Ratio of Electric Quadrupole to Magnetic Dipole Amplitudes in the Nucleon-Delta Transition

R. Davidson, Nimai C. Mukhopadhyay, and R. Wittman
Department of Physics, Rensselaer Polytechnic Institute, Troy, New York 12181
(Received 14 May 1985)

We have analyzed available data on magnetic dipole and electric quadrupole amplitudes for the photoproduction of pions around the delta region, in the framework of a phenomenological $\gamma N \Delta$ interaction and background. The unitarity constraint via Watson's theorem limits severely the range of allowed gauge couplings in the $\gamma N \Delta$ vertex. We obtain $(-1.5 \pm 0.2)\%$ as the ratio of the electric quadrupole to magnetic dipole resonant amplitudes; this value is of the same order of magnitude as predicted in the skyrmion model of baryons. This ratio suggests a deformed structure of the nucleon and the delta isobar.

PACS numbers: 13.60.Le, 13.60.Rj, 14.20.Dh, 14.20.Gk

Since the early days of the quark model, the ratio of electric quadrupole to magnetic dipole transition amplitudes (EMR) in the processes $\gamma + N \rightleftharpoons \Delta(1232)$ has been recognized[1] to be a crucial quantity to test theories for effective forces between quarks needed to understand hadron structure. This ratio is particularly powerful to test the one-gluon-exchange prediction[2] for tensor interaction between quarks, inspired by quantum chromodynamics, and other phenomenological approaches[3] of deforming hadron shapes. Unfortunately, the experimental determination of the EMR is not clearcut, as nonresonant background effects[4] must be taken into account before one extracts resonance contributions to the multipole amplitudes. From the recent tabulation[5] of the electromagnetic helicity amplitudes, we can obtain many values for the EMR, from -0.9% to -2.4%, giving an average of $(-1.4 \pm 0.6)\%$. In this Letter we want to determine this ratio more precisely so that it can serve as a powerful discriminant of current hadron models.[2,3]

We report here the result of our analysis to obtain the EMR from the available $M1^+$ and $E1^+$ multipole data[6] on the photoproduction of pions from nucleons, from pion production threshold through the Δ region. We use standard theoretical pictures[4,7-9] for pion photoproduction through background and Δ-resonance mechanisms. Thus, we take pseudovector theory for the nonresonant nucleon Born sector, add leading vector-meson contributions in the t channel (ω contribution being the only important one in our chosen energy range), make use of a general $\gamma N \Delta$ Lagrangean,[7] and insist on unitarity[8] via the Watson theorem.[9] We find that the only unknown parameters in the theory crucial to the determination of the EMR are the two gauge couplings in the $\gamma N \Delta$ vertex. These are determined by a fit to experimental multipoles within a surprisingly narrow range. This, in turn, yields the following value of the EMR: $(-1.5 \pm 0.2)\%$. The magnitude of it is definitely nonzero, in contrast to its zero value in *spherical* hadron bag models.

We should mention here main points of difference between our approach and some of the previous ones. While Metcalf and Walker[10] *fit* the background contributions smoothly, and Crawford[5] and Crawford and Morton[10] apply dispersion relations to determine the background, we use the effective pseudovector Lagrangean theory, plus the vector-meson contributions, for the background, testing our theory in other independent nonresonant channels, for which observed multipoles are nicely reproduced.[11] Metcalf and Walker do not demand a rigorous satisfaction of the Watson theorem, as we do. Finally, we follow the resonance parametrization procedure due to Olsson.[7] This does not require the use of a Breit-Wigner form of the resonance propagator, as is common with most of the previous works. Our improvement over Olsson is in the use of *both* $\gamma N \Delta$ gauge couplings allowed rather than one.

