Fractal Shape of Hail Clouds

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We present a fractal analysis of severe convective storms using data collected during a recent hail-suppression experiment "Grossversuch IV" in Switzerland. For larger clouds, a fractal shape is observed; an analysis of the perimeter-to-area relation shows the fractal dimension to be $D_s = 1.36 \pm 0.1$, in agreement with the value 1.35 from the theory of relative turbulent diffusion and an earlier analysis of cloud shapes. Moreover, a particle drift, present in strong lateral winds as well as in the center of hail clouds, is presumably responsible for a rather sharp crossover to a smooth behavior ($D_s = 1$) showing the existence of an apparent characteristic length.

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The notion of fractal (or, equivalently, Hausdorff) dimension[1] has proved useful for an analysis of various experimental data (e.g., from polymer solutions[2] and melts, adsorbate domains,[3] electric discharges,[4] and aggregation systems,[5] etc.). Basically, the fractal dimensionality D characterizes any self-similar system: Upon a change of the linear dimensions by a scale factor f, a fractal quantity (e.g., a surface or a contour) will be changed by the factor f^D, for any value of f. The value of the surface dimension D_s is comprised between 2 and 3 (where $D_s = 2$ describes a smooth surface), whereas for the dimension of a contour line $1 \leqslant D_l \leqslant 2$ ($D_l = 1$ for smooth lines). A planar section of a fractal surface has a dimensionality $D_l = D_s - 1$ in general[6] (if pathological cases are disregarded). Up to date there are only a few rather academic models known which allow an exact evaluation of their fractal dimension, and hence the usefulness of the latter stems mainly from numerous experimental observations.[7]

In this work we present an analysis of radar echo data from severe convective clouds showing (i) a nontrivial fractal behavior of larger cloud sections, and (ii) the existence of an apparent characteristic length. The radar information is available as a plan position indication (PPI) (i.e., an XY plot) of nearly horizontal curves of constant radar reflectivity (intensity of the reflected beam). The PPI's were obtained after a full azimuthal (i.e., 360°) revolution of the antenna at a fixed elevation angle of 5.5°. The measuring time was 1 min for 1 PPI, and the radar resolution was 1° in azimuth, 0.3 km in distance, and 1 dBZ in reflectivity. The latter unit is defined as follows: Let

$$Z = \int N(D) D^6 \, dD, \tag{1}$$

where $N(D)$ denotes the number distribution of diameters of the hydrometeors (i.e., Rayleigh scatterers such as rain drops and hailstones) per unit volume and diameter interval D to $D + dD$; Z has units of volume. The validity of the Rayleigh approximation for the distribution of hailstones has been verified recently.[8] (Note that this approximation fails for the determination of the particle shape or composition.) Conveniently, the quantity

$$I = 10 \log_{10}(Z/Z_0) \tag{2a}$$

is used; I is given in dBZ when $Z_0 = 10^{-18}$ m^3, i.e., 1 mm^6/m^3. For a Poisson distribution $N(d) \sim \exp(-\lambda d)$, which describes well the observations, it follows that

$$I = \text{const} - 70 \log_{10} \lambda, \tag{2b}$$

i.e., the values of I are monotonically increasing with the particle size, $\lambda D_{\text{median}} = \text{const}$. Details of the radar data processing, etc., have been described recently by Waldvogel and Schmid.[9]

A typical PPI is shown in Fig. 1, where contours for $I = 45$, 55, 60, and 65 dBZ are plotted. In particular, the 45-dBZ curve represents a heavy rain area (with a precipitation rate $R \gtrsim 20$ mm/h) whereas the 65 dBZ indicates a very heavy rain (with $R \simeq 500$ mm/h) including hail precipitation.

The fractal behavior of the observed hail clouds manifests itself in the fractal properties of the contours with constant I values. The area F within a contour depends fractally on the contour length (i.e., the perimeter) U as

