Chiral Symmetry and Chiral Anomaly in an Incommensurate Charge-Density-Wave System

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It is shown that chiral symmetry and chiral anomaly are inherent to the incommensurability of a quasi one-dimensional charge-density-wave system. This chiral anomaly induced by the applied electric field is interpreted as an acceleration mechanism of the sliding charge-density wave and is connected with the Thomas-Fermi screening effect. The explicit breaking of chiral symmetry due to the external potential is proved to have a sinusoidal dependence on the phase order parameter. Possible observable effects are also discussed.

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Recently extensive studies of anomalies in field theory, especially the chiral anomalies, were reported in the literature.[1] The anomalies are essentially due to the quantum fluctuations which break a symmetry of a classical Lagrangean. A natural question is then, "Are there phenomenon associated with the chiral anomalies in condensed matter physics?"[2] On the other hand, the quantum mechanical aspects of the dynamical behavior of the moving charge-density wave (CDW) still attract considerable interest.[3] In this paper we point out the existence of chiral symmetry and chiral anomaly in the incommensurate quasi one-dimensional CDW sytsem. We then interpret the effect of chiral anomaly in a $(1+1)$-dimensional Abelian gauge-field theory as an acceleration mechanism of the sliding CDW. It is also connected with the Thomas-Fermi screening effect as a quantum fluctuation of the electrons in the filled valence band. Furthermore, we show that the breaking of chiral symmetry due to the external potential (explicit breaking) as well as the anomaly plays an essential role in understanding the dynamics of the incommensurate CDW (ICDW) system. Especially, the sinusoidal nonlinear coherent response, which has been interpreted as a Josephson-type quantum oscillation intuitively,[4-6] is derived as an effect of explicit symmetry breaking. Charge-current expression and some possible experimentally observable effects are also discussed. We expect that introducing the existent chiral symmetry and chiral anomaly will provide a new insight for understanding the quantum nature of the ICDW system.

It is well-known that a quasi one-dimensional electron-phonon interacting ICDW system is described by a two-component Fermi field coupled with a complex scalar field in $1+1$ dimensions:

$$\psi(x)\equiv\begin{bmatrix}\psi_R(x)\\ \psi_L(x)\end{bmatrix}, \tag{1}$$

$$\phi(x)\equiv\tfrac{1}{2}\sqrt{2}[\phi_1(x)+i\phi_2(x)] = \tfrac{1}{2}\sqrt{2}\eta(x)e^{i\chi(x)}. \tag{2}$$

These fields are related respectively to the nonrelativistic electron Schrödinger wave field $\Psi(x)$ (the spin degree of freedom is kept implicitly) and the phonon wave field $u(x)$ (i.e., ion displacement) by

$$\Psi(x)=\psi_R(x)e^{iQx/2\hbar}+\psi_L(x)e^{-iQx/2\hbar},$$
$$u(x)=\phi(x)e^{iQx/\hbar}+\phi^*(x)e^{-iQx/\hbar}, \tag{3}$$

where $Q/2=p_F$ is the Fermi momentum which is incommensurate with the lattice spacing and the acoustic part of the phonon field has been neglected. Following the main line of Lee, Rice, and Anderson,[7] namely inserting (3) into the Frölich Lagrangean and neglecting the terms involving the factor $\exp[\pm iQx/\hbar]$ (incommensurability), one arrives at the following quasi one-dimensional Lagrangean density for the ICDW system in an external electric field:

$$L=\phi^*\hat{\Delta}_0^{-1}\phi+\psi^\dagger[i\hbar\,\partial/\partial t-e\Phi-\tau_3 v_F(-i\hbar\,\partial/\partial x-eA/c)]\psi+(G/\sqrt{2})\psi^\dagger(\tau_1\phi_1+\tau_2\phi_2)\psi, \tag{4}$$

where τ_1, τ_2, and τ_3 are Pauli matrices, v_F is the Fermi velocity, G is the electron phonon coupling constant, and

$$\hat{\Delta}_0^{-1}=\rho_0\left[-\frac{\partial^2}{\partial t^2}+v_Q^2\frac{\partial^2}{\partial x^2}-\omega^2[Q]\right]. \tag{5}$$

