# Neutron-Star Masses as a Constraint on the Nuclear Compression Modulus

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The observed masses of neutron stars are used to shed some light on the current controversy concerning the value of the nuclear compression modulus. We find that values less than about 200 MeV are incompatible with the observed masses.

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There is currently some controversy concerning the compression modulus of nuclear matter at saturation. For a long time, a careful analyses of the giant monopole resonance in nuclei has been accepted as providing this property. The indicated value is around 220 MeV.[1] More recently, Brown and Osnes have claimed that it is much smaller, perhaps as small as 100 MeV.[2] In this note we shall use the observed masses of neutron stars to indicate a lower bound on the compression modulus.[3]

The matter of a star will be arranged in accord with the condition of hydrostatic equilibrium, that is to say the pressure of the matter everywhere balances the force of gravity whose source is the energy density of the star. Thus both ingredients of the equation of state, pressure and energy density, enter the equations of star structure. In a neutron star the concentration of energy is so high that the metric of space-time is curved, and the condition of equilibrium has to be framed in terms of the general theory of relativity, which was done long ago for static spherically symmetric stars by Oppenheimer and Volkoff (OV). The equations are

$$(4\pi r^2)\,dp(r) = -\frac{GM(r)\,dM(r)}{r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}, \quad (1)$$

$$dM(r) = 4\pi r^2 \epsilon(r)\,dr. \quad (2)$$

The first one expresses the balance of net pressure $dp$ acting on a spherical shell with the force of gravity acting on the mass $dM$ of the shell, and the second tells how the mass is to be calculated from the energy density $\epsilon$. For a given equation of state, the OV equations can be integrated outward for an arbitrary central density, until the pressure is zero. The radial coordinate at that point is the radius of the star, and its gravitational mass is given as the solution of the OV equations. By choosing a sequence of central densities, one can generate the star mass as a function of central density for the given equation of state. The mass will have a maximum value, known as the limiting mass. An acceptable equation of state must yield a limiting mass at least as large as the largest known neutron-star mass, about 1.4 solar masses.

In the following we shall fit the parameters of a theory of matter to the known bulk properties of nuclear matter, except for the compression modulus, which is allowed to take a sequence of values between 100 and 285 MeV. We then calculate the equation of state for dense neutron-star matter, which includes hadron degrees of freedom that are not found in the nuclear ground state. For each of these we solve the equations of star structure, to learn which are compatible with the observed star masses.

Since the cores of neutron stars are very compact, having a density of a few times nuclear density, the matter is relativistic. We shall therefore employ a theory of matter that is relativistically covariant. This is the relativistic hadron field theory involving baryons interacting through the exchange of scalar ($\sigma$) and vector ($\omega$) mesons, and in isospin-asymmetric matter, also the isovector $\rho$ meson. This theory, solved in the mean-field approximation, is known to account for the bulk properties of nuclear matter, as well as a large number of single-particle properties of finite nuclei.[4]

For neutron stars we must generalize the theory. Stars are essentially charge neutral because the repulsive Coulomb force is so much stronger than the gravitational one. A star composed solely of neutrons satisfies this condition but is unstable against $\beta$ decay. The neutron at the top of the Fermi sea has enough energy to decay into a proton and electron. So pure neutron stars cannot exist. As the density increases other baryon thresholds will be reached and hyperons also will be present, and perhaps the $\Delta$. Therefore we should allow for a generalized $\beta$ equilibrium in dense neutron-star matter, allowing whatever baryons are dictated by their masses and the interactions to participate. This generalization was carried out previously, and we do not recount the details.[5] The Lagrangean is

$$\mathcal{L} = \sum_B \bar{B}(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu)B - g_\rho\rho_{\mu 3}J_3^\mu + \mathcal{L}_\sigma^0 + \mathcal{L}_\omega^0 + \mathcal{L}_\rho^0 + \mathcal{L}_\pi^0 - \tfrac{1}{3}bm_n(g_\sigma\sigma)^3 - \tfrac{1}{4}c(g_\sigma\sigma)^4 + \sum_\lambda \Psi_\lambda(i\gamma_\mu\partial^\mu - m_\lambda)\Psi_\lambda. \quad (3)$$

Here $B$ denotes a baryon spinor and is summed over $n, p, \Lambda, \Sigma^{-,0,+}, \Xi \ldots$ to convergence. The $\rho$ meson is coupled to the total isospin current $J$, the $\mathcal{L}^0$ are the free Lagrangeans of the mesons, and the last term is summed over the leptons. Another possible coupling for the $\rho$ meson is gauge coupling, which we do not investigate.

