Drastic Increases of Frequency and Damping of a Superconducting Vibrating Reed in a Longitudinal Magnetic Field

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Drastic increases of resonance frequency (factor of 7) and damping (factor of >500) of a superconducting reed are observed when a longitudinal magnetic field is applied. The frequency change is much larger than expected from the pole effect (10^5 times) or from the flux-line-tilt modulus (20 times). We give a quantitative explanation with no adjustable parameter. Unpinning of flux lines leads to frequency corrections and to damping. This allows novel precision measurements of extremely weak pinning in amorphous alloys. First experiments demonstrate the feasibility of this method.

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Pinning of flux lines (FLs) in type-II superconductors has remained a subject of active research in the last two decades. In spite of its fundamental and practical importance both in strong-pinning imperfect crystals[1,2] and in extremely weak-pinning amorphous alloys,[3] the quantitative interpretation of experiments in terms of elementary pinning forces[1,4] and their statistical summation[1,5] still poses problems. It is, therefore, desirable to conceive of novel experiments supplementing the standard measurements of current-voltage and magnetization curves[1-3] and the damping of a torsional pendulum studied some time ago.[6]

In this Letter we suggest a highly sensitive precision method to measure the elastic interaction of the FLs with the pins, the FL viscosity, and hysteretic losses of drifting FLs by the frequency and damping of a cantilevered reed performing flexural vibration. A drastic increase of resonance frequency ω (Fig. 1) and damping (Fig. 2) of a vibrating reed ($l \times w \times d = 10 \times 1.4 \times 0.06$ mm^3) of amorphous $Pd_{30}Zr_{70}$ was discovered accidentally when we measured changes of sound velocity and damping caused by two-level systems[7,8] and applied a longitudinal magnetic field in order to suppress superconductivity. These increases vanished abruptly at the upper critical field of our specimen, $B_{c2} = 2.16$ T at $T = 1.3$ K. First "obvious" explanations failed: Berry's magnetomechanical pole effect[9] vanishes in our case since the magnetization is practically zero.[10] Stiffening of the reed by the FL-tilt modulus[1] $c_{44} = B_a^2/\mu_0$ yields an enhancement of ω^2 which is too small by a factor of 0.05 ($= 4d/\pi w$, see below). Finally, the observed superconducting damping, Γ_{sc}, obtained by subtracting from the total damping Γ the small damping Γ_0 measured in the normal state ($\Gamma_0/\Gamma \approx 0.05$ at $B_a = 1$ T), becomes larger than the upper limit Γ_{max} which results for large B_a when

