Shapes of Rotating Free Drops: Spacelab Experimental Results

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The experimental observation of the behavior of acoustically rotated free drops in the microgravity environment of low Earth orbit has yielded the first data which meet the restrictions of existing theoretical treatments. Rotating drops displayed both "axisymmetric" and "two-lobed" shapes in the presence and absence of applied torques. Their dimensions and general development confirm the existence of the distinct shape families and the bifurcation point between them. The data also indicate that the secular instability in the axisymmetric family occurs at a lower rotation velocity than predicted.

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The surface-tension–controlled shapes of liquid drops in gyrostatic equilibrium have been theoretically considered as part of the more general problem of treating the dynamics of rotating masses under the dominant effects of gravitational, electrical, or even nuclear forces. The modeling of the behavior of planets, distant stars, and atomic nuclei is the obvious motivation for undertaking the solution of this problem, and an elegant summary of the subject has been presented by Swiatecki.[1] Until very recently, however, experimental verification of these theoretical predictions has not been possible to obtain even for the prosaic case of a liquid drop under the influence of surface tension and not submitted to an overwhelming gravitational field. The availability of experimental instrumentation aboard the NASA Space Shuttle and the implementation of acoustic positioning techniques[2] have allowed the investigation of the surface-tension–dominated equilibrium shapes of rotating drops with gravitational acceleration reduced by a factor of at least 10^{-3}.

For this restricted case, the shape of the surface of the spinning drop will be dictated by the usual Young-Laplace equation including a term reflecting the centrifugal acceleration,

$$P(I)-P(O)=\tfrac{1}{2}\rho(I)r^2(1-\cos^2\theta)\Omega^2+T\nabla\cdot\mathbf{n}.$$

Here $P(I)$ denotes the pressure inside the drop, $P(O)$ the pressure outside, $\rho(I)$ the drop density, Ω the rotation velocity, T the surface tension, $\mathbf{n}$ the normal unit vector at the surface, and r and θ the radial and polar-angle coordinates in a spherical system with the origin at the drop center and with the polar axis in the direction of the rotation axis. Only the drop is assumed to rotate and the inertia and the viscosity of the outside medium are neglected.

The problem of the stability of the various possible equilibrium shapes has been treated by both analytical[3,4] and numerical[5] methods. The latter technique, however, has provided predictions about the existence of a number of equilibrium shapes having a decreasing order of symmetry. For low rotation rates, a drop in gyrostatic equilibrium is symmetric in shape with respect to the rotation axis, flattening at the poles as the rate increases. At a well-determined normalized rotation rate, this axisymmetric drop becomes secularly unstable to shape perturbations leading to the two-lobed configuration; axial symmetry is lost and the drop develops along the new branch. Similarly, other branches characterized by three- and four-lobed shapes will separate from the axisymmetric branch at higher rotation velocities before the axisymmetric shape ceases to exist.

Experimental evidence of these equilibrium shapes has been obtained in the laboratory with use of immiscible liquids in neutral-buoyancy systems which diminish the effects of gravitational acceleration. Plateau[6] was the first to gather evidence for the existence of axisymmetric and two-lobed shapes, as well as toroidal configurations. These experiments were later repeated, and three- and four-lobed shapes were observed.[7] These results cannot be correlated with theoretical predictions, however, because the basic condition of gyrostatic equilibrium cannot be satisfied in neutral-buoyancy systems because of the presence of viscous stresses and inertial drag exerted by the outer suspending liquid.

In the present experiment, drops of water or glycerin-water mixtures of various viscosities were deployed inside an acoustic positioning chamber while orbiting the Earth inside the Spacelab module carried in the payload bay of the Space Shuttle. The average level of residual acceleration was around 10^{-3} times the standard Earth gravitational field. The drops were held within a small region inside the chamber by three orthogonal acoustic standing waves generating steady-state forces on the order of 1 dyn, and were driven into rotation by acoustic torques.[8]

The drop volumes varied between 0.5 and 10 cm^3 with viscosities of 1, 10, 20, and 100 cS (centistokes), densities between 1.0 and 1.22 g/cm^3, and surface tensions from 55 to 65 dyn/cm.

An initially nonrotating drop was subjected to an acoustically generated torque which caused it to spin up with increasing velocity. The rotational velocity of the liquid was determined through the motion of immiscible tracer particles suspended within the primary drop. The time variation of the shape of the rotating drops was determined by recording of the drop profile along three orthogonal views on 16-mm motion-picture film.