If pion condensation occurs, the equation of state will be softened, and the limiting mass will be lowered. Negative pions will condense when the electron chemical potential attains the value of the effective pion mass in the medium. We will consider two limiting cases, one for which the pions do not condense, because of an assumed large effective mass. This will provide an upper bound on the lowest nuclear compression modulus that is comparable with observed neutron stars. The other limiting case will allow free pions to condense with their vacuum mass. They will do so when the electron chemical potential attains the value of the pion mass. This case will be the opposite extreme, because the attraction of pions and nucleons occurs in the $p$ wave, and requires momentum. In a related work which studied the fully developed condensate in neutron star matter, having however only neutron and protons in the baryon population,[6] the effective mass was about 170 MeV, which means that pions would condense at a density threshold somewhat higher than for free negative pions. Allowing the condensation of free pions will therefore provide a lower bound on the acceptable nuclear compression modulus, as concerns the role of pions.

When the field equations following from Eq. (3) are solved with the subsidiary conditions of charge neutrality and chemical equilibrium, we obtain a solution corresponding to neutron-star matter. When they are solved with the subsidiary condition of isospin symmetry, we obtain the solution for symmetric nuclear matter. The parameters of the theory are chosen to reproduce the bulk properties of uniform symmetric matter, except that the compression modulus is treated as unknown and allowed to take a sequence of values. There are five parameters:

$$g_\sigma/m_\sigma, \quad g_\omega/m_\omega, \quad g_\rho/m_\rho, \quad a, \quad b.$$

The bulk properties are $B/A = 15.95$ MeV, saturation density $\rho = 0.145\ \text{fm}^{-3}$, symmetry energy coefficient $a_{\text{sym}} = 36.8$ MeV,[7] the compression modulus $K$, and one additional parameter, say the nucleon effective mass at saturation, which we assume to be around 0.8, and to which the results are insensitive within reasonable variation.

The energy density and pressure are given in this theory by the expressions

$$\epsilon = \tfrac{1}{3} b m_n (g_\sigma \sigma)^3 + \tfrac{1}{4} c (g_\sigma \sigma)^4 + \tfrac{1}{2} m_\sigma^2 \sigma^2 + \tfrac{1}{2} m_\omega^2 \omega_0^2 + \tfrac{1}{2} m_\rho^2 \rho_{03}^2 + n_\pi m_\pi + \sum_B \frac{2J_B+1}{2\pi^2} \int_0^{k_B} [p^2 + (m_B - g_{\sigma B}\sigma)^2]^{1/2} p^2\, dp + \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} (p^2 + m_\lambda^2)^{1/2} p^2\, dp, \tag{4}$$

$$p = -\tfrac{1}{3} b m_n (g_\sigma \sigma)^3 - \tfrac{1}{4} c (g_\sigma \sigma)^4 - \tfrac{1}{2} m_\sigma^2 \sigma^2 + \tfrac{1}{2} m_\omega^2 \omega_0^2 + \tfrac{1}{2} m_\rho^2 \rho_{03}^2 + \frac{1}{3} \sum_B \frac{2J_B+1}{2\pi^2} \int_0^{k_B} \frac{p^4}{[p^2 + (m_B - g_{\sigma B}\sigma)^2]^{1/2}}\, dp + \frac{1}{3} \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} \frac{p^4}{(p^2 + m_\lambda^2)^{1/2}}\, dp. \tag{5}$$

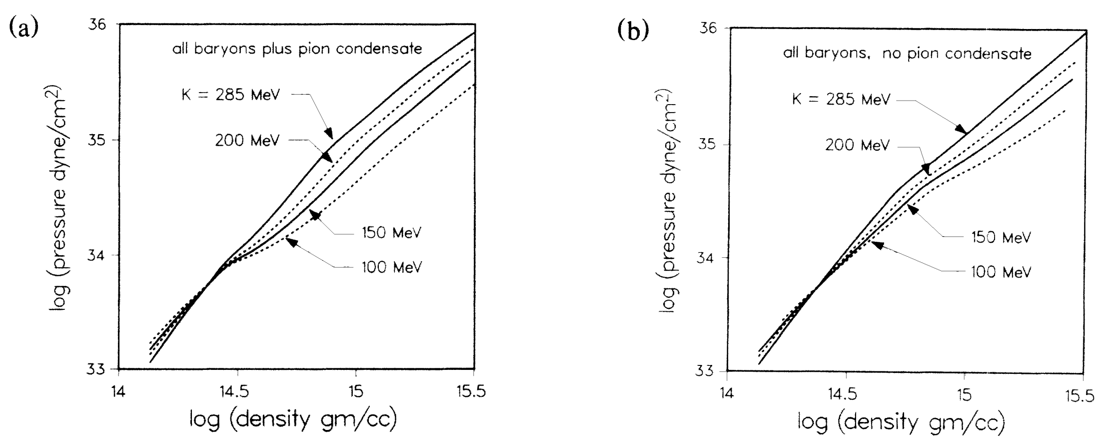


FIG. 1. The equation of state for various compression moduli (a) when free pions condense, and (b) when they do not. These are limiting cases for the role of pions.

