$B \to Kl^+l^-$ and Other Rare B-Meson Decays

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We calculate the rate and the CP-nonconservation asymmetry for the rare decay $B \to Kl^+l^-$ ($l = e$ or μ). In the standard model with three generations we find the branching ratio to be of the order of 10^{-6}, and the CP-nonconservation asymmetry at most about 1%. The decay proceeds via the flavor-changing loop structure of the standard model, and unlike $K \to \pi e^+ e^-$ suffers few uncertainties and can therefore be used as a probe of new physics. We also comment on other rare decay modes of the B meson.

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One interesting type of physics that the Superconducting Super Collider (SSC) as well as other upcoming machines will allow is the study of rare-B-decay branching ratios down to the level of 10^{-7} to 10^{-8}.[1,2] Furthermore, new detection techniques,[2] such as vertex detectors, and the fact that the B meson has a relatively large lifetime[3] will be helpful in these experiments.

The interest in rare B decays[1,4-9] stems from the possibility of measuring hitherto unmeasured higher-order corrections to the standard model, including non-Abelian couplings. Kaon decays are similar in these and in other respects, but suffer from uncertainties due to long-distance effects which are very difficult to disentangle reliably from short-distance contributions.[10] In this Letter we consider a rare B decay, namely $B \to Kl^+l^-$ ($l = e$ or μ) which seems to be the most promising candidate for a measurable rare decay process, and which proceeds via the flavor-changing weak transition in the standard model. With three generations, we find that the branching ratio for the inclusive decay $B \to Kl^+l^- + X$ ranges from around 2×10^{-6} to 3×10^{-6} as m_t changes between 40 and 240 GeV, with very little sensitivity to unknown mixing angles. The CP-nonconserving asymmetry which depends of course on mixing angles is also considered, and is found to be limited to about 1.5%. The exclusive mode $B \to Kl^+l^-$ ($l = e$ or μ) is about a factor of $\frac{1}{2}$ as abundant as the inclusive one and exhibits a similar asymmetry. Any significant deviation of the experimental results from our predictions would indicate a departure from the standard model[11] indicating interesting new physics such as the existence of a fourth generation, supersymmetry, flavor-changing neutral currents, etc. Other rare decay modes of B mesons,[12] including purely nonleptonic ones, will also be briefly discussed.

The matrix element for the quark-level process (see Fig. 1) is[13]

$$\mathcal{V}^{\text{inclusive}} = -i(G_F/2\sqrt{2})(\alpha/\pi)G_1\bar{u}_2\gamma^\mu(1-\gamma_5)u_1\bar{u}_4\gamma_\mu v_3, \tag{1}$$

where

$$G_1 = A_c(F_1^c - F_1^t) + A_u(F_1^u - F_1^t), \tag{2}$$

with $A_j = U_{sj}U_{jb}^\dagger$, the U's being elements of the Kobayashi-Maskwa matrix[14] ($j = u,c,t$), and $A_u + A_c + A_t = 0$. In general two classes of diagrams, denoted by R and Λ (see Fig. 1) contribute to the form factors F_1^j. We use the calculation reported elsewhere,[15,16] and immediately observe that for the top-quark contribution, $k^2 \approx 0$ can be safely assumed. The resulting values[17] of F_1^t for $m_t = 40$, 60, 80, 160, and 240 GeV are respectively 0.62, 0.28, 0.05, -0.45, and -0.70. Let us now discuss the calculation of $F_1^j(k^2)$ with $j = c$ or u. First, note that if $\hat{m}_j^2 << 1$ with $\hat{m}_j = m_j/M_w$, the contribution of the unphysical Higgs scalar is negligible.[18] Furthermore, it is straightforward to show that unlike the t-quark case, only the Λ diagrams contribute dominantly for $j = u,c$, since $|\ln \hat{m}_j^2| >> \hat{m}_j^2$, and the logarithm is present in Λ, but absent from R. We then obtain

$$F_1^j(k^2) = -2e_j\int_0^1 dy\int_0^{1-y} dx[y(x-1)+2x(x-1)]/[y+\hat{m}_j^2(1-y)-\hat{k}^2x(1-x)], \tag{3}$$