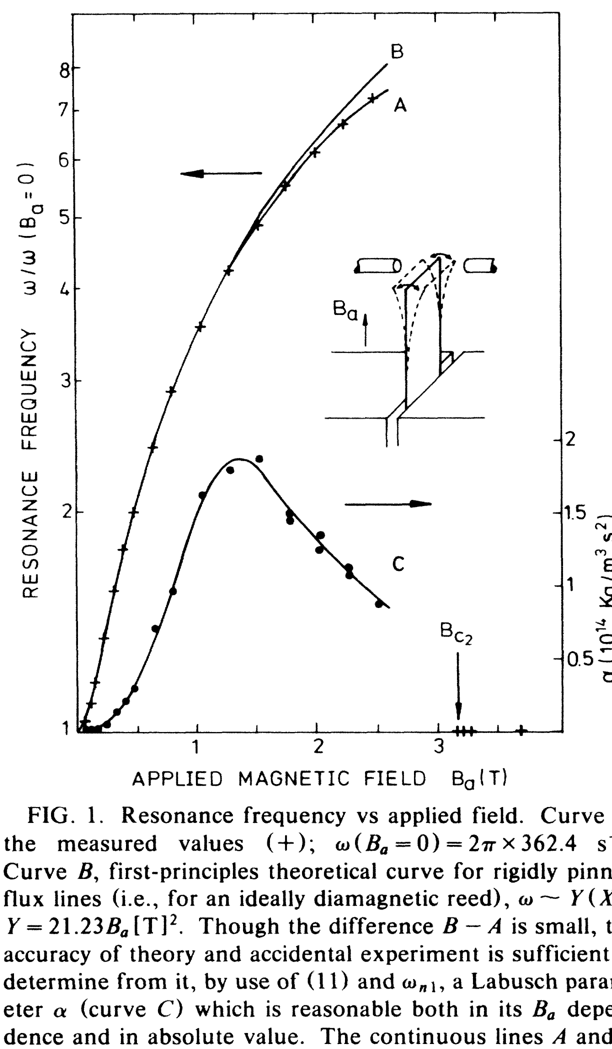

FIG. 1. Resonance frequency vs applied field. Curve A, the measured values (+); $\omega(B_a = 0) = 2\pi \times 362.4$ s^{-1}. Curve B, first-principles theoretical curve for rigidly pinned flux lines (i.e., for an ideally diamagnetic reed), $\omega \sim Y(X)$, $Y = 21.23 B_a[\mathrm{T}]^2$. Though the difference $B - A$ is small, the accuracy of theory and accidental experiment is sufficient to determine from it, by use of (11) and ω_{n1}, a Labusch parameter α (curve C) which is reasonable both in its B_a dependence and in absolute value. The continuous lines A and C are only guides to the eye. The inset shows the vibrating reed.

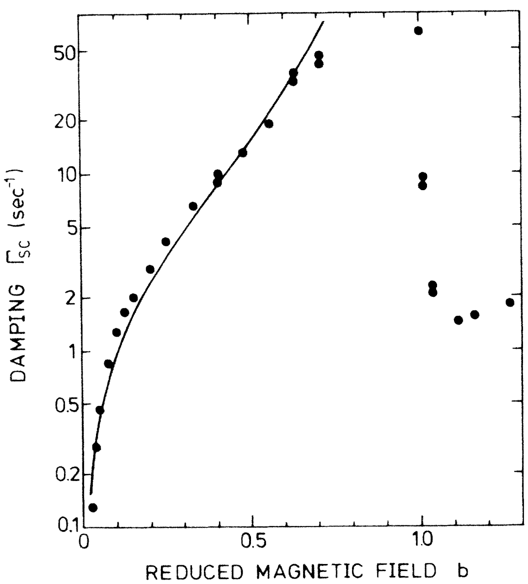

FIG. 2. The measured damping due to superconducting effects Γ_{sc} vs the reduced magnetic field $b = B_a/B_{c2}$; $B_{c2} = 3.16$ T. The curve guiding the eye represents the function $7.5b/(1-b)^2$. A quantitative interpretation of this damping by the explicit version of (10) in Ref. 11 is given in Ref. 7.

the FLs are straight (stretched by B_a) and move viscously relative to the curved vibrating matrix. These apparent paradoxes are solved by the inclusion of surface currents, stray field, and irreversible FL jumps. For ideal pinning our theory contains no adjustable parameter, and small deviations from this ideal case are explained successfully by unpinning (Fig. 1). Here we give the basic ideas and main results.[11]

The essential physics of our problem is as follows: (a) *Statically*, the amorphous reed with very weak pinning and a large Ginsburg-Landau parameter $\kappa_{GL} \approx 60$ behaves like a *nonmagnetic* material: Immediately above B_{c1} the internal field B_i equals B_a and the magnetization is zero [more precisely, $B_a - B_i \approx (B_{c2} - B_a)/2\kappa_{GL}^2$]. Demagnetizing effects are thus completely negligible and the FLs are parallel to the applied field $\mathbf{B}_a \parallel \hat{\mathbf{z}}$. (b) *Dynamically*, for our small amplitude, the reed behaves like a perfect *diamagnet*, with shielding currents $\mathbf{I}_s$ flowing on both surfaces within the effective London penetration depth[12] $\lambda_L \sim 0.5$ μm. These surface currents exert a magnetic restoring force which enhances the vibration frequency, and they drive the FLs, thus contributing to the damping. This is the decisive effect we calculate in the following.

