Precise Test for the Unitarized Pion Photoproduction Impulse Amplitude in Exclusive Nuclear Reactions

R. Wittman and Nimai C. Mukhopadhyay
Department of Physics, Rensselaer Polytechnic Institute, Troy, New York 12181
(Received 2 June 1986)

We construct an impulse operator for pion photoproduction from threshold through the Δ region, suitable for nuclear use, with improved Born dynamics and unitarity. Taking the example of the reaction $^{14}\mathrm{N}(\gamma,\pi^+)^{14}\mathrm{C}$(g.s.), we demonstrate superior features of the amplitude at different energies. At or near the Δ resonance, very sizable effects arise both from the unitarity and from the refined treatment of the pionic final-state interaction in the Δ-hole approach.

PACS numbers: 25.20.Lj, 13.60.Le

Because of recent strides at medium-energy electron "factories," precise angular-distribution experiments[1] on photoproduction of pions from light ($A \leq 16$) nuclei, from threshold through the $\Delta(1232)$ resonance region, are now possible. Since the nuclear structure here is generally well understood through other probes, mechanisms for pion production and propagation in the nuclear medium are being explored, complementing research at meson factories.

Theoretical works[2,3] on nuclear charged-pion photoproduction are primarily[4] based on the Blomqvist-Laget (BL) nonunitary construction[5] of the impulse operator. Besides lack of unitarity[6] specific to this nonunitary model, and neglect of one $\gamma N\Delta$ gauge coupling in the resonant Δ channel, the BL approach suffers from the omission of the u-channel Δ contribution, essential[7] in accounting for the $M_{1^-}(T=\frac{1}{2})$ multipole, and a crude nonrelativistic reduction of the magnetic Born terms. Our main objective in this Letter is to provide for a theoretically satisfactory impulse amplitude, free from these difficulties of the BL approach, still suitable for nuclear application. We also avoid direct use of experimental nucleon multipoles to construct the nuclear impulse amplitude. We illustrate the improved features of our elementary amplitude by applying it to the exclusive $^{14}\mathrm{N}(\gamma,\pi^+)^{14}\mathrm{C}$(g.s.) transition, extraordinary[8] because of the suppression of the nuclear Gamow-Teller matrix element, at several energies. Like Tiator and Wright,[3] we do calculations in momentum space, retaining the ease of treating various momentum operators. Unlike them, we do not need to ignore the Coulomb piece of the pion-nuclear optical potential, of some importance at low energy in the ^{14}N example. We also demonstrate very sizable differences between predictions based on the optical potential generated by empirical fits[9] and that by the Δ-hole theory,[10] in our application to the charged-pion photoproduction at high energy. We conclude with an assessment of the adequacy of the distorted-wave impulse approximation in the theory of these reactions.

In our construction of the amplitude t for the process $\gamma N \rightarrow \pi N'$, N and N' being nucleons, we use the pseudovector theory for the nonresonant Born sector, with four Feynman diagrams, having essentially no free parameters, and the most important vector meson contribution in the t channel, that of ω^0. This guarantees agreement with predictions[11] from low-energy theorems, partial conservation of axial-vector current, and current algebra, and is in accord with the near-threshold pion-production data. We also include the u-channel Δ contribution to the background, neglected by BL. This piece is crucial[7] in the reproduction of the experimentally measured structure of the $M_{1^-}(T=\frac{1}{2})$ multipole. The invariant amplitude for this contribution is

$$A_{fi}^{u\Delta} = (e/2M)\bar{u}_f \tau_3 \gamma^5 [g_1(\mathbf{k}\epsilon_\nu - \epsilon k_\nu) + (g_2/2M)(p_\Delta \cdot \epsilon\, k_\nu - p_\Delta \cdot k\, \epsilon_\nu)] 2M_\Delta \times \sum_{s_\Delta} [u^\nu(p_\Delta, s_\Delta)\bar{u}^\mu(p_\Delta, s_\Delta)/(u - M_\Delta^2)](i g_{\pi\Delta N} q_\mu \tau^+ u_i), \quad (1)$$

