Initial Temperature and Thermalization Time in Heavy-Ion Collisions

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The problem of thermalization of quarks and gluons in relativistic nuclear collisions is considered in a way that minimizes the explicit dependence on the details of the soft QCD interaction. With nuclear transparency and distributed contraction as essential inputs, it is found that, when the collision energy is asymptotically high, the average initial temperature is in the range 300–500 MeV and the thermalization time has a lower bound of order 0.15 fm/c.

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A central question in heavy-ion collisions for the formation of quark-gluon plasma is the initial temperature that can be attained, and closely related to it is the question of the length of thermalization time. To treat the problem from first principles is difficult because of the intractable nature of soft QCD interactions. Attempts have been made in the past to estimate the values from the phenomenology of hadron-hadron and hadron-nucleus collisions or by hydrodynamical considerations.[1] We shall give in this paper a consideration of the problem via a very different approach, viz., on the basis of partons from the beginning of collision to the time when thermal equilibrium is achieved.

In order to elucidate the issues involved as clearly as possible, we shall make simplifying assumptions on those physics inputs that are not crucial, such as flat and limiting parton distribution in pp collisions, no cooperative nuclear effects, very high collision energy, etc., all of which can be relaxed upon fine tuning. The two basic inputs that are directly responsible for the result are nuclear transparency and distributed contraction (to be explained below) *at the parton level.* These two properties are themselves subject to quantitative improvements, but the main thrust of our observation is that to first order they lead to a limiting temperature at asymptotic energies. Furthermore, its value depends on the nuclear size according to $A^{1/6}$.

During the thermalization phase there is evolution toward local equipartition of longitudinal and transverse degrees of freedom in each small space-time cell, but no significant global transverse motion. We may therefore consider the one-dimensional problem of an average longitudinal tube colliding with another, the proper length of which, when averaged over impact parameter, is $L=4R_A/3$, where $R_A=1.2A^{1/3}$ fm. Consider the c.m. system for an $A+A$ collision at asymptotically high energy and let the tip of the light cone be located at the intersection of the nuclear trajectories. If Y is the rapidity of the nuclei, then the spatial extension of the overlapping tubes at $t=0$ is $L/\cosh Y$, which we shall take to be vanishingly small even on the scale of a fraction of a fermi.

For $t>0$ there are partons in the forward light cone at all rapidities y: $-Y \leq y \leq Y$. Since hard interactions among the partons are unimportant compared to the soft ones for the dominant process during thermalization, the partons with small y for proper time $\tau<1$ fm/c could not have originated from the high-y region in the nuclei. They must be the wee partons already contained in the representation of the nuclei, and are taken from virtual to real states by the collision. We therefore focus on the initial state which is a space and momentum distribution of the partons obtained by smearing the parton distributions of the nucleons in the nuclei in a way consistent with the uncertainty principle. To prepare that initial state before thermalization, we shall first switch off the interactions among all partons and regard the free partons as emanating from the initial nucleons, after the hadron bags are broken by the collisions, according to a rapidity distribution $F(y)$ per nucleon appropriate for soft processes. The precise form of $F(y)$ is not important, since reasonable variations would not affect the final conclusion. For clarity's sake, we shall assume that $F(y)$ is scaling and flat, i.e., $F(y)=F_0$, a constant. Of more importance is to recognize that the justification for regarding this initial distribution of all partons in the colliding nuclei as being relevant to the starting point is based on nuclear transparency at the parton level.

