Adiabatic Expansion of a Strongly Correlated Pure Electron Plasma

Daniel H. E. Dubin and Thomas M. O'Neil
Department of Physics, University of California at San Diego, La Jolla, California 92093
(Received 13 November 1985)

Adiabatic expansion is proposed as a method of increasing the degree of correlation of a magnetically confined pure electron plasma. Quantum mechanical effects and correlation effects make the physics of the expansion quite different from that for a classical ideal gas. The proposed expansion may be useful in a current experimental effort to cool a pure electron plasma to the liquid and solid (crystalline) states.

PACS numbers: 52.25.Kn, 52.25.Wz

Recent experiments have involved the magnetic confinement of an unneutralized collection of electrons of sufficient density and volume to be called a plasma, that is, a pure electron plasma.[1] Since there are negligibly few ions in the confinement region, such a plasma may be cooled to very low temperature without the occurrence of recombination. Theory predicts that as the plasma is cooled the electrons become strongly correlated, pass into a state which may be described as a liquid, and ultimately experience a phase transition to a solid (or crystalline) state.[2] The particular state in which the plasma exists is determined by the parameter $\Gamma = e^2/akT$, where T is the temperature and $a = (4\pi n/3)^{-1/3}$ is a measure of the interparticle spacing. The parameter regime $\Gamma \ll 1$ corresponds to a weakly correlated plasma, the regime $2 < \Gamma < 170$ corresponds to a liquid, and the regime $\Gamma > 170$ to a crystal.[3]

There is a current experimental effort to realize a pure electron liquid and a pure electron crystal in practice.[4] By simply allowing the electrons in a magnetically confined pure electron plasma (MCPEP) to radiate away their thermal energy (cyclotron radiation), the plasma temperature likely has been reduced to near the 4-K temperature of the environment (walls of the vacuum vessel). For the experimental density achieved, this corresponds to a value of Γ at the lower limit of the liquid regime (i.e., $\Gamma \simeq 2$).

In this paper, we point out that the plasma temperature may be reduced further, and the value of Γ increased further, by adiabatic expansion of the plasma along the direction of the magnetic field. As we will see, the physics associated with such an expansion is unusual.

To see why adiabatic expansion might be an effective method of increasing Γ, let us first consider the expansion of an ideal gas of electrons. An ideal gas corresponds to the limit of weak correlation ($\Gamma \rightarrow 0$). In such an expansion, the final temperature and volume are related to the initial temperature and volume through the well-known relation $T/T_0 = (V_0/V)^{\gamma-1}$, which when combined with $\Gamma \sim 1/TV^{1/3}$ implies that

$$\Gamma/\Gamma_0 = (V/V_0)^{\gamma-4/3}. \tag{1}$$

Here, the parameter γ is given by $\gamma = (f+2)/f$, where f is the number of degrees of freedom that share the thermal energy of an electron. For the usual case of $f = 3$, adiabatic expansion is not a particularly effective means of increasing Γ [i.e., $\Gamma/\Gamma_0 = (V/V_0)^{1/3}$]. However, if for some reason $f = 1$, then adiabatic expansion is quite effective [i.e., $\Gamma/\Gamma_0 = (V/V_0)^{5/3}$].

Interestingly, for the cryogenic electron system, f does have a value near unity. The confining magnetic field is sufficiently large and the electron temperature is sufficiently low that $\hbar\Omega > kT$, where Ω is the cyclotron frequency.[4] Most of the electrons are in the lowest Landau level and so the two degrees of freedom that are perpendicular to the magnetic field do not share the thermal energy. Thus, for a weakly correlated and cryogenic MCPEP, we conclude that adiabatic expansion offers an effective method of increasing Γ.

The real question is whether or not this conclusion also holds for a strongly correlated MCPEP. At first glance one may worry that it does not hold. Equation (1) is typically derived by use of the equation of state for an ideal gas ($p = nkT$), and this equation is not valid for a strongly correlated plasma. In fact, the pressure for a strongly correlated plasma is negative.[3] Does this imply that the temperature *increases* during expansion? We will see that the expansion always leads to a reduction in temperature and an increase in Γ. It turns out that for the parameter regime of current experimental interest, adiabatic expansion offers a reasonably effective method of increasing Γ.