We start with the matrix element for the process $\Delta \to N\gamma$:

$$iM_{fi}^{\Delta} = (e/2M)\bar{u}_f \tau_3 \gamma^5 [g_1(k\epsilon_\nu - \epsilon k_\nu) + (g_2/2M)(p_f \cdot \epsilon k_\nu - p_f \cdot k \epsilon_\nu)] u_i^\nu (p_\Delta, s_\Delta), \tag{1}$$

where $u_{i,f}$ are the hadron spinors, ϵ and k photon polarization and momentum, τ_3 the appropriate isospin operator, and g_1 and g_2 the gauge couplings to be determined by a fit to the electromagnetic resonance multipoles in the photoproduction of pions. The $M1$ and $E2$ mulitpole amplitudes, and equivalently, the helicity amplitudes $A_{1/2}$ and $A_{3/2}$, arising solely due to resonance production $N\gamma \to \Delta$, are given by

$$f_{M1} = \frac{e}{12M}\left(\frac{k}{M_r M}\right)^{1/2}\left[g_1(3M_r + M) - g_2\frac{M_r(M_r - M)}{2M}\right], \quad f_{E2} = -\frac{e}{6M}\frac{k}{(M_r + M)}\left(\frac{kM_r}{M}\right)^{1/2}\left(g_1 - \frac{M_r}{2M}g_2\right),$$
$$A_{1/2} = -\tfrac{1}{2}(f_{M1} + 3f_{E2}), \quad A_{3/2} = -\sqrt{3}/2(f_{M1} - f_{E2}), \tag{2}$$

M, M_r being the nucleon and Δ masses. Thus, once we determine the couplings g_1 and g_2, the electromagnetic amplitudes for the resonance production by photon are known. Note that the EMR is zero if $g_1=(M_r/2M)g_2$, corresponding to the simple quark-model prediction of $A_{1/2}=\sqrt{3}A_{3/2}$.

Since the electromagnetic decay of Δ is not directly observable, we must consider the process $\gamma N\to N\pi$ and extract the resonant contribution to the multipoles $M1^+$ and $E1^+$. To do this, we follow the Olsson procedure[7,8] of unitarization, in which the amplitude A in the 33 channel is written as

$$A=A_B\exp(i\delta_B)+N_R\exp(i\phi)/(\epsilon-i\gamma)=|A|\exp(i\delta), \tag{3}$$

where B and R stand for resonance and background, and δ for the πN phase shift in the 33 channel. Here, the theory predicts everything but ϕ;

$$(\epsilon-i\gamma)^{-1}=\sin(\delta-\delta_B)\exp[i(\delta+\delta_B)].$$

Thus, $\phi=\delta_A-\delta_B$, with $\delta_A=\sin^{-1}(A_B/N_R)$. The *Ansatz* (3) is best when A is resonance dominated; it fails if $|A_B|>|N_R|$. It is fine for $M1^+$. For $E1^+$, $|A_B|\approx|N_R|$, and (3) yields an unsatisfactory fit to the data. Two helicity amplitudes in the resonance channel are dominated by the delta contribution, and (3) is directly useful to unitarize them, yielding another set of *unitarized* multipole amplitudes. While there is no theoretical preference for one of these two ways of unitarizing the multipoles over the other, the latter approach gives statistically satisfactory fits to both the $M1^+$ and $E1^+$ data, and hence it is preferable to us.

In the pseudovector theory, the nucleon Born terms for pion photoproduction consist of four Feynman diagrams, if the vector-meson exchanges in the t channel are omitted, with essentially no free parameters. For the energy region of our interest, we have found ω^0 exchange in the t channel to be important, a fact overlooked by some authors. Its amplitude is given by

$$iM_{fi}^{\omega}=-\lambda_\omega\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu k^\nu q^\alpha(t-M_\omega^2)^{-1}\bar{u}_f[g_{1\omega}\gamma^\beta-i(g_{2\omega}/2M)\sigma^{\beta\tau}(p_f-p_i)_\tau]u_i, \tag{4}$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor, and $M_\omega=782.6$ MeV. From the radiative decay of ω^0, we get $\lambda_\omega=0.36(4\pi\alpha)^{1/2}$, α being the fine-structure constant. From fits to the nonresonant photoproduction multipoles, we obtain $g_{1\omega}\simeq 10$–14, $g_{2\omega}\simeq 0$, in excellent agreement with their literature values.[5,12] The role of ω^0 in the π^0 photoproduction was first stressed by Berends, Donnachie, and Weaver.[13] However, dispersion-theoretic vector-meson treatments require[13] a way of attenuating their contributions near threshold. We avoid this problem by using the effective-Lagrangean approach.