In this expression ρ_0 is the linear density of ion masses, $\omega[Q]$ is the optical phonon frequency, and $v_Q^2=\hbar^2\partial^2\omega^2[Q]/2\,\partial Q^2$. In the derivation of (4) from the standard nonrelativistic second-quantized Schrödinger field theory, only the electrons near the Fermi surface are taken into account. In practical systems the first two terms on the right-hand side of Eq. (5) are normally much smaller than the third term. But in this paper we keep them since, as we will see, they contribute to produce the chiral current of the optical phonon field.

It is amusing to note that the resulting Lagrangean is invariant under the following global chiral transformation:

$$\psi(x) \to \psi'(x) = \exp[i\tau_3\Lambda]\psi(x),$$
$$\phi(x) \to \phi'(x) = \exp[2i\Lambda]\phi(x). \tag{6}$$

Since the Fermi momentum is incommensurate to the lattice spacing, one cannot distinguish the transformed system from the original system with a shift in phase by Λ for the electron and by 2Λ for the optical-phonon wave function:

$$\Psi(x) = \psi_R(x)e^{i(Qx/2+\Lambda)/\hbar} + \psi_L(x)e^{-i(Qx/2+\Lambda)/\hbar},$$
$$u(x) = \phi(x)e^{i(Qx+2\Lambda)/\hbar} + \phi^*(x)e^{-(Qx+2\Lambda)/\hbar}.$$

This is the origin of the chiral symmetry. We should emphasize a delicate difference between the commensurate system and the incommensurate system, the former having no chiral symmetry. The chiral symmetry is a symmetry associated with zero-mass Fermi fields with left- and right-handed helicity in relativistic fields.[8] However, the analog of this symmetry and its spontaneous breaking appeared already in the BCS theory of superconductivity.[9] In the present model the chiral symmetry is also broken to produce the energy gap near the Fermi surface (Peierls transition).[7] The Goldstone mode associated with the spontaneous chiral symmetry breaking describes precisely the current-carrying sliding CDW.[10] The chiral symmetry in the ICDW system was first noted by Barnes and Zawadowski[6] although they did not take into account the phonon field.

The charge and current densities of the system are given by

$$\rho(x) = \psi^\dagger(x)\psi(x),$$
$$j(x) = v_F\psi^\dagger(x)\tau_3\psi(x), \tag{7}$$

which can be derived from the charge and current densities of Schrödinger theory by use of the same approximation. Since according to the symmetry principle of the general dynamics, i.e., the Noether theorem, every continuous symmetry of the system leads to a corresponding conserved current,[8] we note that the charge and current densities, Eq. (7), are also the Noether currents of the Lagrangean (4) associated with the gauge transformation $\psi(x) \to \psi'(x) = \exp[i\Lambda(x)] \times \psi(x)$. Next we introduce a Noether chiral current density, which consists of two parts, the electron part and the phonon part: $\rho^{(5)} = \rho_{el}^{(5)} + \rho_{ph}^{(5)}$, $j^{(5)} = j_{el}^{(5)} + j_{ph}^{(5)}$, with the phonon part as

$$\rho_{ph}^{(5)}(x) = \frac{2}{\hbar}\rho_0\eta^2\frac{\partial\chi}{\partial t}, \quad j_{ph}^{(5)} = -\frac{2}{\hbar}\rho_0 v_Q^2\eta^2\frac{\partial\chi}{\partial x}, \tag{8}$$

and the electron part of the chiral current being related to the usual current as

$$\rho_{el}^{(5)}(x) = v_F^{-1}j(x), \quad j_{el}^{(5)} = v_F\rho(x), \tag{9}$$

while the phonon part of the chiral current is one of the specific features of the ICDW system, where the optical phonon field participates in the chiral transformation.