We now turn to a derivation of the analog of Eq. (1) for a strongly correlated MCPEP. The confinement geometry is shown in Fig. 1. A conducting cylinder is divided into a series of sections, and is immersed in a uniform axial magnetic field. Initially the plasma resides in section 2, which is grounded; radial confinement of the plasma is provided by the magnetic field and axial confinement by electrostatic fields (due to negatively biased sections 1 and 3). As the potential

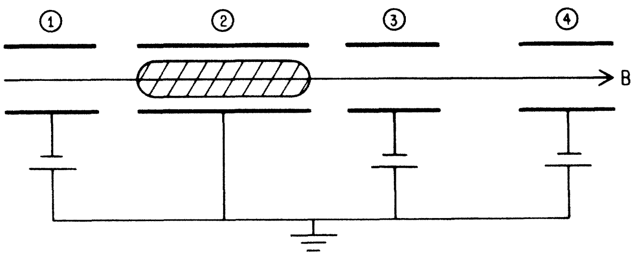

FIG. 1. Confinement geometry.

on section 3 is raised to ground, the plasma gradually expands into section 3. Next the potential on section 4 is raised to ground, and so on. In this way, the plasma expands along the magnetic field against a moving electrostatic piston. We assume that the expansion is quasistatic and that the plasma is thermally isolated. The term "quasistatic" refers to an expansion which occurs slowly enough that the plasma remains near thermal equilibrium at all times. By "thermally isolated" we mean that negligible heat is transferred between the plasma and the external world during the expansion (although energy may be transferred through work). These two conditions imply that there is no net increase in entropy[5]; such an expansion is called "adiabatic."

It is well known that electrons confined in such a geometry come into thermal equilibrium with each other.[2,6] The thermal-equilibrium plasma is characterized by various parameters: the total number of electrons N, the voltages on the cylindrical sections, the magnetic field strength, the temperature T, and the plasma rotation frequency ω.

The rotational flow is characterized by a single frequency since there can be no shear in a thermal-equilibrium flow. It is rotation through the magnetic field which provides a radial confining force to balance the electrostatic force of expansion. In this sense, rotation through a magnetic field is equivalent to neutralization by a positive charge. In fact, the N-electron thermal distribution for the magnetically confined electrons differs only by rotation from that for electrons confined and neutralized by a cylinder of positive charge.[2,6]

For the case of a weakly correlated plasma, it has been shown[7] that the electrons match their density to the density of the effective positive charge out to some surface of revolution where the supply of electrons is exhausted. At this surface, the electron density drops off exponentially on the scale of a Debye length. For a strongly correlated plasma, one expects the density to drop off on the scale of the interparticle spacing. We assume that both these scale lengths are small compared to the dimensions of the plasma; so the bulk plasma is characterized by a nearly uniform density n and a well-defined volume V.

The density and the rotation frequency are related through the equation $\omega_p^2=2\omega(\Omega-\omega)$, where $\omega_p^2=4\pi e^2 n/m$ is the plasma frequency.[7] For typical experimental conditions,[4] the density is sufficiently low that $\omega_p \ll \Omega$, which implies that $\omega \simeq \omega_p^2/2\Omega \ll \Omega$.

The low density also means that certain quantum effects are relatively unimportant. It is typically the case that $\hbar\omega_p \ll kT$, even though $\hbar\Omega \gtrsim kT$, and that $\lambda_d \ll a$, where $\lambda_d=\hbar/(mkT)^{1/2}$ is the thermal de Broglie wavelength and a is the interparticle spacing. In a previous paper,[6] the free energy was calculated to order $(\hbar\omega_p/kT)^2$ and $(\lambda_d/a)^2$. The derivation allows for full quantization of the cyclotron and spin dynamics, but other quantum effects are neglected completely or are treated only approximately. In particular, exchange effects are neglected; they enter as an exponentially small correction at order $\hbar^3$. Also, diamagnetic and relativistic effects are very small and are neglected.

