How Does the SU(2) ⊗ U(1) Symmetry Break in the Early Universe?

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The effect of perturbative infrared singularities on the finite-temperature effective potential in the standard model with a light Higgs boson is investigated. Together with the Coleman-Weinberg term, it promotes the full-gauge-symmetry breaking. The transition to the U(1) vacuum via quantum tunneling is expected to occur at some temperature ~100 MeV. However, without knowledge of dynamical details, one cannot conclusively decide whether the chiral or the (full) gauge symmetry breaks first.

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Because of several theoretical interests, symmetry breaking by the Coleman-Weinberg mechanism has been studied extensively for phase transitions in grand unified theories (GUT's) and electroweak theories. Although some aspects of the models have turned out to be problematical, details still seem to be obscure.[1]

In this paper, we are interested in the nature of the cosmological SU(2) ⊗ U(1) phase transition which has also been investigated previously by several authors. Guth and Weinberg[2] and Witten[3] concluded that, as long as the transition is induced by quantum tunneling, an extreme supercooling cannot be avoided. If so, the phase transition would produce too much entropy to be compatible with observation.

Witten[3] noticed that the chiral-symmetry breaking (CSB) by the strong interaction will rescue this situation: A nonzero vacuum expectation value $\langle \bar{\psi}\psi \rangle$ produces a linear term in the effective potential via the Yukawa coupling. Simultaneously, the gauge-symmetry breaking (GSB) also occurs with a small mass scale because $\bar{\psi}\psi$ is the SU(2) doublet. Then, the nearly symmetric state "rolls" down to the asymmetric state rather smoothly. This occurs at a somewhat moderate temperature of about $T = 200$ MeV, and the entropy is increased by a factor of order 10^5–10^6 at this stage of the phase transition.

However, it seems still possible that the SU(2) ⊗ U(1) symmetry breaks not by the strong interaction but by the electroweak interaction itself. The basis of this anticipation is the property of higher-order contributions to the finite-temperature effective potential. In previous work,[4] the present author noticed that higher-order infrared singularities inherent in massless models in general produce contributions which are smaller than the usual leading thermal-fluctuation term ($e^2T^2\phi^2$) but are larger than the Coleman-Weinberg term ($e^4\phi^4 \ln\phi$), ϕ being the classical Higgs field. Such contributions yield terms with sign opposite to that of the leading thermal term. Accordingly, the width of the potential barrier is decreased and phase transitions will be advanced. As the temperature gets much lower, such an effect is expected to have increasing significance for phase transitions in which very-small-field regions of the effective potentials are relevant.

In the following, we study the feature mentioned above in the SU(2) ⊗ U(1) model with a Higgs field of zero bare mass. The usual one-loop effective potential is given by the sum of finite-temperature and zero-point-energy (i.e., Coleman-Weinberg) terms:

$$V_1(\phi,T) = \frac{3T}{2\pi}\int_0^\infty p^2\,dp\Big(2\ln(1-\exp\{-T^{-1}[p^2+(e\phi/2\sin\theta)^2]^{1/2}\}) + \ln(1-\exp\{-T^{-1}[p^2+(e\phi/2\sin\theta\cos\theta)^2]^{1/2}\})\Big) + \frac{3e^4}{512\pi^2}\frac{1}{\sin^4\theta}\left(2+\frac{1}{\cos^4\theta}\right)\phi^4\left(\ln\frac{\phi}{\sigma}-\frac{1}{4}\right). \quad (1)$$

Here θ is the Weinberg angle, and $\sigma = 247$ GeV is the vacuum expecation value of the Higgs field at zero temperature. As usual, I have taken into account only the heavy vector bosons, assuming the Yukawa and ϕ^4 couplings to be small. The leading thermal-fluctuation term is $O(e^2)$, and the Coleman-Weinberg term is $O(e^4)$.

Now, we evaluate the $O(e^3)$ contribution which

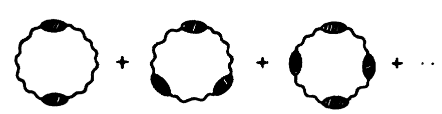

FIG. 1. Gauge-boson ring diagrams. Blobs represent one-loop polarizations.

emerges from two-loop diagrams and ring diagrams (Fig. 1) of zero-energy heavy vector bosons. It is given by

$$V_R(\phi,T) = -\left(\frac{T}{4\pi}\right)\mathrm{Tr}[(\tfrac{1}{3}\Pi + M^2)^{3/2} - M^3], \quad (2)$$

$$\mathrm{diag}(\Pi) = (\Pi_2, \Pi_2, \Pi_2, \Pi_1),$$

$$M^2 = \frac{g_2^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\tan\theta \\ 0 & 0 & -\tan\theta & 1 \end{pmatrix}\phi^2.$$

Π_1 and Π_2 are the 0-0 components of the leading polarization tensors of the U(1) and SU(2) gauge bosons, respectively:

$$\Pi_1 = (\tfrac{5}{18}n_g + \tfrac{1}{6}n_H)g_1^2T^2, \quad (3)$$

$$\Pi_2 = (\tfrac{2}{3} + \tfrac{2}{3}n_g + \tfrac{1}{6}n_H)g_2^2T^2. \quad (4)$$

g_1 and g_2 are U(1) and SU(2) gauge coupling constants and are related to e and θ as $g_1 = e/\cos\theta$ and $g_2 = e/\sin\theta$. n_g and n_H $(=1)$ are the number of generations and the number of Higgs doublets, respectively.