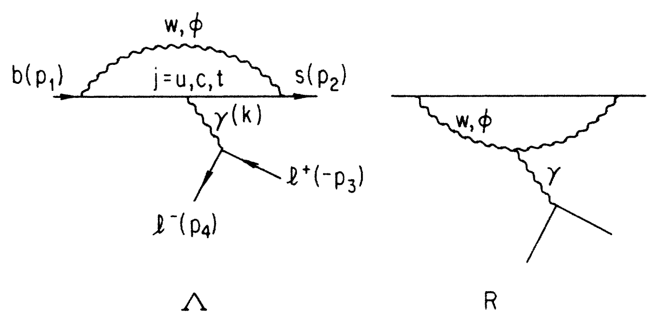

FIG. 1. Diagrams Λ and R (where each stands for a set of diagrams) for $b \to s l^+ l^-$. ϕ is the unphysical Higgs boson required in R_ξ gauges.

TABLE I. Values of the coefficients a_i and integrands I_i appearing in branching ratios and asymmetries. F_1^j given in Eqs. (4) and (5) are to be evaluated at $k^2 = zm_b^2$.

i	a_i	I_i
1	$(\mathrm{Re}A_c)^2$	$(\mathrm{Re}F_1^c - F_1^t)^2$
2	$2A_u\mathrm{Re}A_c$	$(\mathrm{Re}F_1^c - F_1^t)(\mathrm{Re}F_1^u - F_1^t)$
3	A_u^2	$(\mathrm{Re}F_1^u - F_1^t)^2$
4	$(\mathrm{Im}A_c)^2$	$(\mathrm{Im}F_1^c)^2$
5	$-2A_u\mathrm{Im}A_c$	$(\mathrm{Re}F_1^u - F_1^t)\mathrm{Im}F_1^c$
6	$(\mathrm{Im}A_c)^2$	$(\mathrm{Re}F_1^c - F_1^t)^2$
7	$(\mathrm{Re}A_c)^2$	$(\mathrm{Im}F_1^c)^2$
8	$2A_u\mathrm{Re}A_c$	$\mathrm{Im}F_1^c\,\mathrm{Im}F_1^u$
9	A_u^2	$(\mathrm{Im}F_1^u)^2$
10	$2A_u\mathrm{Im}A_c$	$(\mathrm{Re}F_1^c - F_1^t)\mathrm{Im}F_1^u$

where y,x are Feynman parameters, $\hat{k}^2 = k^2/M_W^2$, and $e_j = \frac{2}{3}$ is the charge of j. Equation (3) leads to

$$\mathrm{Re}F_1^j(k^2) = -4e_j \int dx\, x(1-x)\ln|\hat{m}_j^2 - \hat{k}^2 x(1-x)|, \tag{4}$$

and

$$\mathrm{Im}F_1^j(k^2) = [-\tfrac{1}{2}e_j\pi(b_+^2 - b_-^2 - \tfrac{1}{3}b_+^3 + \tfrac{1}{3}b_-^3)]\theta(k^2 - 4m_j^2), \tag{5}$$

with $b_\pm = 1 \pm [1 - 4(m_j^2/k^2)]^{1/2}$.

Let us first consider the inclusive rate $B \to K l^+ l^- + X$ which we take to be equal to the quark-level process $b \to s l^+ l^-$. After squaring V in Eq. (1) and integrating over the three-body phase space one finds

$$\Gamma(B \to K l^+ l^- + X) = (G_F^2 M_b^5/192\pi^3)(\alpha/2\pi)^2 \int_{z_{\min}}^1 dz(1-z)^2(1+2z)|G_1(k^2 = zm_b^2)|^2, \tag{6}$$

where $z_{\min} = 4(m_l/m_b)^2$ and G_1 is given in Eq. (2). We can rewrite the above equation as an expression for the inclusive branching ratio

$$B(B \to K l^+ l^- + X) = \frac{Te^4}{\pi^4}\sum_{i=1}^{10} a_i \int_{z_{\min}}^1 dz(1-z)^2(1+2z)I_i, \tag{7}$$