Even if the chiral symmetry is exact in the classical sense, it is not exact in quantum theory. Namely, the anomaly appears as nonconservation of the chiral Noether current. In this paper we first investigate the chiral anomaly of the ICDW system and its physical implications.

In order to relate Lagrangean (4) to the standard $(1+1)$-dimensional relativistic field-theory expression, we define a set of Dirac γ matrices: $\gamma^0 \equiv \tau_1$, $\gamma^1 \equiv -i\tau_2$, $\gamma_5 \equiv \tau_3$, with $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ and $\gamma_5\gamma^\mu = \epsilon^{\mu\nu}\gamma_\nu$, where $g^{\mu\nu}$ is the Minkowski metric and $\epsilon^{\mu\nu}$ is the Levi-Civita symbol. As usual we define $\bar\psi = \psi^\dagger\gamma^0$; then Lagrangean (4) is expressed as

$$L = \tfrac{1}{2}\phi^*\hat\Delta_0^{-1}\phi + \bar\psi\hat D\psi, \tag{10}$$

where $\hat D = \hat D_0 + (G/\sqrt{2})\eta\exp(i\gamma_5\chi)$ with $\hat D_0 = v_F\gamma^\mu \times (i\hbar\,\partial_\mu - eA_\mu/c)$ and $\partial_0 \equiv \partial/\partial v_F t$, $A^\mu(x) \equiv (c\Phi(x)/v_F, A(x))$. With this notation, the usual current, (7), and the axial current, (8)–(9), can be expressed as $j^\mu(x) = v_F\bar\psi(x)\gamma^\mu\psi(x)$ and $j_\mu^{(5)} = j_{el,\mu}^{(5)}(x) + j_{ph,\mu}^{(5)}$ with $j^{el,\mu(5)}(x) = v_F\bar\psi(x)\gamma^\mu\gamma_5\psi(x)$, $j^{ph,\mu(5)}(x) = (v_F\rho^{(5)}(x), j^{(5)}(x))$. Then the relation between the vector and axial-vector electron current, Eq. (9), which is specific for $(1+1)$-dimensional systems, now becomes

$$j_{el,\mu}^{(5)}(x) = -\epsilon_{\mu\nu}j^\nu(x). \tag{11}$$

In analogy with the partition function, for the zero-temperature many-body system, the path-integral representation of generating-functional formalism allows the discussion of conditions for spontaneous breakdown of symmetry which goes beyond the one based on the classical Lagrangean and which is valid to all orders in perturbation theory.[8] Let Z be a generating functional defined by the following functional integral over the fields in Minkowski space:

$$Z[j,j^*] = \int\cdots\int[d\eta][d\chi][d\psi][d\bar\psi]\exp[(i/\hbar)\int d^4x(L + J^*\phi + J\phi^*)]. \tag{12}$$

Although we write every expression in Minkowski space, the actual anomaly calculations are carried out in correspondent Euclidean space. Performing an infinitesimal chiral transformation on the integrand of the generating functional, and using the chiral invariance of the action, we derive the anomalous Ward identify $\partial_\mu \langle j_5^\mu \rangle = -\delta J[\Lambda]/\delta\Lambda(x)$, where $J[\Lambda]$ is the Jacobian of an infinitesimal chiral transformation

$$[d\chi'][d\psi'][d\bar{\psi}'] = J[\Lambda][d\chi][d\psi][d\bar{\psi}].$$

The nonconservation of axial current, the chiral anomaly, is essentially due to the chiral noninvariance of the path-integral measure.[11] Because of the non-Hermiticity of the electron propagator in Euclidean space we are not certain whether the results are regularization dependent or not. So we calculated the anomaly in two different regularization schemes, Fujikawa's[11] and ζ function.[12] We obtained the same result, that

$$\partial_\mu \langle j_5^\mu (x) \rangle = eE(x)/\pi\hbar, \tag{13}$$

where E is the external electric field and $\langle \ldots \rangle$ means average over the path integral, i.e., the quantum average over the ground state. We note that the chiral anomaly is independent of the spontaneous symmetry breaking of the system.

