Bragg Resonance of Light in Optical Superlattices

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A new phenomenon is reported in which light satisfying a Bragg-type resonance condition gets strongly reflected by a weak coarse-period (tens of microns) superlattice that is superimposed on a fine-period (submicron) stratified medium. A coupled-wave theory is developed to describe this interaction.

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If two waves can interfere to produce real interference fringes, then the energy in one of these waves can be Bragg diffracted into the other by modulation of the properties of the medium periodically with a period equal to the fringe spacing. This forms the basis of many effects from x-ray and electron diffraction in crystalline media to distributed-feedback lasers and holography. I have recently shown[1] that the eigenmodes of a medium with fine (submicron) periodic stratifications—the Floquet-Bloch waves—interfere with each other in a number of curious ways, producing relatively coarse (tens of microns) real spatial fringes. Hence one expects that, by superimposing of a coarse-periodic modulation of the correct type (a *superlattice*) on the existing fine lattice, Bragg-type conversion of one Floquet-Bloch wave into the other will result. The doubly periodic structure that one obtains is reminiscent of the quantum-well structures now the subject of intensive study, and indeed the analysis presented here may be of relevance in that field. A Floquet-Bloch wave (like an electronic Bloch wave in an atomic lattice) can to a first approximation be regarded as a stable balance of two strong spectral waves, a balance that can be upset by relatively weak perturbations of the stratified medium in which it exists. Add to this a phase-matched scattering *Bragg-type* condition (i.e., make the perturbation periodic), and highly efficient energy conversion between two resonant Floquet-Bloch waves can result. In this paper, a coupled-wave theory of this phenomenon is developed, and a number of new results outlined, such as *large-angle* acousto-optical and electro-optical deflection of an incident Floquet-Bloch beam. The analysis has wide-ranging implications for all kinds of open and guiding periodic structures.

The Floquet-Bloch waves[2,3] are the simplest electromagnetic disturbances that can exist in a lossless stratified structure whose effective propagation constant β_0 is modulated periodically, with an effective strength M_0 and a spatial period Λ ($=2\pi/|\mathbf{K}|$, where $\mathbf{K}$ is the grating vector pointing perpendicular to the grating planes):

$$\beta_0^2(\mathbf{r}) = k_v^2 N^2 (1 + M_0 \cos \mathbf{K}\cdot\mathbf{r}), \tag{1}$$

where k_v is the vacuum wave vector and N the average effective refractive index of the light. Equation (1) can apply to a variety of different structures, for example, a volume hologram, a crystal used for four-wave mixing, a single-mode optical fiber with a holographic grating written in it, a periodic planar waveguide, or the resonator of a distributed-feedback laser. In the two-wave approximation,[4] a Floquet-Bloch wave in any one of these cases is represented by the superposition of two spectral waves (eigenmodes of the structure for $M_0 = 0$) chosen so that they interfere to produce fringes with a spacing Λ:

$$E(\mathbf{r}) = \sum_{n=-1}^{0} V_n \exp(-j\mathbf{k}_n\cdot\mathbf{r}), \tag{2}$$

where

$$\mathbf{k}_n = \mathbf{k}_0 + n\mathbf{K}. \tag{3}$$

The quantities V_n in Eq. (2) are *scalar* amplitudes of the two spectral waves in the Floquet-Bloch wave. The effective value of M_0 will of course be determined by the geometry of the structure, and the polarization states and directions of phase progression of the two spectral waves. In this way the analysis is kept quite general, the amplitudes V_n being regarded as solutions of a scalar wave equation with β_0 as its propagation constant:

$$\{\nabla^2 + \beta_0^2(\mathbf{r})\}E = 0. \tag{4}$$

Putting Eq. (2) into Eq. (4) and setting the coefficients of like exponentials equal to 0 leads to an eigenvalue problem whose solutions I shall now summarize.[1] The two wave vectors

$$\bar{\mathbf{k}}_{\mathrm{LP}_n} = (K/2)\{\hat{\mathbf{x}}\cot\theta_B + \hat{\mathbf{y}}(-1)^n\}, \quad n = 0, -1, \tag{5}$$

are those of two "Lorentz points" (borrowing a term from x-ray theorists[5]) in wave-vector space. The unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ point respectively parallel and normal to the grating planes (see Fig. 1). They define the first Bragg angle, $\theta_B = \arcsin(K/2k_vN)$. In the presence of finite grating modulation ($M_0 > 0$), the locus of allowed wave vectors in the vicinity of the Lorentz points (obtained by solution of the dispersion relation)