The s-channel Δ photoproduction and its strong decay are given by two electromagnetic gauge couplings g_1 and g_2 [in Eq. (1)], the strong coupling $g_{\pi N\Delta}$, and the delta mass M_r. We obtain a good fit to the 33 phase shift by using $g_{\pi N\Delta}^2/4\pi\simeq 0.314$ and $M_r=1218.4$ MeV, values close to those of Olsson.[7] This leaves g_1 and g_2 to be determined from the $M1^+$ and $E1^+$ multipole data (Fig. 1).

Some remarks regarding our use of the existing experimental multipole data are in order. We use the multipole data sets of Ref. 6, extracted from the analyses of $N(\gamma,\pi)N'$ cross-section and polarization experiments. The $M1^+$ data are of good quality, with excellent overlap between the two sets. However, the errors of the $E1^+$ data are large, and the two sets do not overlap above the c.m.s. energy of 1250 MeV, while below 1150 MeV there exists only one set of data. For an energy common to both sets, we take as effective $E1^+$ multipole value $(y_1+y_2)/2$, where $y_1=\max(x_1+\sigma_1, x_2+\sigma_2)$ and $y_2=\min(x_1-\sigma_1, x_2-\sigma_2)$; (x_i,σ_i) are the quoted experimental multipole value and error, respectively, for a set i $(i=1,2)$. At such energies, we conservatively estimate the error to be $y_1-(y_1+y_2)/2$. At isolated data points, we estimate the error by a linear interpolation of the errors at two nearest energies with data common to both sets. We feel that this conservative approach is sensible for our fit, given the disagreement between the two multipole data sets. The inclusion of the first two data

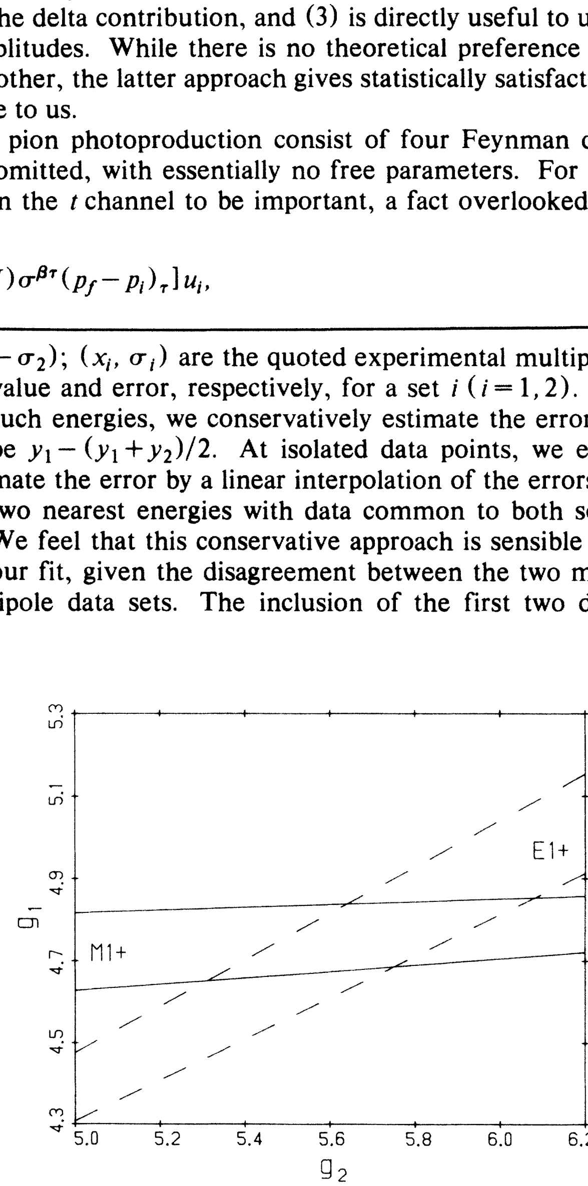

FIG. 1. Range of $\gamma N\Delta$ gauge couplings g_1 and g_2 allowed by fits to the $M1^+$ and $E1^+$ data at the 1% confidence limit, solid and dashed lines, respectively.