The next issue of great importance is the spatial distribution of the partons. While the nuclei are each contracted to a longitudinal length of $L/\cosh Y$, the partons cannot all be contained in such contracted spatial quarters since most partons have $y \ll Y$. For $y \approx 0$ we expect the uncertainty in longitudinal position l to be of order 1 fm, the typical hadronic length scale. If we apply the Lorentz factor consistently for each value of y in the central region, then all partons with the same y occupy a spatial extension of $l/\cosh y$ at $t=0$. In the no-interaction case, the free partons with rapidity y have parallel world lines spanning a world sheet which at $t=0$ is bounded between $\pm l/2\cosh y$. Evidently, except for $y \approx Y$, most partons occupy a spatial extension outside the contracted nuclear disks, as first pointed out by Bjorken.[2] This is as required by the uncertainty principle. We refer to the spatial smearing described above as "distributed contraction," again a property of the partons.

Consider now a space-time cell at proper time τ and spatial rapidity η, defined such that $t=\tau\cosh\eta$ and $z=\tau\sinh\eta$. Furthermore, we shall switch off the parton interaction and consider first the case of free streaming. Because of its particular location in space-time, there is only a limited range of y in which free partons can pass through the cell. From the geometry of the problem, it is straightforward to show that the lower and upper limits of that range are $\eta-\Delta$ and $\eta+\Delta$, respectively, where

$$\Delta = \sinh^{-1}(l/2\tau). \tag{1}$$

Note that the range is independent of η; hence, we have Lorentz invariance along the τ hyperbola.

The energy-momentum tensor at the (τ,η) cell can be expressed in terms of the kinematics of the partons that pass through the cell[3,4]:

$$T^{\mu\nu}(\tau,\eta) = \sum_n \int d\tau' \, k_n^\mu (d/d\tau') Z_n^\nu \delta^2(Z - Z_n(\tau')), \tag{2}$$

where $Z^\nu = (t,z)$, $Z_n^\nu(\tau') = (\tau'\cosh y, \tau'\sinh y + \zeta)$, $k_n^\mu = m_T' v^\mu$, $v^\mu = (\cosh y, \sinh y)$, and

$$\sum_n = \int dy \, F(y) \frac{N}{l/\cosh y} \int_{\zeta_-}^{\zeta_+} d\zeta. \tag{3}$$

Here ζ is the displacement of the world lines from the tip of the light cone and $\zeta_\pm = \pm l/2\cosh y$. N is the average number of nucleons in a tube of proper length L. Depending on whether y is positive or negative we write $F(y) = F_0\theta(\pm y)\theta(Y \mp y)$. From (2) and (3) we get (with the use of a prime to denote the free-streaming case)

$$T'^{\mu\nu}(\tau,\eta) = m_T' n_N (L/l) \int_{-Y}^{Y} dy \, P(\tau,\eta,y) v^\mu(y) v^\nu(y), \tag{4}$$

where $n_N = N/L$, the nucleon density, and

$$P(\tau,\eta,y) = F_0 \theta(y-\eta+\Delta)\theta(\eta+\Delta-y). \tag{5}$$

Similarly, for the current density we have

$$J'^\mu(\tau,\eta) = (n_N L/l) \int_{-Y}^{Y} dy \, P(\tau,\eta,Y) v^\mu(y). \tag{6}$$

In the following we shall have need to consider the transverse components of $T'^{\mu\nu}$. For that, the more complete expression for $T'^{\mu\nu}$ should involve an integration over the three-momentum $\mathbf{k}$ so that k_T can be averaged as well. To enable the partons to have finite rapidities, they are assumed to be massive, $k^2 = m^2$; after the averages are performed, the limit $m \to 0$ may be taken, if desired. Omitting the details, we assert here that the result can be put in the one-dimensional integral form of (4) and (6), if $T'^{\mu\nu}$, J'^μ, and n_N are given their meanings as densities in three-space and if

$$v^\mu = (\cosh y, v^1, v^2, \sinh y), \tag{7}$$

where $v^i = \langle (k^i)^2 \rangle^{1/2}/m_T'$, $m_T' = (m^2 + \langle k_T^2 \rangle)^{1/2}$. Note that $v^\mu v_\mu = m^2/m_T'^2 \to 0$ as $m \to 0$; thus $T'^\mu_\mu \to 0$ in that limit. From (4) and (6) we have the energy density ϵ' and parton density ρ' (in the co-moving frame) for the free-streaming case

$$\epsilon' = T'^{00} = 2m_T' n_q \Delta B(\Delta), \qquad \rho' = J'^0 = 2n_q \sinh\Delta, \tag{8}$$

$$n_q = n_N F_0 L/l, \quad B(\Delta) = \tfrac{1}{2}[1 + (\sinh 2\Delta/2\Delta)]. \tag{9}$$