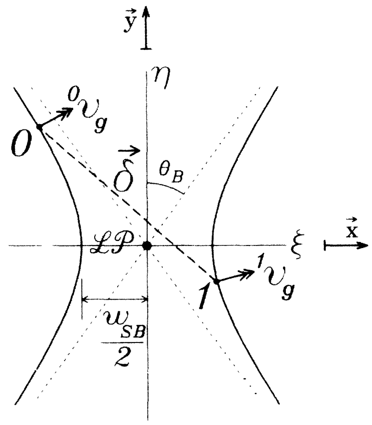

FIG. 1. The stop band in the vicinity of a Lorentz point (labeled LP) in wave-vector space. The unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are oriented respectively parallel and normal to the grating planes in real space. The wave-vector difference $\boldsymbol{\delta}$ between the two arbitrary tie points 0 and 1 determines the orientation and spacing of the coarse grating planes necessary for exact fulfillment of the Bragg condition $\theta=0$ [from Eq. (20)]. The double-headed arrows indicate the directions of the two group velocities.

has the form of a stop band. To evaluate the shape of the stop-band branches, a new set of axes (ξ,η) is defined with its origin at either of the Lorentz points. When we put $\mathbf{k}_n=\bar{\mathbf{k}}_{\mathrm{LP}_n}+\xi\hat{\mathbf{x}}+\eta\hat{\mathbf{y}}$ into Eq. (3) and neglect terms of order higher than 2 in $\xi/k_v N$ and $\eta/k_v N$, the locus of the stop band is the pair of hyperbolae

$$(2\xi/w_{\mathrm{SB}})^2=1+(2\eta\tan\theta_{\mathrm{B}}/w_{\mathrm{SB}})^2, \tag{6}$$

where the parameter w_{SB} is the minimum stop-band width, given by

$$w_{\mathrm{SB}}=M_0 k_v N/2\cos\theta_{\mathrm{B}}. \tag{7}$$

This is depicted in Fig. 1, where the stop band is sketched and w_{SB} indicated. The normalized mode shape $\{\mathbf{V}\}$ of a Floquet-Bloch wave with a *tie point* [i.e., the location on a stop-band branch with coordinates (ξ,η) associated with a given Floquet-Bloch wave] on one of the stop-band branches is

$$\begin{pmatrix} V_0 \\ V_{-1} \end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix} +[1-(\eta\tan\theta_{\mathrm{B}}/\xi)]^{1/2} \\ \pm[1+(\eta\tan\theta_{\mathrm{B}}/\xi)]^{1/2} \end{pmatrix}, \tag{8}$$

where the plus sign refers to a Floquet-Bloch wave with a tie point on the *slow* branch and the minus sign to one with a tie point on the *fast* branch [for $(\mathbf{k}_{\mathrm{LP}_0}\cdot\hat{\mathbf{x}})>0$ these branches are respectively on the right-hand and left-hand sides of the Lorentz point].

The group velocity $\mathbf{v}_g$ of any particular Floquet-Bloch wave follows from the dispersion relation, pointing normal to the stop-band branches, and can lie anywhere between $\pm\theta_{\mathrm{B}}$ (see Fig. 1). A straightforward analysis[1] yields

$$\mathbf{v}_g=(c/N)\cos\theta_{\mathrm{B}}[\hat{\mathbf{x}}-\hat{\mathbf{y}}(\eta\tan^2\theta_{\mathrm{B}}/\xi)], \tag{9}$$

where c is the velocity of light *in vacuo*. It can also be expressed in terms of the modal amplitudes (it lies parallel to the spatially averaged Poynting vector). Within the approximations of the analysis this leads to

$$\mathbf{v}_g=(c/k_v N^2)\{V_0^2\bar{\mathbf{k}}_{\mathrm{LP}_0}+V_{-1}^2\bar{\mathbf{k}}_{\mathrm{LP}_{-1}}\}. \tag{10}$$

From Eq. (9), the angle α which the group velocity makes with the x axis (i.e., with the grating lines) is

$$\alpha=\arctan[-(\eta/\xi)\tan^2\theta_{\mathrm{B}}]. \tag{11}$$

Our representation of a Floquet-Bloch wave consists of a periodic transverse amplitude distribution (period Λ) progressing in phase along the stratifications, and traveling in the ray direction of its group velocity $\mathbf{v}_g$ at an angle α to them. If the energy in a Floquet-Bloch wave has a finite diffraction-limited angular spectrum (so that its tie point in wave-vector space becomes an arc), a Floquet-Bloch *beam* will appear in real space, traveling in the direction of its central group velocity. It is most important in the further development of this paper that a reader retains this physical picture of the Floquet-Bloch waves. Any attempt to think more conventionally in terms of spectral waves coupled together by Bragg diffraction will obscure the relatively simple conceptual framework in which the following results are based.