$$F = F_0 U^E, \tag{3}$$

where the exponent E is related to the fractal dimen-

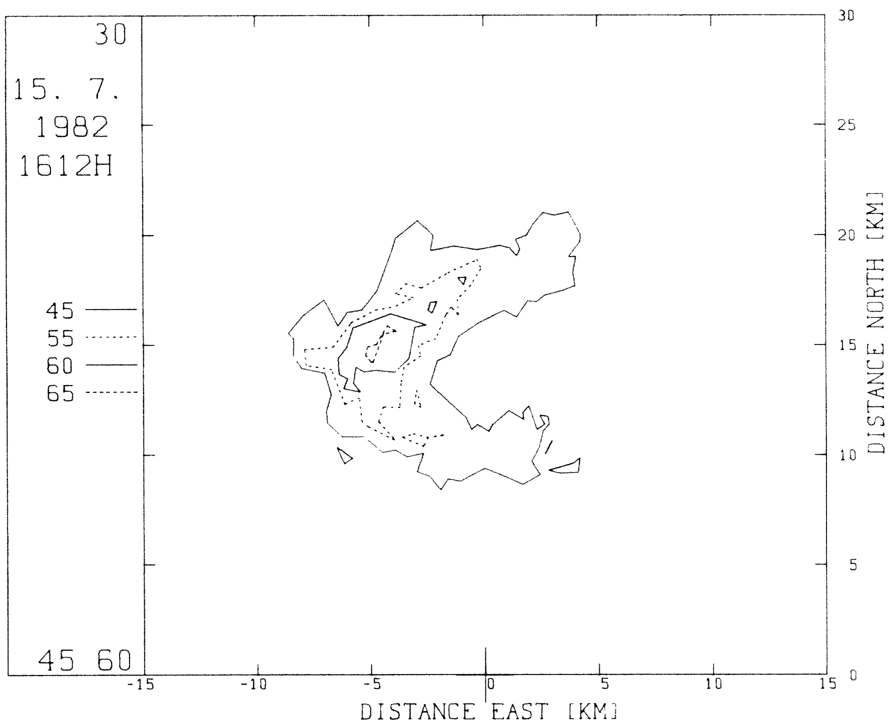

FIG. 1. Example of a horizontal cross section of constant radar reflectivity surfaces of an instant hail-cloud shape, characterized by their I values (see text).

sion D_l of the contour through

$$E = 2/D_l \qquad (4)$$

and, correspondingly, to the fractal dimension $D_s = D_l + 1$ of the surface of constant I values, and thus of the hail cloud.

In Figs. 2(a) and 2(b) the areas F of various contours are plotted versus their perimeters U, in a log-log plot, for the fixed I values 45 and 55 dBZ, respectively, for a typical hailstorm during its entire time evolution (in intervals of 1 min). Data on the higher I values 60 and 65 dBZ, if available, were considered too. Depending on the size of the contours, a plot can be fitted by one or two straight lines.

The results from the evaluation of 24 hail events are summarized as follows:

(1) For *larger* values of the perimeter U a fractal dimension

$$D_l^{(l)} = 1.36 \pm 0.1$$

$$(\text{i.e., } D_s^{(l)} = 2.36 \pm 0.1) \quad (U > U_0) \qquad (5)$$

is obtained. For *smaller* values of U,

$$D_l^{(s)} = 1.0 \pm 0.1$$

$$(\text{i.e., } D_s^{(s)} = 2.0 \pm 0.04) \quad (U < U_0) \qquad (6)$$

describes a smooth behavior.

(2) The rather sharp crossover around $U \cong U_0$ indicates the existence of a characteristic length $L_0 = U_0/\pi = 3 \pm 1$ km.

(3) In the presence of strong horizontal winds the crossover value is apparently shifted toward higher U values and larger contours appear smooth.

The fractal dimension of larger contours $U > U_0$ [Eq. (5)] is, within the error, equal to the value of $\sim \frac{4}{3}$ found in a recent analysis of large and very large clouds by Lovejoy[10] in 1982. His value $D = 1.35 \pm 0.05$ is valid in the range $10 \leq U \leq 10^4$ km. Interestingly, the lower end of this range corresponds approximately to our value U_0 for storms with low horizontal wind strength. Hence, various cloud types such as "fair weather clouds," rain, and hail clouds have the same fractality above L_0. A first attempt to interpret the fractal dimension D of clouds in the frame of the relative turbulent diffusion theory has been published recently.[11]

The observed apparent crossover at U_0 defines a characteristic length U_0 in the cloud problem. L_0 is approximately independent of I, i.e., of the particle size, but seems to depend on the wind strength. Stronger winds cause a smoothing of the contour shape. For small contours, the strong vertical winds which are known to be present near the center of thunderstorms and severe convective storms are