The important feature in Eqs. (2)–(4) is that all contributions from gauge bosons, fermions, and a Higgs boson work on the $O(e^3)$ term in a cooperative way. This is in contrast with the case for the lowest-order β function in the renormalization-group equation (RGE), in which the contributions from non-Abelian gauge bosons and other particles countervail each other. Therefore, if the lowest-order $O(e^2)$ term only is adopted as the boundary condition, Eq. (2) will never be obtained by solution of the RGE even with the full β and γ functions. (Recall that, e.g., in the zero-temperature ϕ^4 model, the one-loop effective potential can be obtained by solution of the RGE with adoption of the lowest-order tree potential as the boundary condition.[5]) In this sense, (2) is regarded not to represent merely a higher-order correction to $O(e^2)$ terms in (1), but rather, together with the $O(e^2)$ term, to provide the boundary condition for the RGE. In fact, the $O(e^3)$ term peculiar to the finite-temperature field theory has appeared as a result of the presence of nonzero (electric) masses of gauge bosons, and the information about the masses can be incorporated by the boundary condition.[5]

When the finite-temperature part in (1) is expanded around $\phi = 0$, an $O(\phi^3)$ term appears and cancels the corresponding M^3 term in (2). From Eqs. (2)–(4), we can easily evaluate $O(e^2)$ terms, but here I do not give their explicit forms. In the small-field region, the total effective potential without CSB is given by the sum of (1) and (2).

We have known[3] that, for the tunneling processes considered below, the region $(e\phi/T)^2 \sim O(1)$ will be the most relevant, at least at the one-loop level. On the other hand, when the number of particle species, and therefore Π, is not small, (2) will also play a significant role in the same region. In fact, $\Pi_2/3T^2$ is about 0.3–0.4 in the present model. Because of this rather large value of $\Pi_2/3T^2$, Eq. (2) as well is expected to give an important contribution for tunneling. (Such a situation will also apply to GUT's with rich particle content.) Figure 2 gives effective potentials with and without ring-diagram corrections. (The temperature is $0.001\sigma = 247$ MeV.) Apparently, the ring-diagram effect is large. This example is one manifestation of our anticipations.

We evaluate the bounce action which determines the tunneling probability at each temperature. Before entering into precise numerical calculations, we qualitatively estimate the effect of the ring diagrams from Fig. 2. For this purpose, we use the approximate formula for the three-dimensional bounce action, $A \sim D^3/\sqrt{H}$, where D and H are the zero and the height of the effective potential. Then, from Fig. 2, we readily see that the ring diagrams will cause a 40% decrease of the bounce action.

Now, the stationary and least action $A(T)$ is expected to be produced by the spherical and nodeless solution of the equation[2,3,6]

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right) - \frac{\partial}{\partial\phi}(V_1 + V_R) = 0, \quad (5)$$

with the boundary condition $\partial\phi/\partial r|_{r=0} = 0$, r being the radial coordinate. In solving (5), I render the coupling constants effectively temperature dependent, since the g's reveal nonnegligible changes of about 20% in a momentum range of 100 GeV–100 MeV. The parametrizations used are given by

$$\frac{1}{g_i^2} = \frac{1}{g_i^2(M_W)} + \frac{B_i}{2\pi}\ln\frac{M_W}{T}, \quad i = 1, 2,$$

$$B_1 = (1/12\pi)(4n_g + \tfrac{3}{10}n_H), \quad (6)$$

$$B_2 = -(1/12\pi)(22 - 4n_g - \tfrac{1}{2}n_H).$$

I have set $M_W = 100$ GeV. The result of computer calculations is given in Fig. 3. (Also performed were calculations in which the coupling constants in the Coleman-Weinberg term were fixed to zero-temperature values. Discrepancies of only about 10% appeared.)