The medium is now taken to have superimposed on its basic periodicity a coarse grating with period Λ_1 $(=2\pi/|\mathbf{g}|$, where $\mathbf{g}$ is its grating vector). This grating might be created by an acoustic wave, by an electro-optical interdigital electrode system, or by interferometry. The resulting propagation constant $\beta_1(\mathbf{r})$ is

$$\beta_1^2(\mathbf{r})=\beta_0^2(\mathbf{r})+k_v^2N^2[M_0 m_a\cos(\mathbf{g}\cdot\mathbf{r})\cos(\mathbf{K}\cdot\mathbf{r})+m_b\cos(\mathbf{g}\cdot\mathbf{r})]. \tag{12}$$

This definition allows for modulation both of the fine grating strength M_0 (via m_a) and of the average propagation constant (via m_b). Consider now two Floquet-Bloch waves (distinguished by superscripts 0 and 1 in front of their parameters) of the unperturbed finely stratified medium described by Eq. (1). Their electric fields take the form

$$^mE(\mathbf{r},{}^m\tilde{\mathbf{k}}_0)=\sum_{n=-1}^{0}{}^mV_n\exp(-j\,{}^m\tilde{\mathbf{k}}_n\cdot\mathbf{r}),\quad m=0,1, \tag{13}$$

where tildes are used to indicate that the medium is as yet unperturbed (i.e., m_a and m_b are zero). Between them exists, for all values of n, a small constant wave-vector difference given by

$$\boldsymbol{\delta} = {}^0\tilde{\mathbf{k}}_n - {}^1\tilde{\mathbf{k}}_n. \tag{14}$$

It is this wave-vector difference that generates coarse interference fringes. I shall now assume that $\boldsymbol{\delta}$ is approximately equal to $\mathbf{g}$, and invoke Floquet's theorem to derive a suitable *Ansatz* for the perturbed wave equation

$$\{\nabla^2 + \beta_1^2(\mathbf{r})\}E = 0 \tag{15}$$

in the form

$$E(\mathbf{r}) = {}^0A(\mathbf{r})\,{}^0E(\mathbf{r}, {}^0\mathbf{k}_0) + {}^1A(\mathbf{r})\,{}^1E(\mathbf{r}, {}^1\mathbf{k}_0), \tag{16}$$

where

$${}^m\mathbf{k}_0 = {}^0\tilde{\mathbf{k}}_0 + m\,\mathbf{g}, \tag{17}$$

and ${}^mA(\mathbf{r})$ is the spatially varying coupled-wave amplitude of the mth Floquet-Bloch wave. On substitution of Eq. (16) into Eq. (15), one of two coupled differential equations can be obtained by our multiplying the result with ${}^1E^*(\mathbf{r}, {}^1\mathbf{k}_0)$ (where the asterisk denotes the complex conjugate), using Eq. (10), neglecting second-order derivatives of ${}^1A$, and setting the coefficients of nonexponential terms to zero. The other coupled differential equation follows if $m=1$ is replaced by $m=0$ in the last sentence, yielding

$$\begin{aligned} &\nabla\,{}^0A\cdot{}^0\mathbf{p} + j\kappa\,{}^1A\,(|{}^1\mathbf{v}_g|/|{}^0\mathbf{v}_g|)^{1/2} = 0,\\ &\nabla\,{}^1A\cdot{}^1\mathbf{p} + j\theta\,{}^1A + j\kappa\,{}^0A\,(|{}^0\mathbf{v}_g|/|{}^1\mathbf{v}_g|)^{1/2} = 0, \end{aligned} \tag{18}$$

where the coupling constant κ is given by

$$\kappa = [k_v c/\{2(|{}^0\mathbf{v}_g|\,|{}^1\mathbf{v}_g|)^{1/2}\}]\{(M_0 m_a/4)({}^0V_{-1}{}^1V_0 + {}^0V_0{}^1V_{-1}) + (m_b/2)({}^0V_0{}^1V_0 + {}^0V_{-1}{}^1V_{-1})\}, \tag{19}$$

and ${}^m\mathbf{p}$ is a unit vector parallel to the group velocity ${}^m\mathbf{v}_g$ of the mth Floquet-Bloch wave. The parameter

$$\theta = (\mathbf{g} - \boldsymbol{\delta})\cdot{}^1\mathbf{p} \tag{20}$$

is a dephasing parameter, describing violations of the Bragg condition. A plot of normalized coupling constants $8\kappa/k_v N M_0 m_a$ and $4\kappa/k_v N m_b$ and deflection angles versus ξ/w_{SB} is presented in Fig. 2 for two special cases: (i) a Floquet-Bloch wave pair whose tie points have equal values of η (curves a and d), and (ii) a Floquet-Bloch wave pair whose tie points have equal values of ξ (curves b and c). Curves a and b are for a grating with $m_b=0$, and curves c and d for one with $m_a=0$. The Bragg angle is 40°, yielding deflection angles (curve e in Fig. 2) lying between 0° and 80°.