We now turn on the QCD interaction and ask what in the foregoing discussion should be changed. The parton distribution, of course, must change because of thermalization. Let us use τ_i to denote the proper time when the quark-gluon system becomes thermalized. Then for $\tau \geqslant \tau_i$, the parton distribution $\mathscr{F}(\tau,\eta,y)$ should be one appropriate for a system in a thermal equilibrium; for simplicity, we take it to be Maxwellian,

$$\mathscr{F}(\tau,\eta,y) = \mathscr{F}_0 \exp[-\beta\cosh(y-\eta)],$$

where

$$\beta(\tau) = m_T/T(\tau). \tag{10}$$

To solve the thermalization problem is to understand how the parton distribution evolves from $P(\tau,\eta,y)$ to $\mathscr{F}(\tau,\eta,y)$. That is a separate issue which we need not consider here.[4,5] The expressions for $T^{\mu\nu}$ and J^μ can be obtained from (4) and (6) by the replacement of P by $\mathscr{F}$ inside the integrals and m_T' by m_T. In the co-moving frame they give

$$\epsilon = T^{00} = 2\mu\{1 + [K_1(\beta)/\beta K_0(\beta)]\}, \qquad \rho = J^0 = 2n_q K_1(\beta)\mathscr{F}_0/F_0, \tag{11}$$

$$T^{11} = T^{22} = \mu, \quad T^{33} = 2\mu K_1(\beta)/\beta K_0(\beta), \tag{12}$$

where the parton mass m has been set equal to 0, and

$$\mu = m_T n_N \mathscr{F}_0 K_0(\beta) L/l = m_T n_q K_0(\beta) \mathscr{F}_0/F_0. \tag{13}$$

$K_\alpha(\beta)$ are the modified Bessel functions.

An important aspect of a thermalized system is that in every co-moving frame there is equipartition in the longitudinal and transverse degrees of freedom. Making that imposition in (12) yields[6]

$$2K_1(\beta)=\beta K_0(\beta), \tag{14}$$

the solution of which is $\beta=2.4$. It follows from (11), (12), and (14) that for a thermal system

$$\epsilon=3\mu, \quad T^{ii}=\mu, \quad i=1,2,3. \tag{15}$$

This is consistent with the hydrodynamical form for $T^{\mu\nu}$,

$$T^{\mu\nu}=(\epsilon+p)u^\mu u^\nu - p g^{\mu\nu}, \tag{16}$$

as it should be, and with $T^\mu_\mu=0$ for ideal fluid.

The normalization factors F_0 and $\mathscr{F}_0$ are related by the requirement $\rho'=\rho$ which is equivalent to the requirement that all the soft partons that are to be created by interaction are contained already in the distribution $P(y)$ and that the switching on of the interaction only redistributes the parton density in rapidity space. We then obtain from (8) and (11)

$$F_0 \sinh\Delta = \mathscr{F}_0 K_1(\beta). \tag{17}$$

We now make the key step in this paper by requiring that at τ_i,when thermalization is complete, we expect $\epsilon(\tau_i)=\epsilon'(\tau_i)$, independent of the detailed properties of soft QCD interaction. This is based on the simple notion that the redistribution of the parton density in rapidity space should be consistent with energy conservation and Lorentz invariance, i.e., η independence of $\epsilon(\tau)$ on a τ hyperbola, on the assumption that by nuclear transparency the partons in the fragmentation region do not diffuse in y space into the central region. Of course, if the interaction strength were zero, there would never be any thermalization. This and other interesting observations will be commented on below, after the consequences of the above requirement are derived.