At a temperature T, the transition probability per unit time and unit volume is roughly estimated to be $T^4\exp[-A(T)]$, while the characteristic space-time volume of the universe is $(M_P/\sqrt{\rho})^4$, where $M_P = 10^{19}$ GeV is the Planck mass and $\rho = (24\ \mathrm{GeV})^4$ is the energy density of the symmetric state at nearly zero tem-

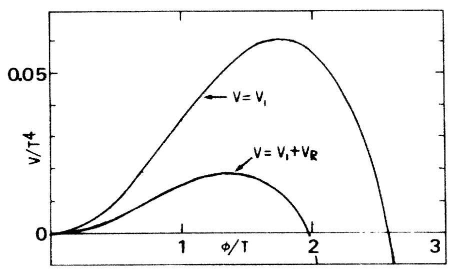

FIG. 2. The effect of ring diagrams on the effective potential at $T=0.001\sigma$. V_1 and V_1+V_R are the one-loop and one-loop plus ring potentials, respectively. In this example, the temperature dependences of the coupling constants are not introduced in the Coleman-Weinberg potential. The values of the parameters are $n_g=3$, $n_H=1$, $g_1^2(M_W)/4\pi=0.016$, and $g_2^2(M_W)/4\pi=0.033$.

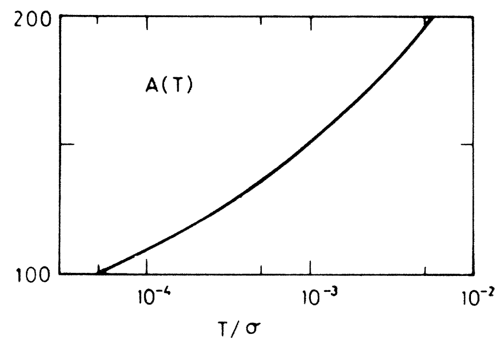

FIG. 3. The temperature dependence of the bounce action. All coupling constants have the temperature dependence given by Eq. (6). If the coupling constants in the Coleman-Weinberg term are fixed, then the bounce action shows about a 10% increase.

perature.[3] (I assume that the universe remains in the supercooled state at that temperature.) The transition to the U(1) vacuum will occur when $(M_P/\sqrt{\rho})^4T^4\exp[-A(T)]\cong 1$, or $A(T)\cong 150$. Taking the roughness of our estimation into consideration, we seek the temperature T^* at which $A(T^*)$ takes values of 130–170. From Fig. 3, we find $90<T^*<500$ MeV. On the other hand, CSB as the origin of the linear potential is expected to take place at some temperature between 100 and 300 MeV.[3] Therefore, not having quantitative knowledge of details of the model including strong interactions, we cannot be decisive about which of the full GSB and the CSB precedes the other. However, the present calculation has confirmed the anticipation that the ring-diagram effects should not generally be neglected in the consideration of tunneling processes.

If full GSB via the formation of bubbles takes place first at a temperature of a few hundred megaelectronvolts, then the final entropy will become 10^5–10^6 times larger than the initial one, and the model can be consistent with cosmological observation. On the other hand, if CSB takes place first, a linear term will appear as a result of the nonzero $\langle\bar{\psi}\psi\rangle$. Furthermore, if the Yukawa coupling, and therefore the heavy-quark contribution which has the tendency to stabilize the $\phi=0$ state, still remains small, then full GSB will occur successively as has been pointed out by Witten.[3] The characteristic mass scale of CSB, ~ 100 MeV, is much smaller than the temperature of ~ 10 GeV attained after the full phase transition, at which almost the same amount of entropy will be produced as in the previous case. Therefore, it seems hardly possible to determine from observation which of the two possibilities actually occurred in our universe.

Finally, I comment on the work by Flores and Sher.[7] They argued that, after CSB, Yukawa coupling to the top quark also grows along with the cooling of the universe. Accordingly, the quark-loop contribution gets large and will modify the effective potential significantly within the context of the perturbation theory ($m_t>65$ GeV). At zero temperature, a potential barrier at $\phi\sim 1$ GeV separates the metastable and the true vacuum, and will again prevent an early transition to the true vacuum. For $m_t<65$ GeV, the barrier is absent. But the perturbative method seems to break down in the small-field region, and one can say nothing about the details of the effective potential. Thus, we might have to give up the picture of the SU(2) ⊗ U(1) transition derived by CSB.

Here, we assume that the top quark is light.[8] If quark-loop contributions are so small that they do not construct any barrier, the transition to U(1) vacuum will be induced by quantum tunneling or by CSB in a way that we have already discussed. Even if a small barrier appears as was shown by Flores and Sher under some hypothetical situation, an instantaneous transition will occur for the following reason. Since CSB at the temperature of ~ 100 MeV will occur abruptly in almost the whole universe (neglecting temperature fluctuations), the classical field may coherently roll down the steep linear slope toward the local minimum, and then begin to climb the opposite slope. It might well be that the height of this hill is not high.[7] If so, the classical field will go over the top and then roll down to the global minimum. The gauge symmetry is broken without extreme supercooling, and not too much entropy will be produced.

Thus we have learned that the standard model with the Coleman-Weinberg mechanism can be compatible with cosmological observation. This conclusion is in some part strongly based on the significance of the ring-diagram effect on the effective potential. Since such an effect is expected to be a universal one, the author would like to propose the reinvestigation of the nature of cosmological phase transitions also in other models (such as GUT's), with incorporation of such contributions.

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