The other thermodynamic functions follow from the free energy. Here, we need the entropy $S=S_I+S_C$, where

$$S_I=-Nk\ln\left[\frac{(2\pi\hbar)^2N\tanh x}{(\Omega-\omega)V(2\pi m^3kT)^{1/2}}\right]+Nk\left[\tfrac{5}{2}+\alpha(x)\right] \tag{2}$$

is the ideal-gas contribution and

$$S_C=-\int_0^{\Gamma}(d\Gamma'/\Gamma')NkC_{\mathrm{OCP}}(\Gamma') \tag{3}$$

is the correlation contribution. In these expressions we have used the definitions

$$x=\frac{\hbar(\Omega-\omega)}{2kT},\qquad \alpha(x)=x[\coth x-\tanh x]-1. \tag{4}$$

The plasma volume, V, is assumed to be well defined in the sense mentioned previously. Of course, V is ultimately expressible in terms of primary variables, such as the voltages on the various cylindrical sections.

The quantity $NkC_{\mathrm{OCP}}(\Gamma)$ which appears in Eq. (3) is the specific heat at constant density for a classical one component plasma (OCP). An OCP is a system of like-point charges embedded in a rigid neutralizing background charge. This system is a favorite theoretical model for the study of correlation effects, and $C_{\mathrm{OCP}}(\Gamma)$ is known from extensive Monte Carlo calculations.[3] It is not surprising that this quantity enters here; recall the previously mentioned equivalence between a MCPEP and a system of electrons confined and neutralized by a cylinder of uniform positive charge.

Equations (2) and (3) can be derived from the free energy of Ref. 5 after some work. However, the form of the expressions should be intuitively obvious: S_I is the contribution to the entropy of an ideal gas of electrons in a magnetic field characterized by a Doppler-shifted cyclotron frequency $\Omega - \omega$ (the Doppler shift occurs because the plasma is rotating with frequency ω), while the remaining contribution, S_C, is due to interelectron correlations which tend to increase the order (i.e., decrease the entropy) of the plasma. The magnetic field appears only through quantum effects; if such effects are negligible the entropy reduces to that of a classical OCP. Note that there is no explicit contribution to the entropy from the mean (or Vlasov) field. This mean field has a significant effect on the internal energy and on the free energy, but only affects the entropy through the self-consistent balance of forces which determines the plasma volume. This is because the mean field can transfer no heat energy from the plasma to the external world during a quasistatic change of state (although it may perform a substantial amount of work).

In the expression for the entropy, the variables T and ω can be eliminated in favor of Γ and V by use of the relations $\omega \simeq 2\pi e^2 N/m\Omega V$ and $kT = (e^2/\Gamma) \times (4\pi N/3V)^{1/3}$. During the expansion, N and Ω are unchanged; so the entropy can be written as $S = S(\Gamma, V)$. The desired analog of Eq. (1) is simply the statement that the entropy is unchanged during the expansion, $S(\Gamma, V) = S(\Gamma_0, V_0)$, which when written out takes the form

$$\ln(\Gamma/\Gamma_0) + 2\int_{\Gamma_0}^{\Gamma} (d\Gamma'/\Gamma') C_{\mathrm{OCP}}(\Gamma') = \tfrac{5}{3} \ln Y, \qquad (5)$$

where

$$Y = \left\{ \left(\frac{V}{V_0} \right) \left[\left(\frac{\Omega - \omega}{\Omega - \omega_0} \right) \frac{\tanh x_0}{\tanh x} \exp[\alpha(x) - \alpha(x_0)] \right] \right\}^{6/5}. \qquad (6)$$

At first glance it looks as though Eq. (5) is not written in a convenient form, since x depends on Γ. However, the value of x increases during an expansion. If the expansion is substantial and if $x_0 \gtrsim 1$, we may use the large-x asymptotic forms $\tanh x = 1$ and $\alpha(x) = -1$ when evaluating Y. Thus, Y is determined by V/V_0 and initial parameters:

$$Y \simeq \left\{ \left(\frac{V}{V_0} \right) \left[\left[\left(1 + \frac{\omega_0}{\Omega} \right) \left(1 - \frac{V_0}{V} \right) \right] \tanh x_0 \exp[-1 - \alpha(x_0)] \right] \right\}^{6/5}. \qquad (7)$$

For the interesting case where $x_0 >> 1$, Y is approximately V/V_0.