where $u_{i,f}$ are the hadron spinors, ϵ and k the photon polarization and momentum, q the pion momentum, and τ the appropriate isospin operator; g_1 and g_2 are the $\gamma N\Delta$ gauge couplings, $g_{\pi N\Delta}$ is the strong coupling, $u=(p-q)^2$; p is the initial nucleon momentum, M_Δ and M are the Δ and nucleon mass. In view of the appearance[12] of g_1 and g_2 also in the s-channel Δ photoproduction and decay, these gauge couplings now contribute to both background *and* resonant multipoles. Thus, these must be determined by a combined fit of the background and resonant multipoles. Because of the nature of the u-channel singularity, the contribution of the Δ width to the amplitude (1) is negligible.

We now discuss the procedure of unitarization of multipoles, *thus far ignored in nuclear calculations.* The

© 1986 The American Physical Society

resonant *s*-channel Δ photoproduction[12] and subsequent Δ decay to πN can be written in terms of the $\gamma N\Delta$ gauge couplings, g_1 and g_2, and the parameters for the strong-decay sector. We can unitarize[7] the resonant helicity amplitudes using the Watson theorem:

$$|A_\lambda| \exp(i\delta) = B_\lambda \exp(i\delta_e) - M_\Delta \Gamma_\Delta N_\lambda \exp(i\phi_\lambda)/[S - M_\Delta^2 + iM_\Delta\Gamma_\Delta \gamma(s)]; \quad (2)$$

here B_λ is the background amplitude for helicity $\lambda/2$ ($\lambda = 1,3$) obtained at the tree level; Γ_Δ is the width of Δ; $\sqrt{s}$ is the c.m. energy; $\gamma(s) = \tan(\delta+\delta_e)(M_\Delta^2 - s)/M_\Delta\Gamma_\Delta$; δ_e is deduced[7] from the background contribution to the elastic scattering in the $J = T = \frac{3}{2}$ channel; the product $\Gamma_\Delta N_\lambda$ is a function of kinematic variables and gauge couplings; δ is the πN phase shift in the Δ channel. We can determine the phase angles ϕ_λ from the equation $\sin(\phi_\lambda + \delta_e) = B_\lambda/N_\lambda$. The unitarity in the nonresonant multipoles can be easily restored by the multiplication of a real nonresonant amplitude B_J obtained at the tree level by the Watson factor $\exp(i\delta_J)$, where δ_J is the appropriate *strong* phase shift.

The reluctance of theorists[2,3] to use a unitarized impulse amplitude t in nuclear applications so far stems from the technical difficulty of expressing the amplitude in arbitrary reference frame, since the multipole decomposition is achieved in the πN c.m. frame. Here we come to a crucial result of this paper: *We can express the dominant piece of the unitarized amplitude in a frame-independent fashion.* It is straightforward to show that t is given by

$$t = t_B + t_R(g_1', g_2') + \sum_J^N [\exp(i\delta_J) - 1] t_B^J, \quad (3)$$

where t_B and t_R are *full* nonresonant and resonant amplitudes, the latter evaluated for the effective $\gamma N\Delta$ gauge couplings g_i'; t_B^J are the nonresonant amplitudes for multipoles J (N of them); $\delta_J = \delta_e$ for M_{1+} or E_{1+}, and nonresonant Watson phases otherwise; $g_i' = c_i(s) g_i$, where $c_i(s)$'s are suitable functions of s, g_1, g_2, ϕ_1, and ϕ_3. Thus, only the third term in Eq. (3) requires frame transformation, which *is* well defined for one-shell nucleons. Our explicit computation shows that this is a small correction for the photoproduction processes in $1p$-shell nuclei. This is not surprising, since the δ_J's in Eq. (3) are small.