Let the reed occupy the space $|x| \leq d/2$, $|y| \leq w/2$, and $0 \leq z \leq l$, with $l \gg w \gg d$, and oscillate along x with an amplitude $u(z) \ll d$. At the clamped end $u(0) = u'(0) = 0$. The FLs are displaced from their equilibrium position by an amplitude $v(x,y,z) = u(z) + s(x,y,z)$ along x, with $s(x,y,z) \approx S_1(x)S_2(y)s(z)$. For $d \ll w$ the x dependence can be separated exactly, $S_1(x) = d\cosh(x/\lambda)/2\lambda\sinh(d/2\lambda)$, where $\lambda = (B_a^2/\mu_0\alpha)^{1/2}$ is Campbell's penetration depth for compressional waves in the FL lattice[1] and α is Labusch's parameter[5] (the elastic coupling of the FLs to the pins). Since our experiment yields $\lambda > d$ we will put $S_1(x) \equiv 1$ throughout this Letter. The y and z dependences can be separated within a local approximation to a stray-field problem; this is exact in the limit $s \ll u$, i.e., if the pinning is not extremely weak, for $\lambda^2 \ll l^2 d/w$, Ref. 11. Then, though the reed is curved, we may calculate $\mathbf{I}_s$ for $l \gg w$ from the two-dimensional theory of potentials (the problem of laminar flow around a thin plate): $I_{s,y} \sim u''(z)(\frac{1}{4}w^2 - y^2)^{1/2}$, $I_{s,z} \sim u'(z)y/(\frac{1}{4}w^2 - y^2)^{1/2}$. The component $I_{s,y}$ compresses the FL lattice; thus $S_2 \sim I_{s,y}$, or $S_2(y) = (8/\pi w)(\frac{1}{4}w^2 - y^2)^{1/2}$. We chose the averages $\langle S_1 \rangle_x = \langle S_2 \rangle_y = 1$ such that $s(z)$ is the mean FL displacement relative to the reed axis.

Equations of motion for $u(z,t)$ and $s(z,t)$, together with the natural boundary conditions, we derive by minimizing the time average of the Lagrangean $L = T - U$ (kinetic minus potential energy) with respect to u and s (Hamilton's principle). After some calculation we obtain

$$L = (\rho lwd/2)\langle \dot{u}^2 - Au''^2 - Bs^2 - C(u' + \sigma s')^2 \rangle, \tag{1}$$

where z, u, and s are expressed in units of l; $\langle \ldots \rangle$ denotes averaging over z, the dot $\partial/\partial t$, and the prime $\partial/\partial z$. The constants (of dimension s^{-2}) are

$$A = Ed^2/12\rho l^4, \quad B = \alpha\sigma/\rho,$$
$$C = \pi B_a^2 w/4\mu_0\rho l^2 d, \tag{2}$$

where ρ is the density and E the Young's modulus of the reed, and $\sigma = \langle S_2^2 \rangle_y = 32/3\pi^2 = 1.08$. The terms in (1) are the kinetic and bending energy of the reed, the elastic pinning energy combined with the compressional energy, and the field energy (a line tension). This term originates from the surface current ($\sim u'$) induced in a (diamagnetically behaving) reed with rigidly pinned FLs when tilted against the applied field, and from the stray field ($\sim s'$) around a reed with FLs not parallel to its axis. One can prove[12] that FLs tilted against a planar surface cause a stray field even if they do *not* cut this surface. A similar term, but with a smaller prefactor $C' = B_a^2/\mu_0\rho l^2$, results if (by error) only the FL tilt energy is considered, and also from the pole effect.[9,10]

From (2) variational calculus gives us

$$\ddot{u} + Au'''' - c(u'' + \sigma s'') = 0, \tag{3}$$

$$Bs - C(u'' + \sigma s'') = 0, \tag{4}$$

and $u=u'=s'=0$ at $z=0$, $u''=u'''=u'+\sigma s'$ at $z=1$. The viscous-force density $-\eta\dot{s}(x,y,z,t)$ exerted on a moving FL lattice (η = FL viscosity) and other damping mechanisms are treated as weak perturbations, thus

$$\frac{u(z,t)}{u(z)}=\frac{s(z,t)}{s(z)}=\cos(\omega t)\exp(-\Gamma_t)$$

with $\Gamma \ll \omega$. We write $\Gamma=\Gamma_0+\Gamma_{sc}$, $\Gamma_{sc}=\Gamma_v+\Gamma_h$, where Γ_v is the viscous damping and Γ_h a possible hysteretic (amplitude-dependent) damping caused by jumping FLs (elastic instabilities) and vanishing at sufficiently low reed amplitudes. We obtain Γ as the dissipation rate divided by twice the total energy, e.g.,

$$\Gamma_v=(\eta\sigma/2\rho)\langle s(z)^2\rangle/\langle u(z)^2\rangle. \quad (5)$$