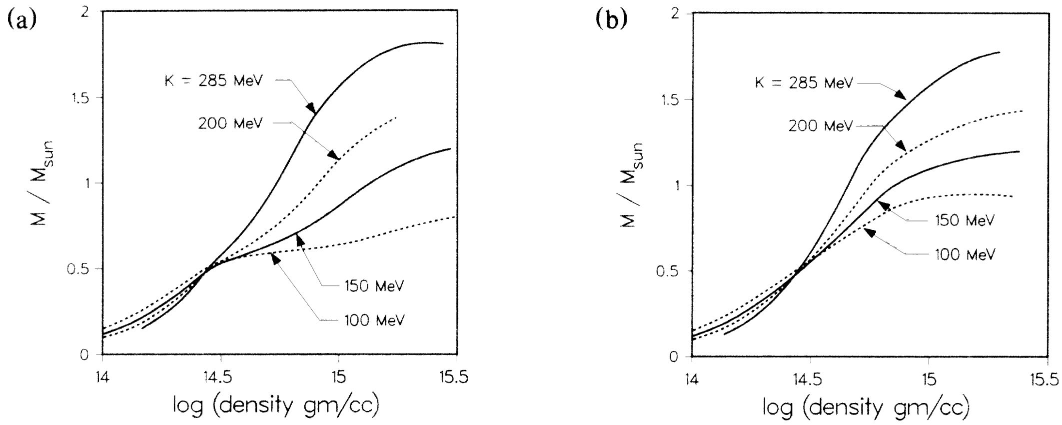


FIG. 2. The masses of neutron stars as a function of their central density for various nuclear compression moduli, in the two cases (a) free pions condense and (b) pions do not condense. These are limiting cases as concerns the effect of pions.

The equation of state showing pressure as a function of energy density is shown in Fig. 1 for several values of the nuclear compression modulus. The parameters of the theory corresponding to the various compression moduli are shown in Table I. The limiting cases regarding the role of pions are shown in the two parts of the figure. By comparison one can see the softening effect of pion condensation. The hyperons also soften the equation of state in comparison to a theory allowing only the neutron and proton.[5] The OV equations for star structure are solved for each of these equations of state, and the results are shown in Fig. 2. Here we see that nuclear compression moduli of less than about 200 MeV are incompatible with the observed neutron-star masses.[8]

The two limiting effects of pion condensation on neutron-star masses are shown in Fig. 3 for $K = 200$ MeV. It can be seen that, in this case, pions do not affect the limiting mass. This is true for $K > 200$ MeV. For $K < 200$ MeV, the softening effect of pions does reduce the limiting mass as can be seen from Fig. 2. In either case, our conclusion that $K < 200$ MeV is incompatible with neutron-star masses is unaffected by the question of whether or not pions condense.

TABLE I. Parameters of the theory that fit the bulk properties of nuclear matter and have an effective nucleon mass about 0.8, but for which the compression modulus takes on the indicated values.

| $K$ (MeV) | $(g_\sigma/m_\sigma)^2$ (fm$^2$) | $(g_\omega/m_\omega)^2$ (fm$^2$) | $(g_\rho/m_\rho)^2$ (fm$^2$) | $a$ | $b$ |
|---|---|---|---|---|---|
| 285 | 9.96 | 5.35 | 6.20 | 0.004 | 0.007 |
| 200 | 8.98 | 3.20 | 6.07 | 0.021 | 0.006 |
| 150 | 8.98 | 1.87 | 6.65 | 0.051 | 0.055 |
| 100 | 8.98 | 0.622 | 6.81 | 0.059 | 1.46 |

In summary, we have solved a relativistically covariant field theory of nuclear matter in the mean-field approximation, for both symmetric nuclear matter and neutron-star matter, involving a generalized beta equilibrium between nucleons, hyperpons, isobars, and leptons and in two limiting cases in which pions condense at an effective mass equal to the vacuum value and in which they do not condense because of an assumed effective mass which is too large compared to the electron chemical potential. The parameters of the theory were fitted to the bulk properties of nuclear matter except that the comparison modulus was treated as unknown. The mass curves of neutron stars were calculated for various assumed values of the compression modulus, and it was found that values less than about 200 MeV are incompatible with observed neutron-star masses.

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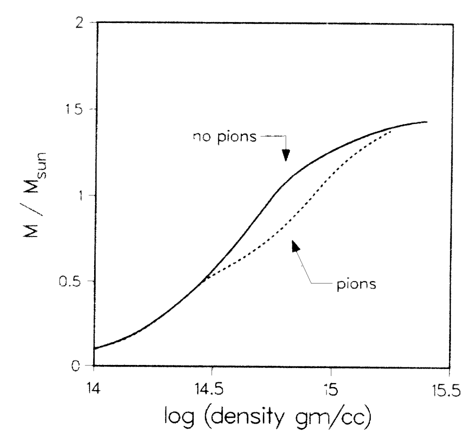


FIG. 3. The effect of pion condensation on neutron-star masses at $K = 200$ MeV.

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