Before we proceed to discuss nuclear applications, we comment on some limitations of the nonrelativistic reduction scheme of BL.[5] Retaining all terms in the amplitude while putting it in the Pauli form, and comparing it with the nonrelativistic form of BL, we discover important contributions of the magnetic Born terms lost in the BL approximation. We find these to be included by replacing $\boldsymbol{\sigma}\cdot\mathbf{q}$ with $\boldsymbol{\sigma}\cdot[\mathbf{p}_a - (p_a^0/M)\mathbf{p}']$ in $\boldsymbol{\sigma}\cdot\mathbf{q}\,\boldsymbol{\sigma}\cdot(\mathbf{k}\times\hat{\boldsymbol{\epsilon}})/2E_a(p_a^0 - E_a)$, and $(\boldsymbol{\sigma}\cdot\mathbf{q})$ with $-\boldsymbol{\sigma}\cdot[\mathbf{p}_b - (p_b^0/M)\mathbf{p}]$ in $\boldsymbol{\sigma}\cdot(\mathbf{k}\times\hat{\boldsymbol{\epsilon}})\boldsymbol{\sigma}\cdot\mathbf{q}/2E_b(p_b^0 - E_b)$, in Eqs. (4)–(9) of BL, with the same notation.[5] These corrections are $O(\mu_p - \mu_n)$, the difference of proton and neutron magnetic moments, and thus not always negligible.

The differential c.m. cross section for the process $\gamma + A \to \pi + A'$, A and A' being the target and daughter nucleus, respectively, is given in the impulse approximation[3] by

$$d\sigma/d\Omega = \langle T_f \Lambda_f T_i - \Lambda_i | 1\Lambda\rangle^2 \textstyle\sum_{\eta\eta'} \psi_{fi}^{\eta} \psi_{fi}^{\eta'} \sigma_{\eta\eta'} \times 2/[3(2J+1)(2J_i+1)], \quad (4)$$

where i and f are the subscripts indicating initial and final nuclear states; $\eta = [(LS)J]$, the quantum numbers of the nuclear transition operator; and ψ_{fi}^{η} are nuclear reduced density matrix elements in LS coupling. $\sigma_{\eta\eta'}$ are partial cross sections in this representation, proportional to $\sum_m V_{\eta 1}^{m\Lambda} V_{\eta' 1}^{m\Lambda *}$ in the impulse approximation, the V's being the isovector single-particle matrix elements containing the six-dimensional integral[3]

$$I_{(LS)J,1}^{m,\Lambda} = \int d^3q'\, d^3p\, \phi_\pi^\dagger(\mathbf{q}',\mathbf{q}) \rho(p',p) [[Y_1(\hat{\mathbf{p}}')\times Y_1(\hat{\mathbf{p}})]_L \times K_S^\Lambda]_J^m. \quad (5)$$

Here $\phi_\pi^\dagger$ is the pion wave function, ρ is the nuclear single-particle overlap functional, and $\mathbf{p}' = \mathbf{p} + \mathbf{k} - \mathbf{q}'$. All of our improvements in the impulse amplitude are contained in K_S^Λ, with $\Lambda = \mp 1$ for $\pi^\pm$, $\sum_{S=0,1} \sigma_S K_S^\Lambda = L^\Lambda + i\boldsymbol{\sigma}\cdot\mathbf{K}^\Lambda \equiv A$. We can treat the Coulomb contribution in the pion-nuclear final-state interaction, neglected by Tiator and Wright,[3] *exactly* by doing an additional integration. At low energies, where the additional integration due to the Coulomb piece of the pion-nuclear optical potential needs to be done, we have found an approximation to the amplitude A that allows us to do the Fermi momentum integration *analytically* for harmonic-oscillator overlap functionals. This is $A \cong A_{\mathbf{p}=\mathbf{p}_0} + (\mathbf{p}-\mathbf{p}_0)\cdot(\nabla_p A)_{\mathbf{p}=\mathbf{p}_0}$, with $\mathbf{p} = -(A-1)(\mathbf{k}-\mathbf{q}')/2A$, A being the nuclear mass number for the $1p$-shell transition; this works very well up to pion energies of 60 MeV. Fortunately, the effect of the Coulomb piece is only important at low energies (Fig. 1), and is entirely negligible at high energy. So, at high energy, one needs to do the full six-dimensional integration in (5), without using this last trick.