Data from a typical experimental sequence involving several spinups and spindowns of a 1-cm^3, 100-cS drop are shown in Fig. 1, where the rotation speed is plotted along the ordinate axis, and time is measured along the abscissa. In the first phase of the spinup the drop shape is still axisymmetric with respect to the rotation axis: The initially spherical drop deforms into a spheroid with increasing aspect ratio, a/b, where a is the equatorial and b the polar radius, respectively. At a given normalized rotation velocity, denoted by $\omega(\mathrm{II})$ in Fig. 1, the drop shape changes from axisymmetric to two lobed. This is the point of bifurcation. At this point the rotation velocity of the drop starts to decrease as a result of the larger deformation. If the acoustic torque continues to act on the drop, fission will eventually take place. On the other hand, if the torque is turned off, the two-lobed deformation will decrease, and a transition back to the axisymmetric configuration will take place (430–490 sec, Fig. 1). The rotation velocity at which this reverse transition takes place is very close to $\omega(\mathrm{II})$. In the axisymmetric regime, the rotation velocity of the drop will slowly decay if only drag is acting on the liquid. A much faster decay to zero rotation rate can be induced when a reverse acoustic torque is applied (480 sec). Because of this reverse torque the drop will slow down quickly and start to rotate in the other direction, eventually repeating the same cycle. This sequence ends in the fission of the drop. In this particular instance, the reverse torque was of a higher magnitude than in the forward direction. This was accomplished by purposefully raising the sound pressure level.

Of particular interest here are both the value of $\omega(\mathrm{II})$, the rotation velocity at bifurcation, and the deformation of the drop as a function of rotation speed, since these two characteristics have been precisely determined theoretically.

One of the main features of these experimental results was the absence of configurations other than the axisymmetric and the two-lobed shapes. Comparison with theoretical results will thus be restricted to the quantitative results obtained for these two shapes.

Most of the experiments were performed by application of the acoustic torque and by observation of the evolution of the shape as the rotation rate changed. The magnitude of the torque was kept low to make the angular acceleration quite small. For high enough liquid viscosities, this experimental approach can be considered a close approximation to the ideal system

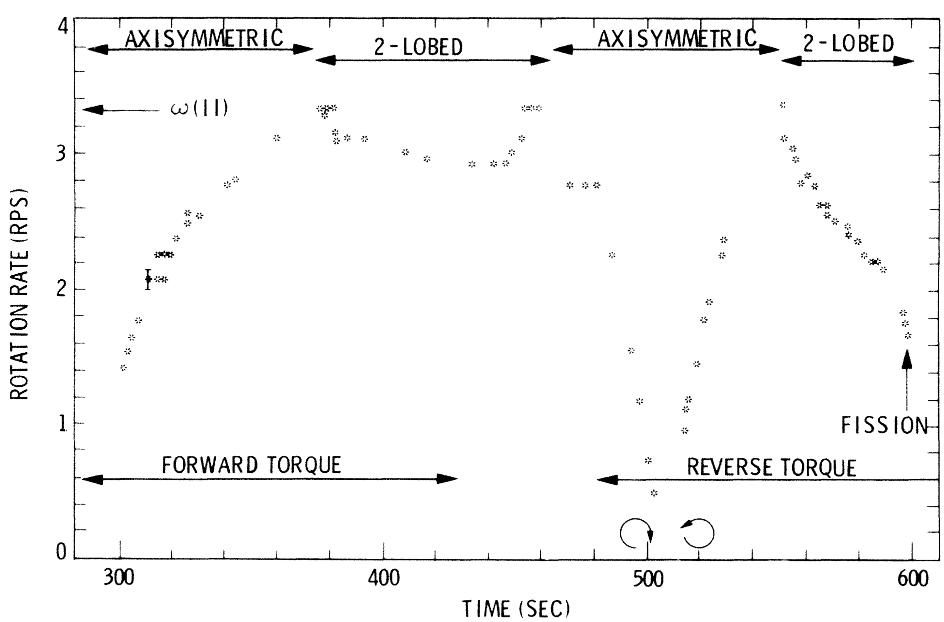

FIG. 1. Experimental sequence studying drop rotation using a 1-cm^3 drop of a 100-cS mixture of glycerin and water. The times when torques were applied and the general shapes of the drop are indicated. During this experiment the drop went from an axisymmetric shape to a two-lobed one under an applied acoustic torque (372 sec); from two-lobed to axisymmetric when the torque was removed (462 sec); under an opposite torque decelerating and accelerating axisymmetrically, becoming two-lobed and then splitting into two equal volumes. $\omega(\mathrm{II})$ denotes the experimentally measured velocity at bifurcation.

of a drop with constant rotation velocity and in gyrostatic equilibrium. Because the rotation speed changed quite slowly it is reasonable to assume that the speed remained constant on the time scale of the relaxation time. For example, with use of a viscous length, $(\nu t)^{1/2}$, of 0.5 cm for a 100-cS drop 1 cm in diameter, the characteristic relaxation time is 0.25 sec. For a typical acceleration of 0.014 rev/s^2 the velocity changed by 0.0035 rev/s during the relaxation time. For a 1-cS drop, the relaxation time would be 25 sec, and the velocity would have increased by 0.35 rev/s. For this reason, data for the more viscous fluids conform more closely to the model.

