Can the Skyrmion Model be a Good Description of the Nucleon?

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The nucleon mass is computed in the skyrmion model. Starting with a parameter-free nonlinear chiral Lagrangean containing the effects of heavy mesons (ρ, σ, ω), we find that the computed mass is a factor of 2 too large compared with the measured value.

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In 1960, in a very original paper, Skyrme proposed a unified field theory of mesons and bosons by considering baryons as soliton solutions of a nonlinear meson field theory.[1] His main motivation was to give a proper treatment of an extended object. Skyrme's idea was recently revived by a number of people.[2-4] The real impetus in this direction is due to Witten[4] who pointed out that in the limit of large number of colors, N_c, quantum chromodynamics (QCD) may be approximated by an effective theory of mesons in the form of a nonlinear chiral Lagrangean and baryons could emerge as soliton solutions of this effective Lagrangean. If this idea is correct, static properties of baryons can be computed in terms of a few parameters completely determined by low-energy meson physics.[5]

The purpose of this note is to test Skyrme's idea by calculating baryon masses and some other static properties, using the low-energy effective Lagrangean which, as mentioned above, contains no free parameters. Our effective Lagrangean, apart from the standard minimal terms (which can be derived from the nonlinear σ model), in the SU(2) ⊗ SU(2)-symmetry limit contains the two most general quartic terms in derivatives of the pion field (denoted as the Skyrme and the non-Skyrme terms) and a term having six powers of the field derivative obtained by the contribution of the ω field in the Lagrangean. In a previous paper,[5] it was pointed out that the strength of the Skyrme term can be evaluated easily from the heavy-ρ-meson term in the Lagrangean. The non-Skyrme term corresponds to the contribution of the σ meson in the Lagrangean. In the same manner, because of its nearly degenerate mass with the ρ, the ω terms produce an important contribution and must be included in the Skyrme Lagrangean. As will be clear below, the ρ and ω terms stabilize the soliton and make a positive contribution to its mass while the σ destabilizes it. In previous works the soliton is stabilized either by the Skyrme term[6] or by the ω term[7] alone. This is not a correct treatment because all the heavy-meson contributions are equally important and must be included in the full Lagrangean as explained above.

Our approach takes into account the effects of the coupling of the ρ, ω, and σ fields with the pion system. It is essentially a pole approximation with the momentum dependence in the propagator (which gives rise to a higher-derivative expansion) neglected. In this way the energy of the skyrmion is obtained by solving the usual Skyrme Lagrangean without the ρ, ω, and σ fields.

Throughout this note we neglect the A_1 contribution to terms with six powers of the derivative of the pion field, because the $A_1 \to \rho\pi$ squared coupling constant is 1 order of magnitude smaller than that of $\omega \to \rho\pi$ and because of the higher A_1 mass [for this purpose we use $A_1\rho\pi$ coupling in the form $\boldsymbol{\rho}_{\mu\nu} \cdot (\partial_\mu \mathbf{A}_1 \times \partial_\nu \boldsymbol{\pi})$].

The main conclusion of this article is that, contrary to the usual claims, the skyrmion model, in its present form, gives only a qualitative description of the baryon: The calculated baryon mass is a factor of 2 too large compared to the experimental values.

We begin by writing down the effective Lagrangean taking into account the effect of the interaction ρ, ω, and σ fields with the pion systems:

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_\rho + \mathscr{L}_\sigma + \mathscr{L}_\omega, \tag{1}$$

where

$$\mathscr{L}_0 = \tfrac{1}{8} f_\pi^2 \operatorname{Tr}(\partial_\mu M\, \partial^\mu M^\dagger) + \tfrac{1}{4} m_\pi^2 f_\pi^2 (\operatorname{Tr} M - 2), \tag{2a}$$

$$\mathscr{L}_\rho = (1/32e^2) \operatorname{Tr}\{[\partial_\mu M\, M^\dagger, \partial_\nu M\, M^\dagger]^2\}, \tag{2b}$$

$$\mathscr{L}_\sigma = (\gamma/8e^2)[\operatorname{Tr}(\partial_\mu M\, \partial_\mu M^\dagger)]^2, \tag{2c}$$

$$\mathscr{L}_\omega = -(\beta^2/m_\omega^2)(1/24\pi^2)^2 [\epsilon^{\mu\nu\rho\sigma} \operatorname{Tr}(M^\dagger \partial_\nu M\, M^\dagger \partial_\rho M\, M^\dagger \partial_\sigma M)]^2, \tag{2d}$$

and $M = \exp[i(\sqrt{2}/f_\pi)\boldsymbol{\pi}\cdot\boldsymbol{\tau}]$ with $f_\pi = 135$ MeV. Note that f_π is the pion decay constant measured in the π_{l2} decays. This interpretation follows from the chiral Lagrangean regardless of whether this Lagrangean admits a classical soliton or not. Hence the parameter f_π together with other parameters in the Skyrme Lagrangean are fixed *a priori* from low-energy meson physics. The nucleon mass and other static quantities such as G_A/G_V or nucleon

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magnetic moments are then computed in terms of these parameters within the framework of the skyrmion model. In previous works[6,7] Adkins, Nappi, and Witten turn the problem around and try to predict f_π in terms of the nucleon mass. We do not see any reason for doing this.