points in the $E1^+$ set lowers the quality of fit; however, the extracted value for the EMR is not significantly affected.

In Fig. 1, we show the regions in the g_1-g_2 plane that yield an acceptable fit to the $M1^+$ and $E1^+$ data at the 1% confidence limits for the χ^2 statistic. The acceptable ranges of g_1 and g_2 for the 1% limit are $g_1 = 4.8 \pm 0.1$, $g_2 = 5.7 \pm 0.2$. These yield

$$\mathrm{EMR} = (-1.5 \pm 0.2)\%. \tag{5}$$

Figure 2 displays our fits to the experimental multipoles. Regarding the role of ω exchange in the t channel, the large uncertainty in the ωNN coupling constants $g_{1\omega}$ and $g_{2\omega}$ does not significantly influence the results (5). Around the value $g_{2\omega} \simeq 0$, favored by most analyses,[12] a large variation of $g_{1\omega}$ (in the range 6–12) produces acceptable χ^2, with about 15% variation in the EMR. We recall that the nonresonant background multipole fits restrict $g_{1\omega}$ in the range 8–14, with $g_{2\omega} \simeq 0$.

We summarize, in Table I, the results of our determination of the $N \rightleftharpoons \Delta$ helicity amplitudes and the EMR, comparing them with literature values. The errors quoted in our values generously take into account the uncertainties of our analysis. We stress that applying Olsson's *Ansatz* for unitarization directly on the multipoles would have yielded poorer quality of fit to the $E1^+$ multipole. This would have given[14] somewhat different helicity amplitudes and the EMR extracted would have been $\pm 5\%$. This can be taken as an upper bound for the EMR in our approach. This is not surprising since the empirical electric quadrupole transition amplitude in the $N \rightleftharpoons \Delta$ process, and hence the EMR extracted from the photoproduction data, are very *sensitive* to the treatment of background, resonance, and interference contributions, and the differences in Table I are attributable to the built-in differences in these in various approaches.

In summary, we have analyzed the available data on magnetic dipole and electric quadrupole amplitudes in