where a_i, I_i are given in Table I, and T is defined through the B lifetime,[19]

$$\tau_B = 10^{-12}T \text{ sec}, \tag{8}$$

with $1 < T < 2$.[3] It is obvious that since from experiment[20]

$$B(b \to u) = 0.04y, \quad y \leq 1, \tag{9}$$

then the c and t quarks contribute more than the u quark. Although T has the indicated uncertainty at present, the branching ratio in Eq. (7) is practically independent of T, since T^{-1} appears in Kobayashi-Maskawa angles in the numerator of Eq. (7) through a_i.[21] In Fig. 2 the inclusive branching ratio is shown (solid line) as a function of m_t for[22] $s_8 = 0.1$, $c_8 > 0$, $y = 0.5$, and $T = 1$. s_2 and s_3 are given by[23]

$$s_3 = (10^{-3}y/T)^{1/2},$$
$$s_2 = (10^{-3}/T)^{1/2}[-c_8 y^{1/2} + (3 - s_8^2 y)^{1/2}]. \tag{10}$$

As s_8, y, T vary over their allowed region (including a reversal of the sign of C_8) the branching ratio changes

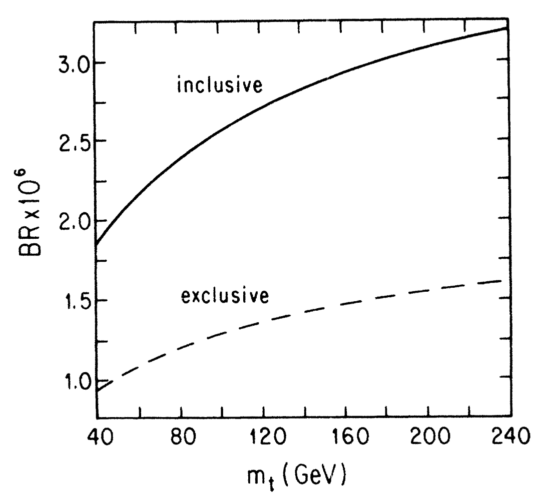

FIG. 2. Branching ratios for the inclusive $B \to K l^+ l^- + X$ (solid line) and exclusive $B \to K l^+ l^-$ (dashed line) processes.

by at most 10%.

Let us now consider the CP-nonconserving asymmetry which should exhibit itself through a rate difference between B^+ and B^-:

$$a=\frac{\Gamma_{\bar{b}}-\Gamma_b}{\Gamma_{\bar{b}}+\Gamma_b}=\frac{a_5\int dz(1-z)^2(1+2z)(I_5-I_{10})}{a_5\int dz(1-z)^2(1+2z)(I_5-I_{10})-\Gamma_b},\qquad(11)$$

where Γ_b is the inclusive width for $B\to Kl^+l^-+X$ [Eq. (7)], and a_i,I_i were given in Table I. Since a_5 is proportional to A_u, only when $A_u\neq 0$ is there a possibility of CP asymmetry ($a\neq 0$), and when we vary s_δ, y, and T over their allowed region the asymmetry changes accordingly. From Eqs. (10) and (11) we obtain

$$a=\tfrac{2}{3}s_1^2y^{1/2}s_\delta R[-c_\delta y^{1/2}+(3-s_\delta^2y)^{1/2}],\qquad(12)$$

where

$$R=\frac{-\int dz(1-z)^2(1+2z)(I_5-I_{10})}{\int dz(1-z)^2(1+2z)I_1}.\qquad(13)$$

The maximum value allowed for the asymmetry is obtained for $s_\delta=0.87$, $c_\delta<0$, and $y=1$. It is given by

$$\max(a)=0.06R.\qquad(14)$$

In Fig. 3 we plot the value of max(a) (solid line) as a function of m_t; it is of the order of 1%–1.5%.