In discussing the nuclear application of our impulse

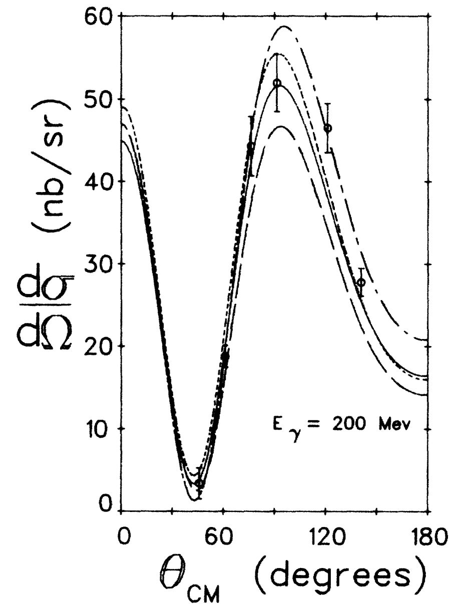

FIG. 1. Comparison of the c.m. cross section for $T_\pi^{\text{c.m.}}=58$ MeV. Data: Cottman *et al.* (Ref. 1). Calculations: solid line, our amplitude; dotted line, our amplitude without Coulomb piece in the pionic final-state interaction (FSI); dashed line, our amplitude without *u*-channel Δ contribution; dot-dashed line, Blomqvist-Laget nonunitary amplitude (Ref. 5).

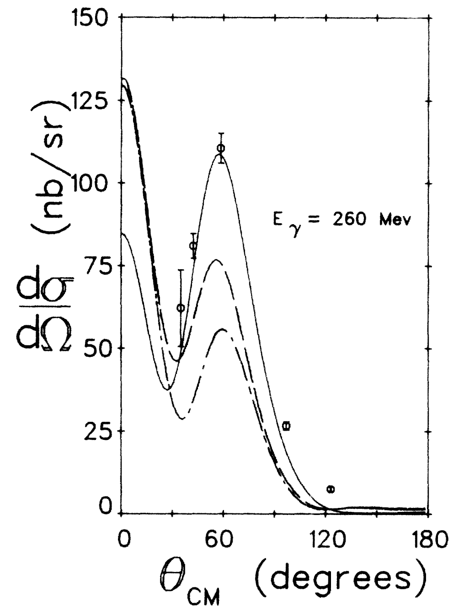

FIG. 2. $T_\pi^{\text{c.m.}}=116$ MeV. Data : Stoler and Seneviratne (Ref. 14). Calculations: solid line; our full amplitude with pionic FSI from the Δ-hole approach (Ref. 10); dashed line, our full amplitude with pionic FSI from Stricker, McManus, and Carr (Ref. 9); dot-dashed line, BL amplitude with the Stricker, McManus, and Carr FSI.

formalism, we consider the charged-pion photoproduction reaction,[1,8]

$$^{14}\text{N}(J^\pi T=1^+0)+\gamma\rightarrow {}^{14}\text{C}(\text{g.s.})(0^+1)+\pi^+,$$

in detail. This process has been shown[8] to be remarkable, because of the suppression of the dominant Gamow-Teller matrix element involved in the β decay and μ capture between the same nuclear states. It has turned out to be excellent in the display of the subtle features of our improved amplitude. It is also quite discriminatory of various pion-nuclear optical-potential models for the final-state interaction. We discuss, in turn, pion photoproduction at low, intermediate, and resonance energies (Figs. 1–3), using effective nuclear one-body amplitudes[13] from fits to other electroweak observables.

At *low energy*[1] ($T_\gamma=200$ MeV, $T_\pi^{\text{c.m.}}=58$ MeV, Fig. 1), calculations with our amplitude without the *u*-channel Δ contribution, and those of BL show a very large difference ($\sim 30\%$) at the peak. This is entirely due to the crude approximation of the magnetic Born terms in the BL approach, as we have checked.