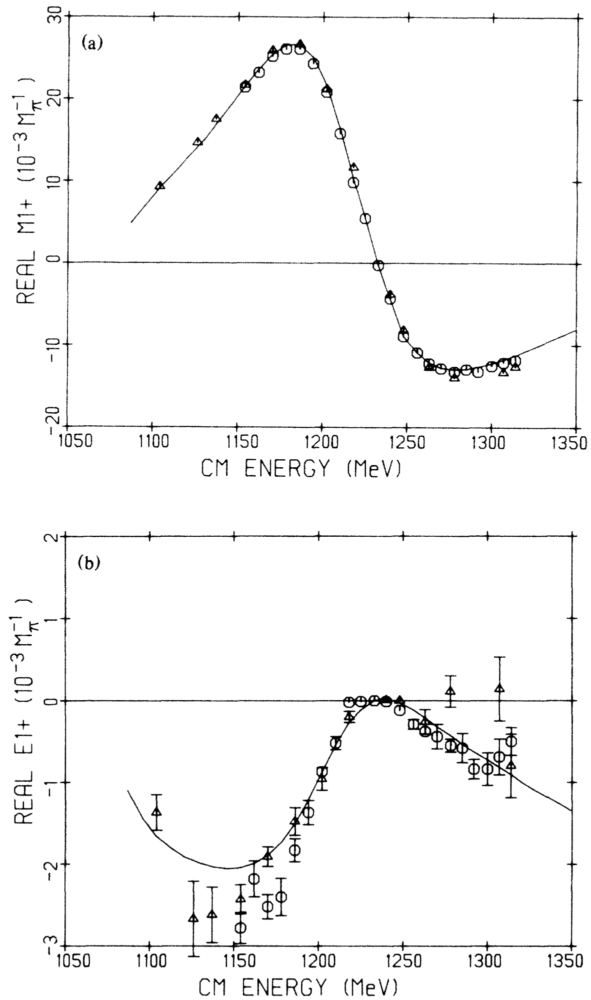

FIG. 2. The real part of (a) $M1^+$ and (b) $E1^+$, in units of $10^{-3} m_\pi^{-1}$, as a function of the c.m.s. energy in megaelectronvolts. Data are from Pfeil and Schwela (Ref. 6) (triangles), and Berends and Donnachie (Ref. 6) (circles). Our best fits are with $g_1 = 4.8$, $g_2 = 5.7$. Error bars are not shown if the error is approximately the size of the figure around the data point.

TABLE I. A comparison of the helicity amplitudes, $A_{3/2}$ and $A_{1/2}$, and the $E2/M1$ ratio (EMR), extracted by various authors used in the analysis of Ref. 5: (B) Barbour *et al.*, (ARI) Arai (Solution 1), (AW) Awaji *et al.*, (CM) Crawford and Morton (energy-dependent analysis), Ref. 5 (PDG) average, and this work.[a] Helicity amplitudes are in units of 10^{-3}.

Observable	B	ARI	AW	CM	PDG avg.	This work
$A_{3/2}$ ($\mathrm{GeV}^{-1/2}$)	-271 ± 10	-264 ± 2	-259 ± 6	-247 ± 10	-258 ± 11	-229 ± 6
$A_{1/2}$ ($\mathrm{GeV}^{-1/2}$)	-142 ± 7	-247 ± 1	-138 ± 4	-136 ± 6	-141 ± 5	-125 ± 3
EMR (%)	-2.37	-0.90	-1.97	-1.2	-1.4 ± 0.6	-1.5 ± 0.2

[a]The errors on our helicity amplitudes and EMR values are obtained by direct propagation of errors of g_1 and g_2, hence the higher precision of our obtained EMR.

the framework of pseudovector Born theory and $\gamma N \Delta$ interaction, insisting on a rigorous constraint of the Watson theorem. Our analysis avoids the Breit-Wigner-type resonance parametrization and yields a narrow range of allowed gauge couplings in the $\gamma N \Delta$ vertex giving the ratio of electric quadrupole to magnetic dipole amplitudes for the $N \rightleftharpoons \Delta$ transition to be $(-1.5 \pm 0.2)\%$. This magnitude is significantly higher than what most hadronic models[2,3] currently predict and is definitely inconsistent with the spherical structure of nucleon and delta isobar, or with the SU(6) and $SU(6)_W$ theorems.[2,15] Only in the skyrmion model[16] of nucleon and isobar does an EMR as large as 5% emerge quite naturally.

Future improvements on the quality of the experimental $E1^+$ multipole data will be very welcome. They will have a strong bearing on a better determination of the EMR.

We thank Professor M. K. Banerjee, Professor J. M. Eisenberg, Professor A. S. Goldhaber, Professor N. Isgur, Professor R. L. Jaffe, Professor F. Lenz, Professor M. Locher, Professor G. A. Miller, and Professor E. J. Moniz for helpful discussions. We thank Professor M. G. Olsson for a useful communication. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-83ER40114-A002.

Note added.—Since the completion of our work, we have received two preprints on related subjects, from S. N. Yang and from H. Tanabe and T. Ohta, now published.[17] Both attempt to model the strong-interaction rescattering, and hence the background-resonance interference effects, introducing additional theoretical parameters. The t-channel ω exchange is either ignored (first work) or approximately treated (second). The extracted EMR are model dependent and are $\sim -4\%$ and $\sim +4\%$, respectively. Note the difference in sign between the two. We thank these authors for communicating to us their results prior to publication.

