so that one is not far from the unitarity or somewhat larger, so that one is not far from the unitarity limit and the effects we have discussed would be important.[10] In heavy-fermion compounds there is an additional reason for expecting significant phase shifts. These systems bear a similarity to Kondo ones, where phase shifts can be close to $\pi/2$ in many circumstances,[11] and it would therefore not be surprising if scatterers in heavy-fermion systems had large phase shifts.

One of the important conclusions of this paper is that qualitative features of transport coefficients for heavy-fermion systems are rather insensitive to the particular superconducting state, in particular whether it has nodes of the gap at points or lines on the Fermi surface. The Born approximation predicts mean free paths that increase with decreasing temperature, and hence leads to results for the thermal conductivity and probably for the ultrasonic attenuation which are much larger than those observed experimentally. It is encouraging that when the effects of multiple scattering are taken into account, we obtain results for the ultrasonic attenuation, α, and κ/T which fall off with decreasing temperature, in qualitative agreement with experiment.

Interpretation of ultrasonic-attenuation measurements is difficult because there is no way of separating contributions intrinsic to the sample from end terms and other extrinsic effects. At T_c the extrinsic contribution is comparable in size with the intrinsic one. Consequently it is not known experimentally whether or not α is finite at $T=0$. Our calculations show that for the anisotropic superconducting states, in contrast to a BCS superconductor, one would expect a finite attenuation at $T=0$. Moreover, in comparing theory with experiment it is important to keep in mind that in the temperature range, $0.1T_c \lesssim T \lesssim T_c$, over which most measurements have been performed, there may be significant corrections to the limiting forms of the temperature dependence derived here.

At sufficiently low temperatures one expects the mean free path to increase. Eventually it will become larger than the wavelength of sound, and the sound will no longer be hydrodynamic. The fact that the attenuation measured experimentally has an ω^2 dependence is strong evidence that all experiments to date have been in the hydrodynamic limit.

The temperature dependence we predict for the thermal conductivity is in disagreement with that observed experimentally for UPt_3 and UBe_{13}. This is perhaps not surprising for UPt_3, since the simple model we have used here is unable to account for the rather complicated temperature dependence of its specific heat in the superconducting state.[12] For UBe_{13}, the low-temperature specific heat[13] varies as T^3, as one would expect for an ABM-like superconductor in which the gap has nodes at points. However, both its specific heat and transport properties in the normal state are anomalous, in that C/T is not constant and the electrical resistivity shows a peak at $T \sim 2$ K,[14] while $\kappa_N(T)$ is not proportional to T.[2] Since the twin assumptions of normal-Fermi-liquid theory and impurities as the dominant scattering mechanism fail in the normal state of UBe_{13}, it would not seem unreasonable that the straightforward approach presented here fails to describe the thermal conductivity in the superconducting state; what is perhaps surprising is that it is consistent with the ultrasonic attenuation at low temperatures.

In conclusion we emphasize that the Born approximation may be expected to fail in all calculations of impurity-limited transport in heavy-fermion systems, be they normal or superconducting. We have presented a framework for the inclusion of multiple-scattering processes in calculations of heavy-fermion transport processes in both normal and superconducting states. The extension of our approach to scattering by magnetic impurities, which is likely to be important for UPt_3 and other heavy-fermion systems which display a Fermi-liquid enhancement of the static magnetic susceptibility, to magnetotransport phenomena, and to dirty superconductors, should prove of interest. The inadequacy of a description of transport processes as arising solely from elastic scattering by impurities is clear from the disagreement between theory and experiment for the thermal conductivity of both UPt_3 and UBe_{13}; whether the impurity scattering is energy dependent, or whether additional scattering mechanisms are required, remains an open question.

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