An obvious situation where one might expect to see evidence of the Bragg resonances described by Eqs. (18) occurs when an acoustic wave is superimposed on the finely stratified medium. This would most likely produce a grating of type $m_b > 0$, the stress field of the sound causing variations in average propagation constant. The reflection efficiency would be a function of the acoustic power and wavelength, and extraordinarily large deflection angles could be observed under the correct conditions, if we consider that the conventional Bragg angles for acousto-optical scattering are less than a degree. In a typical single-mode corrugated Ta_2O_5 waveguide,[1] with $M_0 = 0.01$, $\theta_B = 40°$, $\Lambda_1 = 17.7$ μm, an effective wavelength (in the guide) of 350 nm, and symmetrical tie points on one side of the stop band (i.e., ${}^0\eta = -{}^1\eta$), a deflection angle of 76° at a coupling rate of $\kappa = 0.37$ mm^{-1} can be achieved for $m_b = 0.0001$. I should also comment on the validity of Eqs. (18). If the rays of the two coupled Floquet-Bloch waves (parallel to the vectors ${}^0\mathbf{p}$ and ${}^1\mathbf{p}$) cross a large number of the coarse grating planes on their way through the perturbed medium, then the two-wave approximation is likely to be valid. This condition can be expressed as

$$({}^m\mathbf{p}\cdot\delta)\mathrm{L}^2/2\pi({}^m\mathbf{p}\cdot L) >> 1,$$

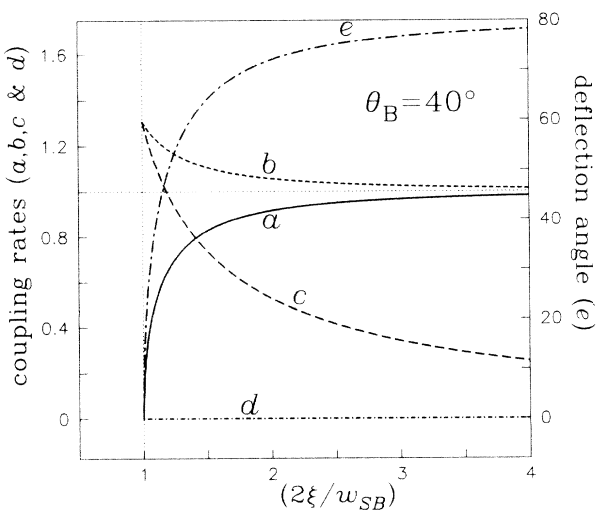

FIG. 2. Coupling rates (*normalized* quantities; see text) and deflection angles vs ξ coordinate; see Fig. 1. If we define the coordinates of the two tie points as $({}^0\xi, {}^0\eta)$ and $({}^1\xi, {}^1\eta)$, the curves labeled a, b, c, and d represent respectively the following different situations: $m_b=0$, ${}^0\eta={}^1\eta$; $m_b=0$, ${}^0\xi={}^1\xi$; $m_a=0$, ${}^0\xi={}^1\xi$; $m_a=0$, ${}^0\eta={}^1\eta$. The curve labeled e yields the deflection angle (the angle through which the group velocity of the light is turned). Because of the symmetry of the stop band this angle behaves identically in all four cases.

where $\mathbf{L}$ is the width vector of the supposedly slab-shaped grating. If, however, the angle between the waves is very small, very few grating planes will be crossed, and the two-wave approximation will be invalid because of the presence of many high-order Floquet-Bloch waves neglected in the *Ansatz* in Eq. (16). This diffraction regime has similarities with the Raman-Nath[6] regime in conventional theories of Bragg diffraction. Mathematical techniques for the solution of Eqs. (18) already exist, having been developed both in the realm of x-ray diffraction[7] and in the theory of volume gratings.[4,6] Solutions of Eqs. (18) will be explored in subsequent publications. A final comment is in order as to the possible relevance of these results to quantum-well structures. It might be of interest to use the analysis for the investigation of the effect of very small-period superlattices ($\sim$ 100-Å period, such as those encountered in multiple quantum-well structures) on the electronic band structure. It seems likely that quantization of the energy levels would result, together with changes in the refractive index.

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