Now, we use (11) to obtain

$$\left\langle \frac{1}{v_{\rm F}} \frac{\partial j(x)}{\partial t} - v_{\rm F} \frac{\partial \rho(x)}{\partial x} \right\rangle = \frac{eE(x)}{\pi\hbar} - \frac{2}{\hbar}\rho_0 \left\langle \frac{\partial}{\partial t}\left[\eta^2(x) \frac{\partial \chi(x)}{\partial t}\right] - v_Q^2 \frac{\partial}{\partial x}\left[\eta^2(x) \frac{\partial \chi(x)}{\partial x}\right] \right\rangle. \tag{14}$$

This equation is a direct consequence of the chiral anomaly in ICDW system. The second term on the right-hand side of Eq. (14), which is contributed by the spatial and temporal derivative part of the phonon Lagrangean (5), is usually negligibly small. Therefore, according to Eq. (14), the anomaly can be interpreted as an acceleration mechanism of the current, which is essentially carried by the sliding CDW in the zero-temperature limit. It is also worthwhile to note that the response coefficient of $\partial j/\partial t$ to E is $ev_{\rm F}/\pi\hbar$ without any phenomenological parameter.

Moreover, in a quasi one-dimensional system, the Maxwell equation is written as

$$\partial E(x)/\partial x^\mu = 4\pi e N^{(2)} \epsilon_{\mu\nu} j^\nu (x)/v_{\rm F},$$

where $N^{(2)}$ is the two-dimensional electron density perpendicular to the nesting direction within the transverse quantum coherence length. Thus, combining (14) and the Maxwll equation we obtain

$$\left[\frac{1}{v_{\rm F}^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right] E(x) = -\frac{4e^2 N^{(2)}}{\hbar v_{\rm F}} E(x) + \frac{8\pi e \rho_0 N^{(2)}}{\hbar v_{\rm F}} \left\langle \frac{\partial}{\partial t}\left[\eta^2(x) \frac{\partial \chi(x)}{\partial t}\right] - v_{\rm F}^2 \frac{\partial}{\partial x}\left[\eta^2(x) \frac{\partial \chi(x)}{\partial x}\right] \right\rangle. \tag{15}$$

The first term on the right-hand side of Eq. (15) shows that the anomaly contributes to a macroscopic screening effect for the external electric field with the screening length

$$\xi = (\hbar v_{\rm F}/4e^2 N^{(2)})^{1/2}. \tag{16}$$

In deriving the Lagrangean (4) we neglected the spatial variations which correspond to integral multiples of $2k_{\rm F} = Q/\hbar$ and our result is valid only for spatial variations much larger than $1/k_{\rm F}$. In an ordinary CDW system $4e^2/\hbar v_{\rm F}$ is order of 1. Therefore, even if the second term on the right-hand side of Eq. (15) is negligible, the macroscopic screening effect such as perfect static diaelectricity can be achieved only when the spacing of the coherent electrons along the nesting direction is much smaller than that in the transverse direction.

We derived the anomaly from the chiral noninvariance of the path-integration measure following Fujikawa.[11] The discussion is quite abstract. In order to have some physical intuition, let us discuss the Thomas-Fermi screening of the one-dimensional electron gas. Let n and n_0 be the linear densities of electrons with and without small perturbative external electric field. Then,

$$n = \int_{-p_{\rm F}}^{p_{\rm F}} \frac{dp}{2\pi\hbar} = \frac{p_{\rm F}}{\pi\hbar} \tag{17}$$

and $n_0 = p_{\rm F0}/\pi\hbar$, where $p_{\rm F}$ and $p_{\rm F0}$ are the corresponding Fermi momenta. They are related by $p_{\rm F}^2/2m + e\delta\Phi(x) = p_{\rm F0}^2/2m = \mu$, where μ is the chemical potential of the system. Then the Poisson equation becomes