Figure 2 shows experimental data for a 3-cm^3, 100-cS water-glycerin drop. This particular experiment involves the spinup of the drop, its transition to the two-lobed shape, and finally its relaxation to the axisymmetric shape as the acoustic torque is turned off. The rotational velocity is normalized by the frequency of the nonrotating fundamental oblate-prolate resonant mode of shape oscillation. The relative deformation is the largest dimension in the equatorial plane of the drop divided by the diameter of the nonrotating spherical drop. Both theoretical predictions and experimental results are shown in the plot.

The most obvious difference is in the location of the bifurcation: Ω(II) was experimentally determined to be 0.47 ± 0.04 while it is calculated to be 0.56.[5] In addition, the experimental rate of change of the relative deformation with rotation rate appears less than that predicted by calculations for the two-lobed shape, while the agreement appears to be quite good for the axisymmetric shape. One might also note that, within experimental uncertainty, there is no clear evidence for the existence of different paths for spinup and spindown to the two-lobed regime (although the scatter of the experimental results increases when the data for spindown are added). These results are representative of the data gathered for several drops of various volumes.

The fact that no higher-number-lobed configurations were obtained does not necessarily preclude the possibility of the existence of higher-order bifurcation points at larger velocity. It is conceivable that the low rotational acceleration used in this particular investigation or the small translational drop oscillations within the potential well adversely affected the probability of bypassing the first bifurcation point to reach higher-order lobed shapes. A more detailed investigation with a systematic variation of the relevant parameters is needed to provide a definitive answer.

When the deformation of the drop in the two-lobed regime is increased by the imposition of a larger torque, fission becomes inevitable beyond a certain value. Fission occurred experimentally after a monotonic increase in the two-lobed deformation and a short decrease in that deformation. The drops split symmetrically, occasionally leaving much smaller droplets at the center of mass. Figure 3 reproduces experimental data for a rotation sequence leading to the splitting of the drop. Fission occurred at a normalized rotation velocity of about 0.21 (± 0.04). There is no simple prediction of a critical velocity at fission, but one could compare the data to the velocity calculated for maximum drop deformation (0.26). Calculations also indicate that the rotational kinetic and surface en-

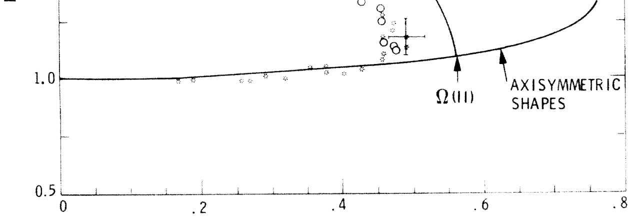

FIG. 2. Comparison of theoretical and experimental values for the relative deformation $R(\max)/R(0)$ as a function of the normalized rotation speed for a 3-cm^3 drop (100-cS water-glycerin mixtre).

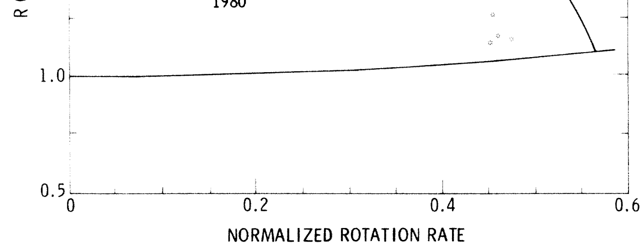

FIG. 3. Theoretical and experimental relative deformation in the two-lobed shape up to fission as a function of rotation speed. Data from the last sequence shown in Fig. 1.

ergies for a single two-lobed drop and for two separate drops match in the region where the normalized velocity is equal to 0.25.[5]

These experimental data obtained in the low-gravity environment of space have yielded a first experimental test in agreement with the various analytical and numerical predictions regarding the axisymmetric equilibrium shapes of rotating drops and evidence for the early onset of secular instability experienced by those shapes. This may mean that the drop shape is less stable to fluctuations than the calculations have predicted. That no mode higher than the two-lobed space was observed also lends support to the diminished-stability argument. These data also yield a good quantitative agreement between the theory and experiment for the velocity at fission in the two-lobed region.

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