1G. Murpurgo, in *Theory and Phenomenology in Particle Physics*, edited by A. Zichichi (Academic, New York, 1969), p. 107.

2See, for example, N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978), and **19**, 2653 (1979); S. S. Gershtein and D. V. Dzhikiya, J. Nucl. Phys. **34**, 870 (1981); N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. D **25**, 2394 (1982); K. F. Liu, Phys. Lett. **114B**, 222 (1982); S. Ohta, M. Oka, A. Arima, and K. Yazaki, Phys. Lett. **119B**, 35 (1982); J. Dey and M. Dey, Phys. Lett. **138B**, 200 (1984); M. V. N. Murti and R. K. Bhaduri, Phys. Rev. Lett. **54**, 745 (1985).

3See, for example, S. L. Glashow, Physica (Amsterdam) **96A**, 27 (1979); V. Vento, M. Rho, E. M. Nyman, J. H. Jun, and G. E. Brown, Nucl. Phys. **A345**, 413 (1980); V. Vento, G. Baym, and A. D. Jackson, Phys. Lett. **102B**, 97 (1981); G. Kälbermann and J. M. Eisenberg, Phys. Rev. D **28**, 71 (1983); A. Araki and A. N. Kamal, Phys. Rev. D **29**, 1345 (1984); W. Broniowski and M. K. Banerjee, Phys. Lett. **158B**, 335 (1985).

4R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972), p. 34.

5C. G. Wohl *et al.* (Particle Data Group), Rev. Mod. Phys. **56**, S1 (1984). Here, and in Table I, we are using the partial-wave analyses employed by the Particle Data Group to get their average helicity amplitude.

6W. Pfeil and D. Schwela, Nucl. Phys. **B45**, 379 (1971); F. A. Berends and A. Donnachie, Nucl. Phys. **B84**, 342 (1975).

7M. G. Olsson, Nucl. Phys. **B78**, 55 (1974), and Phys. Rev. D **13**, 2502 (1976). We obtain $\Gamma \sim (WM_R)^{-1}$, rather than W^{-2} given in Eqs. (29) and (47) of the first paper.

8R. Wittman, R. Davidson, and N. C Mukhopadhyay, Phys. Lett. **142B**, 336 (1984).

9K. M. Watson, Phys. Rev. **95**, 228 (1954).

10W. J. Metcalf and R. L. Walker, Nucl. Phys. **B76**, 253 (1974); R. L. Crawford and W. T. Morton, Nucl. Phys. **B211**, 1 (1983).

11R. Wittman, R. Davidson, and N. C. Mukhopadhyay, in *Proceedings of the Tenth International Conference on Particles and Nuclei, Heidelberg, Germany, 30 July–3 August 1984*, edited by B. Povh and G. zu Putlitz (North-Holland, Amsterdam, 1985).

12O. Dumbrajs, R. Koch, H. Pilkuhn, G. C. Oades, H. Behrens, J. J. de Swart, and P. Kroll, Nucl. Phys. **B216**, 277 (1983).

13F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. **B4**, 103 (1967); M. G. Olsson and E. T. Osypowski, Phys. Rev. D **17**, 174 (1978).

14R. Davidson, N. C. Mukhopadhyay, and R. Wittman, in *Hadronic Probes and Nuclear Interactions—1985,* edited by J. R. Comfort *et al.*, AIP Conference Proceedings No. 133 (American Institute of Physics, New York, 1985), p. 384.

15C. Becchi and G. Morpurgo, Phys. Lett. **17**, 352 (1965); H. Harari and H. J. Lipkin, Phys. Rev. B **140**, 1617 (1965).

16G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).

17H. Tanabe and T. Ohta, Phys. Rev. C **31**, 1876 (1985); S. N. Yang, J. Phys. G **11**, 1205 (1985).