Though a general solution of the system (3) and (4) is possible, we will use a more transparent perturbation method. We first solve (3) with $s(z)=0$ (*limit of rigid pinning*) and appropriate boundary condition $Cu'(1)=Au'''(1)$ derived from (1). Writing $X=C/2A$ and $Y=\omega^2/A$ we obtain the solution

$$u(z)=\cosh(\kappa z)-\cos(kz)-c\sinh(\kappa z)+(c\kappa/k)\sin(kz),$$
$$c=(\kappa^2\cosh\kappa+k^2\cos k)/(\kappa^2\sinh\kappa+k\kappa\sinh k),\quad \kappa^2,k^2=(X^2+Y)^{1/2}\pm X, \quad (6)$$

where $Y(X)$ is obtained from the transcendental equation

$$(2X^2+Y)\cosh\kappa\cos k+X\kappa k\sinh\kappa\sin k+Y=0. \quad (7)$$

$X=\mathrm{const}B_a^2$ is the only variable of our theory. The reduced frequency square $Y(X)$ and the functions

$$F(X)=\langle u''^2\rangle/\langle u^2\rangle,\quad G(X)=u'(1)^2/\langle u^2\rangle \quad (8)$$

required below were obtained numerically. For $X=0$, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, and 500 one has $Y=12.3624$, 16.894, 21.220, 29.385, 51.373, 83.921, 143.44, 310.13, 577.32, 1099.6, and 2637.3; $F=12.3624$, 12.471, 12.749, 13.604, 16.865, 21.662, 28.086, 38.554, 49.036, 63.460, and 91.984; $G=7.5791$, 6.650, 5.880, 4.695, 2.714, 1.424, 0.6382, 0.2034, 0.08786, 0.03942, and 0.01432. In zero field ($X=0$) we get the vibrating-reed solution[13]: $\kappa=k=1.8751$ (the lowest solution of $\cos\kappa=-1/\cosh\kappa$), $c=\coth\kappa+\cot\kappa=0.7341$, $u(z)=u_0(z)$, $u(1)=2$, $\langle u^2\rangle=1$, and $Y(0)=\kappa^4$. Limiting expressions are, at low fields ($X\leq 1$),

$$Y=12.3624+9.292X,\quad F=12.3624+X^2/2,\quad G=7.5791-2.046X;$$

and at high fields (with $K=\pi^2/4$),

$$Y=K[(2X)^{1/2}+1]^2+K^2\quad (X\geq 0.7),\quad F=K[K+3+(2X)^{1/2}],\quad G=(K^2/X)[1+5/(2X)^{1/2}],$$
$$k=\pi/2+\pi(8X)^{-1/2},\quad \kappa=(2X)^{1/2}+(\pi^2/8)(2X)^{-1/2},\quad c=1-(2k/\kappa)\exp(-\kappa)\quad (X\gtrsim 5).$$

The reed shape is $u\approx 3z^2-z^3$ for $X\lesssim 1$, and $u\approx(\kappa/k)\sin(\pi z/2)$ for $X\gg 1$, with a *strong curvature* $u''\approx\kappa^2\exp(-\kappa z)$ near $z=0$ forced by the condition $u'(0)=0$.

Next we determine the perturbation $s(z)$ (*effect of unpinning*) from (4) with $u(z)$, Eq. (6), inserted. The solution $s(z)$ is composed of the particular solution Cu''/B and the homogeneous solution, the latter giving a contribution $u'(1)^2$ to $\langle s^2\rangle$ which originates from the boundary condition $s'(1)=-u'(1)/\sigma$. A similar contribution $u'(1)^2$, with larger prefactor, is due to an edge effect:

The longitudinal component of the surface current flows up one half of the reed and comes down the other half. Near the free end of the reed it performs a U turn; this generates there a transverse surface current of total size $(B_a/\mu_0)u'(1)w/2$ which causes additional FL displacements. No such effect occurs at the clamped end since there $u'(0)=0$. At present we can only estimate the prefactor of this contribution, $p\approx 3$. Its precise value follows from three-dimensional potential theory. As B_a increases, this edge term becomes negligible since $u'(1)$ decreases. We finally get the viscous (linear) damping

$$\Gamma_v=\frac{\eta\sigma}{2\rho}\left(\frac{\Lambda}{l}\right)^4\left[\frac{F(X)}{1+r}+\frac{pl}{w}G(X)\right], \quad (9)$$

where $\Lambda^2=\lambda^2\pi w/4d=l^2C\sigma/B$ is an effective penetration depth for FL tilt waves stiffened by the stray field, and $r=\sigma C/(AB)^{1/2}$. The hysteretic (nonlinear) damping caused by irreversible FL jumps we obtain from the general expression

$$\Gamma_h=\langle A[s(z)]\rangle/2\pi\omega\rho\langle u(z)^2\rangle, \quad (10)$$

where $A(s)$ is the area of a $2s$-wide hysteresis loop in the irreversible force displacement curve $F(s)$ of a FL lattice shifted across the pins.[14] For small shifts s one expects $A(s)\sim|s|^3$, and thus Γ_h increases linearly with the reed amplitude $u(l)$ and the resonance curve is no longer Lorentzian. For explicit expressions see

Ref. 11.