We now discuss the exclusive decay $B\to Kl^+l^-$ which is much easier to reconstruct. The estimate of the exclusive mode is subject to an uncertainty due to the transition from the quark to the meson level. We use

$$\langle K^+|\bar{s}\gamma_\mu b|B^+\rangle=f^+(k^2)(2p_2+k)_\mu+f^-(k^2)k_\mu\qquad(15)$$

(for definition of momenta see Fig. 1). Therefore, the matrix element for the exclusive process is

$$\mathcal{V}^{\text{exclusive}}=-i\left(\frac{G_F}{2\sqrt{2}}\right)\frac{\alpha}{\pi}G_1f^+(2p_2+k)^\mu\bar{u}_4\gamma_\mu v_3.\qquad(16)$$

After squaring, integrating over phase space, and using $f^+(k^2)=m_B^2/(m_B^2-k_2)$, we find

$$B(B\to Kl^+l^-)=(Te^4/2\pi^4)\sum a_i\int_{z_{\min}}^1dz(1-z)I_i,\qquad(17)$$

where a_1,I_i are in Table I. The exclusive branching ratio and the corresponding asymmetry[24] are plotted in Figs. 2 and 3 (dashed lines). The rate is smaller by about a factor of 2 as compared to the inclusive case. Clearly, the largest uncertainty is due to our ignorance regarding the form factor in Eq. (15), i.e., our choice of f^+ as having a form motivated after a similar form which successfully parametrizes kaon[10] and D decays.[25]

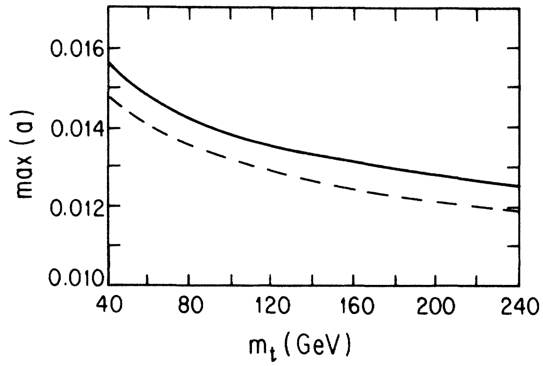

FIG. 3. The maximum value of the CP-nonconservation asymmetry a [defined in Eq. (11)] vs m_t for the inclusive (solid line) and exclusive (dashed line) processes.

We therefore find that the branching ratios are of the order of 10^{-6}, and measurable, while the asymmetries are bounded by 1% or so. As emphasized, we have estimated the long-distance (LD) contributions[26] to $B\to Kl^+l^-$ and find them to be less than 2% of the short-distance (SD) contribution.[27] The calculation involves virtual photon emission preceded or followed by $B\to K$ transition. This transition is estimated from the effective nonleptonic Hamiltonian which is dominated by the penguin graph. Using vacuum saturation we find that the ratio of LD to SD amplitude is less than

$$\frac{2\alpha_s(m_b^2)F_BF_K\ln(m_t/m_b)}{m_\rho^2\ln(m_t/m_c)}<0.02.$$

Details of this calculation will be presented elsewhere. A source of background for the processes we have considered arising from the decay of charmonia produced in $B\to K\psi$ or $B\to K\psi X$ can be easily removed by cutting of the dilepton mass around the mass of ψ.

Finally we note that there are other rare decay modes of B mesons which proceed via the flavor-changing loop.[28] First, there are the $B\to K+\gamma+X$ (no charm) and[29] $B\to K^*+\gamma$ decays[30] which proceed through diagrams similar to Fig. 1.[13] The branching ratios are of the order of 10^{-4}. Then there are exclusive modes which receive no contribution except for higher-order loops in the standard model, for example[5,8,9] $B_d\to\phi K_S$, $B_s\to\phi\phi$. The branching ratios are difficult to estimate reliably. They may be well above 10^{-5}.[8] All these decays should also exhibit CP nonconservation in the standard model (unless $U_{bu}=0$) which will be hard to detect, since it is again expected to be at most of the order of 1% in the standard model as discussed above.

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[19]We then find $\Gamma_B = (G_F^2 m_b^5/192\pi^3)(1/64T)$ GeV, in terms of the measurable quantity T, if $m_b = 4.5$ GeV (substituting m_B instead of m_b will change the constant $\frac{1}{64}$ in the above equation for Γ_B, but will not affect any of our results).

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