From (8) and (11)–(15) we have

$$\epsilon'/\rho' = m_T' \Delta B(\Delta)/\sinh\Delta, \quad \epsilon/\rho = 3T. \tag{18}$$

If we simplify the first expression by the assumption that $\tau_i/l \ll 1$, to be verified *a posteriori*, we have $\epsilon'/\rho' = m_T' l/4\tau_i$. Equating it to ϵ/ρ yields

$$T_i\tau_i = m_T' l/12, \tag{19}$$

where $T_i=T(\tau_i)$. This is our first result that is independent of the hadronic scale, since $m_T' l$ is of order one.

From (10)–(17) we can further obtain $\epsilon_i = 6n_q T_i \sinh\Delta$. Equating this with the Stefan-Boltzmann formula

$$\epsilon_i = 3aT_i^4, \quad a=\tfrac{1}{90}\pi^2(16+\tfrac{21}{2}N_f), \tag{20}$$

we obtain, with the help of (1) and (9),

$$T_i^3\tau_i = n_N F_0 L/a = \frac{1}{\pi R_A^2}\frac{1}{4a}\frac{S}{2Y}, \tag{21}$$

where the second expression follows from considerations of hydrodynamical flow,[7] $S/2Y$ being the comoving entropy of the system. It has been exploited for the diagnostic of the quark-gluon plasma.[7] Here it is related to the parton densities of the incident nuclei. In the following their connection with the multiplicity of particles produced will be discussed. Equation (21) is our second result.

Combining (19) and (21) gives the main result:

$$T_i = \left(\frac{12 n_N F_0 L)}{a m_T' l}\right)^{1/2} = \frac{1}{(m_T' l)^{1/2}}\left(\frac{12F_0}{a\pi}\right)^{1/2}\frac{A^{1/6}}{1.2\ \text{fm}} = \frac{m_T' l}{12\tau_i}. \tag{22}$$

To give numbers we assume that the basic hadron length scale l is $\sim m_\pi^{-1}$, and that the "intrinsic" parton transverse mass m_T' is $\sim 3m_\pi$; thus $m_T' l \sim 3$. Furthermore, if we use the reasonable values $n_N=0.14\ \text{fm}^{-3}$, $F_0=5$, and $N_f=3$ ($a=5.21$), we get the initial temperature and thermalization time

$$T_i = 180A^{1/6}\ \text{MeV}, \tag{23}$$

$$\tau_i = 0.27A^{-1/6}\ \text{fm}/c. \tag{24}$$

The A dependence is very weak.[8] For $16 \leq A \leq 238$, we have $300 \leq T_i \leq 500$ MeV and $0.1 \leq \tau_i \leq 0.2$ fm/c. The values obtained are only averages; large fluctuations are, however, always possible experimentally.

The small value for τ_i justifies the approximation $\tau_i \ll l$ used earlier. It does not imply inconsistency with the uncertainty principle because most partons that give rise to the high temperature in the cell at τ_i and $\eta=0$, say, have $y\sim \sinh^{-1} l/2\tau_i$, so that their contracted uncertainty Δz is approximately τ_i, not l. The partons with $y\sim 0$ and $\Delta z \sim l$ contribute negligible thermal energy to the cell.

Experimentally the combination $T_i^3\tau_i$ in (21) is determined by particle multiplicity. The height of the rapidity plateau at zero impact parameter for $A+B \to \pi + X$ is $dN/dy_\pi(b=0)=S/2cY$ where $c=3.6$, on the assumption of isentropic expansion and hadronization, and $R_A \leq R_B$. When this is averaged over b,[7] we have, by use of (21),

$$\frac{dN}{dy_\pi}(A+B\to\pi+X) = \left(1+\frac{R_A}{R_B}\right)^{-2}\frac{4F_0}{c}A. \tag{25}$$

For $A=B$, we get $dN/dy_\pi = hA$ where $h=F_0/c$. Using $F_0=5$ yields $h=1.4$. This is a very reasonable prediction for an average event, both in height and in A dependence.