Including the *u*-channel Δ contribution in our amplitude enhances the peak by $\sim 10\%$, entering primarily through the dominant $L=2$, $S=1$ partial cross section. Finally, the Coulomb piece of the pion-nuclear optical potential, neglected by Tiator and Wright,[3] has an effect of about 10% at the peak of the angular distribution for the $^{14}\text{N}\rightarrow {}^{14}\text{C}(\text{g.s.})$ reaction at $T_\pi^{\text{c.m.}}\leq 60$ MeV. It is of no importance at higher energies ($T_\pi^{\text{c.m.}}>100$ MeV).

At intermediate energy[14] ($T_\gamma=260$ MeV, $T_\pi^{\text{c.m.}}=116$ MeV), we discover two important effects. First, the angular distributions (Fig. 2), predicted by use of our unitarized amplitude and the BL nonunitary one, are considerably different, the former about 25% higher than the latter at the peak. This is primarily due to the unitarization of the $M_{1^+}(T=\frac{3}{2})$ single-nucleon multipole as is checked by replacement of this nonunitary BL amplitude with our unitary one, $E_{1^+}(T=\frac{3}{2})$ being responsible for less than a 3% effect. Second, the final-state pion distortion, generated by the Δ-hole approach,[10] makes a large difference when compared with the distortion effects obtained from the empirical optical-potential fits.[9] Our distorted-wave impulse-approximation predictions, using our improved impulse amplitude and the Δ-hole model for the final-state interaction, are in excellent agreement with the preliminary data.[14]

At higher energy ($T_\gamma=320$ MeV, $T_\pi^{\text{c.m.}}=173$ MeV), both the effects discussed in the last paragragh are visible, but we are in substantial disagreement with published data.[1] We can understand the demonstrated importance of unitarity and Δ-hole dynamics in the optical potential at intermediate and high energies for the

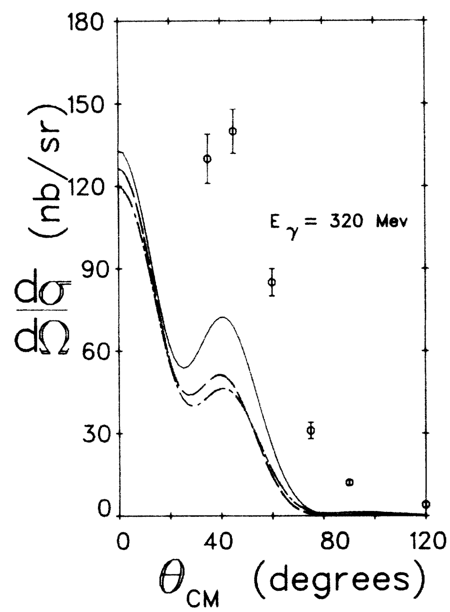

FIG. 3. $T_\pi^{\text{c.m.}} = 173$ MeV. Data, Teng *et al.* (Ref. 1); other legends as in Fig. 2.

$^{14}N \rightarrow {}^{14}C$(g.s.) transition: The normally dominant spin-flip ($L=0$, $S=1$) contribution is suppressed here, and the spin-nonflip contribution ($L=1$, $S=0$), dominated nearly 100% by Δ excitation, interferes strongly with the $L=2$, $S=1$ piece. Can we understand the discrepancy at high energy? We suspect it to be primarily due to a combination of two effects: the inadequacy of the effective[13] one-body nuclear amplitudes, and the neglect of the role of the coherent π^0 photoproduction and subsequent charge exchange. The latter mechanism should be strongly energy dependent, and is under study for a variety of nuclear transitions.