For the frequency reduction caused by the total damping $\Gamma = \Gamma_0 + \Gamma_v + \Gamma_h$ and by unpinning we get $\omega^2 = AY(X) - \Gamma^2 - \omega_{\rm pin}^2$ with

$$\omega_{\rm pin}^2 = [B\langle s^2\rangle + C\sigma^2\langle s'^2\rangle]/\langle u^2\rangle$$
$$= C\sigma\left(\frac{\Lambda}{l}\right)^2\left[\frac{F(X)}{1+\frac{1}{2}r} + \left(\frac{pl}{w}\right)G(X)\right]. \quad (11)$$

Since $\omega_{\rm pin}^2 \sim \Lambda^2 \approx l^2 C/B \sim \alpha^{-1}$ is small for strong pinning, nonlocal corrections to the rigid pinning frequency may become important; the main corrections are[11] the replacement of AY by $AY - \omega_{n1}^2$, where $\omega_{n1}^2 \leqslant \beta CF(X)$ $(\beta \approx w^2/2\pi^2 l^2$; $\omega_{n1}^2 \approx \omega^2 w^2/8l^2$ for $2X\beta >> 1)$ and of $F(X)$ in (11) by $F(x/(1+4\beta X))$. The exact nonlocal treatment of the stray field is under way.

We now compare this theory with our accidental measurements. In Fig. 1 we note excellent overall agreement with the rigid-pinning result. From $\rho = 7.8\times10^3$ kg m^{-3}, $d = 63$ μm, $\omega_0 = 2\pi\times362.40$ s^{-1}, and $A = \omega_0^2/12.3642$ we obtain $X = 21.23 B_a[{\rm T}]^2$. Interpreting the small deviation $AY - \omega_{n1}^2 - \omega_{\rm exp}^2$ as $\omega_{\rm pin}^2$ we get from Eq. (11) a pinning strength α which qualitatively agrees with that obtained for other materials by other methods. For $B_a \to 0$, α vanishes and has a maximum $\alpha_{\rm max} \approx 1.9\times10^{14}$ N m^{-4} at $b = B_a/B_{c2} \approx 0.4$. No precise frequency is available above $B_a = 2.48$ T, where many materials exhibit a peak in $\alpha(B_a)$, since for our reed geometry and amplitude the damping was too high (Fig. 2).

This good agreement demonstrates the possibility to obtain $\alpha(B_a, T)$ (which is usually determined by ac methods[1]) from the resonance frequency of a reed, in particular for weak-pinning materials. Additional information on the dynamics of the FL lattice is gained from the reed's damping at various driving amplitudes. Inserting in (9) the FL viscosity[15] $\eta \approx \sigma_n B_{c2}^2 b$ ($\sigma_n = 3.1\times10^5$ Ω^{-1} m^{-1} is the normal conductivity of the reed) we get $\eta/2\rho \approx 200b$ s^{-1} and 0.013 s^{-1} $\leqslant \Gamma_v \leqslant 0.13$ s^{-1} for 0.12 T $\leqslant B_a \leqslant 2.24$ T. This is smaller than our measured value 0.3 s$^{-1} \leqslant \Gamma_{\rm sc} \leqslant 44.4$ s^{-1}. Thus in our experiment, hysteretic damping dominates and we cannot determine α and η separately from (9) and (11). At smaller reed amplitude [our $u(l)$ was about 30 Å at $B_a = 2$ T] the transition from viscous to hysteretic damping should be observable, possibly even in *one* (non-Lorentzian) resonance curve.

Therefore, by measurement at various reed amplitudes the proposed method in principle offers the determination not only of the elastic pinning force α and the FL viscosity η but even the tracing of the entire force-displacement curve $F(s)$ of the FL lattice, including its initial slope α and its saturation value $j_c B_a$ (the maximum volume-pinning force) or critical current density j_c.

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