By taking the derivative of Eq. (5) with respect to V, it is not difficult to show that Γ is a strictly increasing function of V. However, as correlations increase, the rate of change of Γ decreases. This is borne out in Fig. 2, where Γ is plotted versus Y for various values of Γ_0. Here we have used the expression for C_{OCP} given by Slattery, Doolen, and Dewitt.[3] For a strongly correlated MCPEP adiabatic expansion is not as effective as it is for a weakly correlated MCPEP. Nevertheless, for the parameter range of current interest, adiabatic expansion does offer a reasonably effective method of increasing Γ. For example, an expansion from $Y_0 = 1$ to $Y = 20$ changes the state of correlation from $\Gamma = 5$ to $\Gamma \simeq 40$.

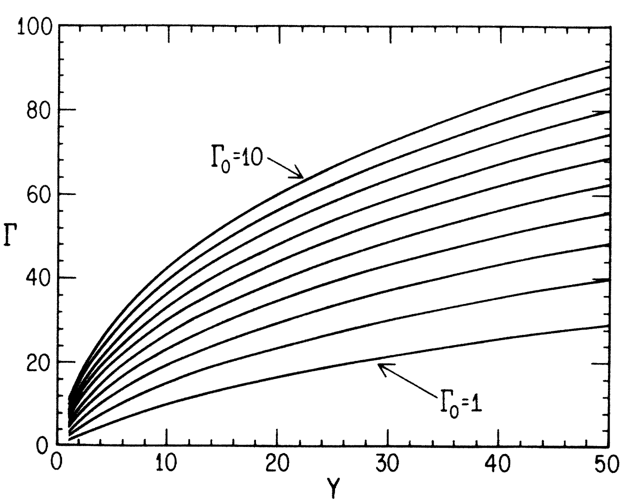

FIG. 2. Plot of Γ vs Y for $\Gamma_0 = 1, 2, \ldots, 10$.

The authors have enjoyed useful discussions with Professors J. H. Malmberg and W. B. Thompson. The work was supported by the National Science Foundation through Grant No. PHY83-06077.

[1]J. H. Malmberg and C. F. Driscoll, Phys. Rev. Lett. **44**, 654 (1980); C. F. Driscoll and J. H. Malmberg, Phys. Rev. Lett. **50**, 167 (1983).

[2]J. H. Malmberg and T. M. O'Neil, Phys. Rev. Lett. **39**,

1333 (1977).

[3]S. G. Brush, H. L. Sahlin, and E. Teller, J. Chem. Phys. **45**, 2102 (1966); E. L. Pollock and J. P. Hansen, Phys. Rev. A **8**, 3110 (1973); S. Ichimaru, Rev. Mod. Phys. **54**, 1017 (1982); W. L. Slattery, G. D. Doolen, and H. E. Dewitt, Phys. Rev. A **21**, 2087 (1980).

[4]J. H. Malmberg, T. M. O'Neil, A. W. Hyatt, and C. F. Driscoll, in Proceedings of 1984 Sendai Symposium on Plasma Nonlinear Phenomena, Tohoku University, Sendai, Japan, 8–9 March 1984 (to be published), pp. 31–37.

[5]L. Landau and E. Lifshitz, *Statistical Physics, Part 1* (Pergamon, New York, 1980).

[6]D. H. E. Dubin and T. M. O'Neil, Phys. Fluids **29**, 11 (1986).

[7]S. A. Prasad and T. M. O'Neil, Phys. Fluids **22**, 278 (1979).