In summary, we have solved here the important theoretical problem of generating an impulse amplitude for pion photoproduction, from threshold through the Δ region, that both is sound in dynamics and is unitary. Our improved amplitude is superior to those developed in the BL approach, both in unitarity and in dynamics. In the example of $^{14}N(\gamma, \pi^+)^{14}C$(g.s.), discussed here, the u-channel Δ contribution and a careful consideration of the magnetic nucleon Born terms are seen important at low energy, while the unitarity effects of the single-nucleon M_{1+} multipole are shown to be paramount at high energy. The Δ-hole approach gives a much better accounting of the pionic final-state interaction effects at the Δ resonance region, compared to the phenomenological approach, as expected.

We thank Professor E. J. Moniz for making the Massachusetts Institute of Technology Δ-hole code (Ref. 10) available to us. We are grateful to our Rensselaer Polytechnic Institute collaborator Mr. R. Davidson and to Professor I. Blomqvist, Professor S. Dytman, Professor J. H. Koch, Professor T.-S. H. Lee, Professor S. Maleki, Professor E. J. Moniz, Professor F. Tabakin, Professor L. Tiator, Professor M. G. Olsson, Professor M. Singham, Professor M. Yamazaki, and Professor L. E. Wright for valuable discussions and communications. We thank Professor K. Min, Professor P. Stoler, Professor E. J. Winhold, and Professor P. Yergin for their enthusiasm and very generous computing support, and for sharing unpublished data. For the latter we also thank Mr. M. D. Seneviratne. This research is supported by the U.S. Department of Energy under Contract No. DE-AC02-83ER40114-A003, and by the Research Corporation. Aspects of this work are in partial fulfillment of the requirements of the Ph.D. degree of one of us (R.W.).

---

[1]B. H. Cottman *et al.*, Phys. Rev. Lett. **55**, 684 (1985); P. K. Teng *et al.*, to be published.

[2]M. K. Singham and F. Tabakin, Ann. Phys. (N.Y.) **135**, 71 (1981); S. A. Dytman and F. Tabakin, Phys. Rev. C **33**, 1699 (1986).

[3]L. Tiator and L. E. Wright, Phys. Rev. C **30**, 989 (1984).

[4]For exceptions see, for example, S. Maleki, Nucl. Phys. **A403**, 607 (1983).

[5]I. Blomqvist and J. M. Laget, Nucl. Phys. **A280**, 405 (1977). This work also provides for an alternative approach, so far unused in nuclear studies, in which the $M_{1+}(T=\frac{3}{2})$ multipole *alone* is unitarized. This follows the unitarization method of M. G. Olsson, Nucl. Phys. **B78**, 55 (1974), and Phys. Rev. D **13**, 2502 (1976), and neglects second gauge coupling to the $\gamma N \Delta$ vertex, the importance of which is discussed by R. Wittman, R. Davidson, and N. C. Mukhopadhyay, Phys. Lett. **142B**, 336 (1984), and R. Davidson, N. C. Mukhopadhyay, and R. Wittman, Phys. Rev. Lett. **56**, 804 (1986), and to be published.

[6]Wittman, Davidson, and Mukhopadhyay, Ref. 5.

[7]Olsson, Ref. 5.

[8]A. Figureau and N. C. Mukhopadhyay, in *Meson-Nuclear Physics—1976*, edited by P. D. Barnes, R. A. E. Eisenstein, and L. S. Kisslinger, AIP Conference Proceedings No. 33 (American Institute of Physics, New York, 1976), and Nucl. Phys. **A338**, 514 (1980).

[9]K. Stricker, H. McManus, and J. A. Carr, Phys. Rev. C **19**, 929 (1979), and subsequent papers.

[10]B. Karaoglu, T. Karapiperis, and E. J. Moniz, Phys. Rev. C **22**, 1806 (1980); B. Karaoglu, thesis, Massachusetts Institute of Technology, 1982 (unpublished).

[11]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, MA, 1972); N. Dombey and B. J. Read, Nucl. Phys. **B60**, 65 (1973).

[12]Davidson, Mukhopadhyay, and Wittman, Ref. 5.

[13]R. L. Huffman *et al.*, Phys. Lett. **139B**, 269 (1984).

[14]P. Stoler and M. D. Seneviratne, private communications.