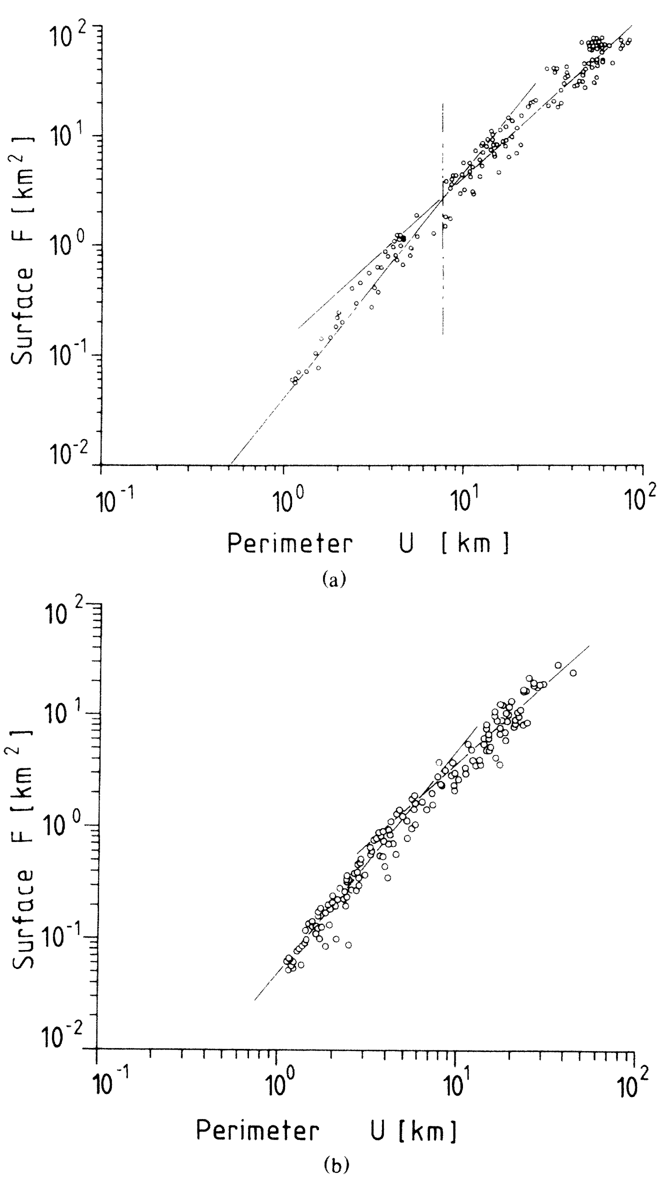

FIG. 2. log-log plot of the surface (F)-to-perimeter (U) relation for the constant I values (a) 45 dBZ and (b) 55 dBZ. Every point corresponds to a particular time during the temporal evolution of a hailstorm. From the linear fits (shown as full lines) of 24 different plots the averaged values of Eqs. (5) and (6) are obtained.

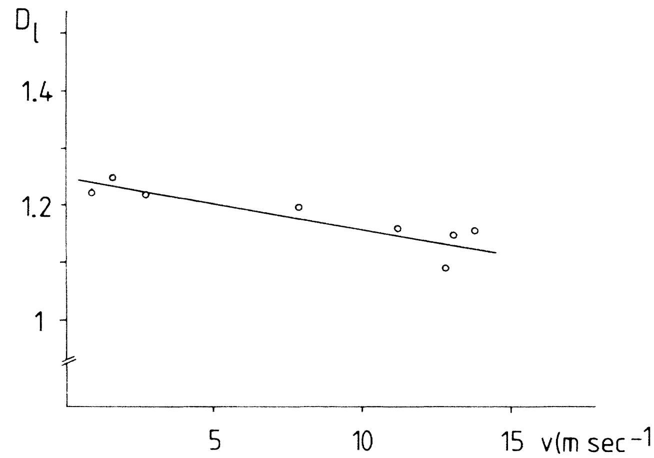

FIG. 3. Effective fractal exponents D_l of the 45-db$Z$ contours of eight particular hailstorms as a function of their mean horizontal center-of-mass displacement velocity v. For simplicity, only one linear fit (instead of two) of the log F vs log U relation have been used here to illustrate the wind strength effect on the contour smoothness.