$$\frac{\partial^2 \delta\Phi(x)}{\partial x^2} = -4\pi e N^{(2)}(n - n^{(0)}) = \xi_{\rm TF}^{-2}\delta\Phi(x),$$

where $\xi_{\rm TF}$ is the Thomas-Fermi screening length $[\hbar v_{\rm F}/4e^2 N^{(2)}]^{1/2}$ which has exactly the same expression as that of the anomaly, (16). It is rather interesting to note that the Thomas-Fermi screening is due to the fact that the Fermi level is imbedded in the continuum of the electronic band; however, for an ICDW system with spontaneous symmetry breaking, the anomaly describes the screening effect for the electrons that are in the filled valence band, but are separated from the conduction band by a Peierls gap. There is an extraordinary collective degree of freedom for the electrons in the ICDW system with spontaneous symmetry breaking. Our path-integral calculation is valid for either case, with or without spontaneous symmetry breaking.

The above discussion indicates that the anomaly is essentially the interaction of the external field with electrons below the Peierls gap (in the negative Dirac sea). That is the very reason why it describes the acceleration mechanism of the "electrons moved bodily through the lattice."[10]

Another important consequence of Eq. (14) is that in pure incommensurate CDW systems a charged kinklike self-localized state is absent, which makes the incommensurate system different from a commensurate system. The argument goes as follows. In case the system is stationary and under no external electric field, Eq. (14) is integrated as

$$\langle\rho(x)\rangle+\frac{2}{\hbar v_F}\rho_0 v_Q^2\left\langle\eta^2(x)\frac{\partial\chi(x)}{\partial x}\right\rangle=0. \tag{18}$$

The second term is negligibly small in a practical system, so that $\langle\rho(x)\rangle\approx 0$.

Next let us consider the case that an arbitrary static potential $V(x)$ is present. We again decompose the potential as

$$V(x)=V_a(x)+e^{(i/\hbar)Qx}V_o(x)+e^{-(i/\hbar)Qx}V_o^*(x), \tag{19}$$

where V_a is real, and $V_o(x)=V_1(x)-iV_2(x)\equiv\zeta(x)\exp[-i\delta(x)]$. Both the acoustic part V_a and the optical part V_o are slowly varying compared to the inverse of the Fermi momentum. The Lagrangean of the system is then given by

$$L_V=L+\psi^\dagger(V_a+V_1\tau_1+V_2\tau_2)\psi, \tag{20}$$

where L is given by Eq. (4) and

$$\psi^\dagger V_a\psi=V_a(\psi_R^\dagger\psi_R+\psi_L^\dagger\psi_L),\quad \psi^\dagger(V_1\tau_1+V_2\tau_2)\psi=V_o\psi_R^\dagger\psi_L+V_o^*\psi_L^\dagger\psi_R.$$

From now on we shall use the usual adiabatic approximation, namely we neglect the time and space derivatives of the optical phonon field in the Lagrangean. Then $\Delta_0^{-1}=-\rho_0\omega^2[Q]$ becomes a c number and the phonon part of the chiral current has to be neglected. By the same procedure as before we can derive the following form of the anomalous Ward identity:

$$\frac{1}{v_F}\frac{\partial\langle j(x)\rangle_V}{\partial t}-v_F\frac{\partial\langle\rho(x)\rangle_V}{\partial x}=\frac{eE}{\pi\hbar}-\frac{2\sqrt{2}\Delta_0^{-1}}{\hbar G}\zeta(x)\langle\eta(x)\sin[\chi(x)-\delta(x)]\rangle_V. \tag{21}$$

This result is derived generally with only symmetry arguments. It describes the coherent response of the observable quantities $j(x)$, $\rho(x)$, which is in general spatial and temporal dependent, to the applied field and pinning potential without phenomenological parameters.[13]

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13Because of lack of space we cannot discuss here the implications of the symmetry-breaking effects, for which see B. Sakita and Z. B. Su, "Phenomenological Lagrangian of Incommensurate Charge Density Wave System," Suppl. Prog. Theor. Phys. (to be published).