

### Comment on "Evidence for a Phenomenological Supersymmetry in Atomic Physics"

In a recent Letter,<sup>1</sup> Kostelecký and Nieto give a very interesting construction of a supersymmetric Hamiltonian for the nonrelativistic radial Coulomb problem. However, their interpretation—for instance, that the Li spectrum is a supersymmetric partner of the H spectrum—is unconvincing.

The radial Schrödinger equation for  $s$  waves in H is

$$\left[-\frac{1}{2} \frac{d^2}{dr^2} - 1/r - E_n\right] \chi_n(r) = 0,$$

with eigenvalues  $E_n = -\frac{1}{2}n^{-2}$ ,  $n = 1, 2, 3, \dots$ . I will use atomic units throughout ( $\hbar = m = e = 1$ , energy unit 27.21 eV). Also, I have removed a linear derivative term in the usual manner by factoring  $1/r$  from the radial function; the true wave function is in fact  $\chi_n(r)/r$ . The supersymmetric partner is

$$\left[-\frac{1}{2} \frac{d^2}{dr^2} - 1/r + 1/r^2 - E'_n\right] \chi'_n(r) = 0.$$

Since this is formally identical to the radial equation for  $l=1$  partial waves, the eigenvalues are clearly  $E'_n = -\frac{1}{2}(n+1)^{-2}$ ,  $n = 1, 2, 3, \dots$ . The authors<sup>1</sup> would like to interpret  $E'_n$  as the  $s$  states [ $1s^2(n+1)s^2S$  states in the atomic notation] of the Li atom, and this set as the partner to the  $ns$  states of H. This would give a binding energy for the ground state of Li (relative to  $\text{Li}^+$ ) of 3.4 eV to be compared to the actual value of 5.39.

Such comparisons of Rydberg series are really appropriately made not in terms of energies but rather in terms of the quantum defect  $\mu$  in the Rydberg formula,<sup>2</sup> which for these Li states would be written  $-\frac{1}{2}(n+1-\mu)^{-2}$ . Particularly at large  $n$ , a single quantum defect describes the entire series. For  $s$  states in Li, the empirical value<sup>3</sup> is  $\mu = 0.4$  (regardless of the value of  $n$ ). The departure of  $\mu$  from zero and the fact that it is positive (so that the energies lie deeper than in H) reflects the effect of the core, that the penetration of the outer electron into the core makes it see an effective potential *more attractive* than the pure  $-1/r$  Coulomb potential. By contrast, the supersymmetric construction models the core by a *repulsive* angular-momentum-barrier-like potential. Because of the peculiar  $l$  degeneracy of the Coulomb problem, this angular-momentum potential gives  $\mu = 0$ . Note, however, that because the supersymmetric partner does not have the lowest eigenvalue of the original Hamiltonian, the unity in the  $n+1$  in the above energy expressions is properly built in.

In their comparison with empirical values, the authors present numerical entries whose seeming coincidence misleadingly hints at the goodness of supersymmetry. What they do is compare energy differences (as stated above, the appropriate variables to use

are quantum defects, not energies or energy differences) of successive  $n$  levels. Here they turn not to  $s$  states of Li but the  $d$  states where the energy differences are in fact closely similar to those in H. But all that this reflects is an elementary atomic fact that higher partial waves are negligible at small  $r$ , therefore do not penetrate the core, and, as a result, are hydrogenic in all atoms. The underlined entries in the tables can, therefore, hardly be considered evidence of supersymmetry.<sup>4</sup> Consistency of their argument requires looking at the  $s$  states of Li, and their quantum defects; such a comparison immediately points to how poor is the evidence for supersymmetry ( $\mu = 0.4$  is a large departure from  $\mu = 0$ ). Furthermore, since their radial function is that of  $l=1$ , it has even the wrong behavior near the origin to describe  $s$  states (and, for example, hyperfine structure) of Li. Their function ( $2p$ ) also lacks a radial node that is present in the ground state of Li. An alternative supersymmetric construction that does not suffer from such deficiencies will be presented elsewhere.<sup>5</sup>

In summary, the construction of the authors can only be used to say the following. The coincidence of energy levels of H for some  $l$  with those of  $l-1$  (except for one extra lowest state in the latter) can be viewed as reflecting supersymmetry of the  $-1/r$  potential. For high  $l$  values ( $l > 3$ ) in any atom, insofar as the spectrum is hydrogenic because of no core penetration, the same coincidence obviously prevails. But there are no grounds for claiming connections between the spectra of one atom and another.

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<sup>1</sup>V. Alan Kostelecký and M. M. Nieto, Phys. Rev. Lett. **53**, 2285 (1984).

<sup>2</sup>H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, Berlin, 1957), p. 134.

<sup>3</sup>As extracted from data in C. E. Moore, *Atomic Energy Levels*, National Standard Data Reference Series NBS 35 (U. S. GPO, Washington, D. C., 1971), Vol. I.

<sup>4</sup>Table II actually adds little to Table I, the underlined entries and their agreement with the numbers in the second column being no more than a repeat of the earlier table except that all numbers are 4 times larger. This is simply an atomic fact that in an isoelectronic comparison, hydrogenic energies scale like the square of the effective charge.

<sup>5</sup>R. W. Haymaker and A. R. P. Rau, unpublished.