thought to be responsible for this smoothing, whereas larger contours are sensibly smoothed *only* in presence of important horizontal wind strengths. This observation is illustrated in Fig. 3, where the effective fractal dimension D_l is plotted as a function of the average horizontal wind speed v for the case of eight different hail events, for which sufficiently reliable data were available. The linear least-squares fit shows clearly the tendency toward the smooth dimensionality $D_l = 1$ for increasing values of v. Further support for our assertion stems from the analysis of a specific hail event during its evolution. A suitable case (16 August 1981, 5:34 p.m. till 6:04 p.m.) was chosen which showed a rather stationary maximal I value of 55 dBZ during the first 30 min at an average wind strength $v \simeq 13$ m sec^{-1}. The corresponding trajectory of the 45-dBZ contour in the plot of log F versus log$U$ is, to a good approximation, linear with $D_l = 1.0 \pm 0.10$ for U values up to 12 km. In contrast, slow wind strengths were observed in some events during their temporal development from the analysis of the dislocation speed of the contour center of mass. Such an example is given by the hailstorm on 8 August 1981, at 4:37 p.m. till 4:45 p.m. For $v = 3$ to 5 m sec^{-1} and (increasing) U values from 4 to 12 km an effective value $D_l = 1.5 \pm 0.15$ was found. Finally, in various other examples, an abrupt decrease (increase) of v is characterized by an increase (decrease) of U at an almost constant F value describing, again, a roughening (smoothing) of the corresponding contour. Note that the value of L_0 is 1 order of magnitude *larger* than the resolution of the radar. Therefore, a smoothing effect due to the finite instrument resolution would affect contours with a diameter of $\lesssim 0.3$ km only (as a fractal contour with $D_l > 1$ involves its whole domain and hence, the "roughness width" is given by the diameter).

Unfortunately, no detailed data on vertical wind strengths are available from the present experiment.

We conjecture that the smoothing of the smaller contours near the cloud center on one hand, and of large contours in presence of strong horizontal winds on the other hand, is caused by a particle-drift effect. A small particle showing a Brownian motion in the drift-free case will follow an oriented and rather smoothed trajectory at higher wind strengths, i.e., in the presence of a particle drift. The latter affects the shape of agglomerates which appear smoothed because of the drift. A similar drift effect has been observed for the diffusion-limited aggregation problem[12] and the electrical-discharge problem[13] recently. This observation could be the basis of a useful phenomenological method for the determination of cloud areas with strong winds by measuring the corresponding fractal cloud dimension. Note that our results [Eqs. (5) and (6)] were obtained from data sampled during the whole hailstorm and, therefore, temporal variations of the wind strength certainly contribute to the deviations from the mean values in the plots shown in Fig. 2. More detailed observations on various scales and at well-defined wind strengths are needed to confirm our findings with better statistics. Moreover, the developed ideas are to be built into a quantitative theory of the cloud fractality in the presence of winds.

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[1]B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, San Francisco, Cal., 1982).

[2]P. J. de Gennes, *Scaling Concepts in Solids* (Cornell Univ. Press, Ithaca, N.Y., 1979).

[3]See, e.g., W. T. Elam, S. A. Wolf, J. Sprague, D. U. Gubser, D. Van Vechten, G. L. Barz, Jr., and P. Meakin, Phys. Rev. Lett. **54**, 701 (1985).

[4]L. Niemeier, L. Pietronero, and H. J. Wiesmann, Phys. Rev. Lett. **52**, 1033 (1984).

[5]T. A. Witten and L. M. Sander, Phys. Rev. Lett. **47**, 1400 (1981).

[6]P. Mattila, to be published.

[7]See, e.g., Proceedings of the Third Conference on Fractals in the Physical Sciences, Gaithersburg, Maryland, 20–23 November 1983, edited by M. F. Schlesinger, J. Stat. Phys. **36**, (1984).

[8]A. Waldvogel, B. Federer, W. Schmid, and J. F. Mezeix, J. Appl. Meteorol. **17**, 1680 (1978).

[9]A. Waldvogel and W. Schmid, J. Appl. Meteorol. **21**, 1228 (1982).

[10]S. Lovejoy, Science **216**, 186 (1982), and in Proceedings of the Twentieth Conference on Radar Meteorology, Boston, Massachusetts, 30 November–3 December 1981 (unpublished).

[11]H. G. E. Hentschel and I. Procaccia, Phys. Rev. A **29** 1461 (1984).

[12]P. Meakin, Phys. Rev. B **28**, 5221 (1983).

[13]H. J. Wiesmann, private communication.