It is known that at high energies the total cross section is increasing and the multiplicity increases faster than log$s$. These nonscaling features are accounted for by letting F_0 increase as log$s$. The initial temperature increases proportionally to $F_0^{1/2}$ and thus nonscaling effects help in the attainment of high temperatures.

The method used to derive the quantitative results is to calculate first the energy density $\epsilon'(\tau)$ by assumption of free streaming; it decreases as τ^{-2}, according to (8). For the thermalized plasma, $\epsilon(\tau)$ decreases as $\tau^{-4/3}$, as is evident from (20) and (21), or from hydrodynamics. We determine τ_i by the intersection of the two curves, reasoning that interaction makes the plasma cohesive and that therefore after thermalization the expansion will follow the slower curve. Ignoring interaction before τ_i evidently results in an underestimate of τ_i. Thus, (24) should be regarded as a lower bound. Notice that if the physical coupling strength were weaker, the hadronic parameters n_N and F_0 would presumably be smaller, while $m_T^{\prime}l$ remains unchanged. It then follows from (22) that T_i would be lower and τ_i longer, as one expects on physical grounds.

It is interesting to compare the above bound with the value of τ for which $T'^{ii}(\tau)$ are equal for all three components, a conceivable condition that approximates local equilibrium. From (4) and (7) we find that the condition implies (for $m=0$) $\sinh 2\Delta = 4\Delta$, whose solution corresponds to $\tau = l/2.67 \cong 0.5$ fm/c. This value is not a reliable estimate of the thermalization time because, for one reason, it is independent of A; nevertheless, it serves as a useful insight into the short time scale in which the longitudinal and transverse components of $T'^{\mu\nu}$ become equalized even in free expansion. Interaction should shorten that time.

A way of understanding the origin of our result that the initial temperature is independent of collision energy as it becomes very large (apart from nonscaling effects) is to recognize the following. In our picture of relativistic nuclear collisions the spatial extensions of the opposite-going fast (valence) quarks are contracted to infinitesimally thin disks, but they do not interact effectively. On the other hand, the slow (wee) partons that do interact strongly are not spatially contracted. Since T is a measure of the thermal energy of an interacting system, there is no mechanism to increase the energy density indefinitely no matter how high Y is. Evidently, nuclear transparency and distributed contraction at the parton level are the basic properties responsible for our result.

To conclude, we have described the $A+A$ collisions in the parton basis incorporating the basic phenomenological features of $h+h$ and $h+A$ collisions, but without assuming any specific thermalization dynamics, and we have obtained a lower bound on the thermalization time on the order of 0.15 fm/c and an average initial temperature in the 300–500-MeV range at large Y. The result is interesting as well as encouraging for the purpose of creating quark-gluon plasma and deserves further study.

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[1]See *Quark Matter Formation and Heavy Ion Collisions '82,* edited by M. Jacob and H. Satz (World Scientific, Singapore, 1982); Proceedings of the Third International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions, Brookhaven National Laboratory, 26–30 September 1983, Nucl. Phys. **A418**, 1c-678c (1984); *Quark Matter '84,* edited by K. Kajantie (Springer-Verlag, New York, 1984).

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[6]This property is automatically contained in a three-dimensional integral form for $T^{\mu\nu}$. The result is the same.

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[8]The $A^{1/6}$ dependence of T_i was also pointed out by L. McLerran at the Workshop on Relativistic Heavy Ion Colliders, Brookhaven National Laboratory, April 1985. His estimate of τ_i is also very small; see Ref. 1, *Quark Matter '84*.