At the classical level, with the standard hedgehog *Ansatz*[1]

$$M_0(\mathbf{r}) = \exp[iF(r)\hat{\mathbf{r}}\cdot\boldsymbol{\tau}],$$

it is straightforward to derive the Euler-Lagrange equation for $F(r)$. Neglecting terms of the order $O(m_\pi)$, we find

$$\left[6\gamma x^2F'^2 + 2(2\gamma-1)\sin^2F - \tfrac{1}{4}x^2 - \Omega\frac{\sin^4F}{x^2}\right]F'' + [4\gamma F'^2 - \tfrac{1}{2}]xF' + \left[\frac{1}{4} + (1-4\gamma)\frac{\sin^2F}{x^2} + (2\gamma-1)F'^2\right]\sin 2F + \Omega\left[2F'\frac{\sin^2F}{x} - F'^2\sin 2F\right]\frac{\sin^2F}{x^2} = 0, \quad (3)$$

where

$$x = \sqrt{2}ef_\pi r, \quad \Omega = \beta^2e^4f_\pi^2/\pi^4m_\omega^2.$$

Since we are interested in the soliton with baryon number $B=1$, $F(r)$ must satisfy the boundary equation[1]

$$F(0) = \pi, \quad F(\infty) = 0.$$

The soliton mass $\mathcal{M}$ is given by $(m_\pi = 0)$

$$\mathcal{M} = H_0 + H_\rho + H_\sigma + H_\omega, \quad (4)$$

where

$$H_0 = \frac{C}{4}\int_0^\infty dx\, x^2\left[F'^2 + \frac{2\sin^2F}{x^2}\right], \quad (5a)$$

$$H_\rho = C\int_0^\infty dx\left[2F'^2 + \frac{\sin^2F}{x^2}\right]\sin^2F, \quad (5b)$$

$$H_\sigma = -\gamma C\int_0^\infty dx\, x^2\left[F'^2 + \frac{2\sin^2F}{x^2}\right]^2, \quad (5c)$$

$$H_\omega = \Omega C\int_0^\infty dx\,\frac{F'^2\sin^4F}{x^2}, \quad (5d)$$

and

$$C = 2\sqrt{2}\pi f_\pi/e.$$

Note that the σ meson makes a negative contribution to the energy and would destabilize the solution for $\gamma \geqslant \frac{1}{3}$ in the absence of the ω term as shown previously.[5]

The quantum correction which comes from the spin-isospin excitation may be obtained by the method of Adkins, Nappi, and Witten.[6] By rotating the soliton around the static classical solution to obtain a time-dependent one in the form

$$M(\mathbf{r},t) = A(t)M_0(\mathbf{r})A^{-1}(t),$$

with

$$A(t) = a_0 + i\mathbf{a}\cdot\mathbf{r}$$

being dynamical variables, we obtain the quantum correction for the lowest excitation with $I=\frac{1}{2}$, $J=\frac{1}{2}$ as

$$\delta H = \frac{-\sqrt{2}\pi}{3e^3f_\pi}\int_0^\infty dx\, x^2\sin^2F\left\{1 + 4(1-2\gamma)F'^2 + 4(1-4\gamma+\Omega F'^2)\frac{\sin^2F}{x^2}\right\}\mathrm{Tr}(\partial_0A\,\partial_0A^{-1}) \quad (6)$$

(neglecting a term quartic in the derivative of A). We now proceed to the evaluation of the coupling constants in the chiral Lagrangean and the solution of the Euler-Lagrange equation (3).

(i) The strength of the Skyrme and non-Skyrme terms are evaluated in Ref. 5. There is little uncertainty in evaluation of the strength of the Skyrme term, because the pion-pion P-wave phase shift is well known.

We have $1/e^2 = f_\pi^2/m_\rho^2$ with $m_\rho^2 \simeq 26m_\pi^2$ (because of the unitarity correction the ρ mass is shifted downward from the experimental value). The non-Skyrme term is found to be $\gamma/e^2 = \frac{1}{3}f_\pi^2/m_\sigma^2$. The S-wave $I=0$ π-π phase shift is not very well measured. Within experimental error, the S-wave π-π phase shift below 900 MeV can be fitted with $m_\sigma^2 = (22\text{–}32)m_\pi^2$; hence

$$\gamma \simeq 0.28\text{–}0.34.$$

The coupling β can be obtained from the $\omega \to 3\pi$ width. Using the experimental width $\Gamma(\omega\to 3\pi) = 8.9$ MeV and taking into account ρ dominance in $\omega\to 3\pi$ decay as supported by the calculation of $\omega^0 \to \pi^0\gamma$ width, we have $\beta \simeq 17$ which gives $\Omega \geqslant 75$, the uncertainty in this value being at most 30%.

It should be stressed that the strength parameter β in the effective Lagrangean for the coupling of ω to 3π receives contributions from both the contact term and the $\omega\rho\pi$ vertex via the coupling of ρ to 2π which, in the limit of small di-pion invariant mass, reduces to the $\omega \to 3\pi$ effective Lagrangean. Since the Gell-Mann–Sharp–Wagner (GSW) model[8] describes well the ratio $\Gamma(\omega \to \pi^0\gamma)/\Gamma(\omega \to 3\pi)$ via the vector-meson dominance, the strength of the contact term is negligible. This result is substantiated by recent theoretical analysis.[9] The parameter β is then determined from the $\omega \to 3\pi$ decay rate as discussed by Adkins and Nappi[7] but must be reduced by approximately a factor of $\frac{2}{3}$ to take into account of the enhancement in the $\omega \to 3\pi$ amplitude due to the ρ propagator.

(ii) *Solution of the differential equation.*—We have searched for numerical solutions both for $\Omega = 0$ and for $\Omega = 75$ and for different values of γ. In fact, when we look closer at Eq. (3), we see that γ may cause trouble, as it appears with a positive sign in the coefficient of F'', so that this coefficient is not negative definite and may vanish. Indeed, we found that this is the case with γ greater than a critical value γ_c which varies with Ω. More precisely, for $\Omega = 0$, we could not find regular solutions of (3) for[10] $\gamma \gtrsim 0.10$, and for $\Omega = 75$, the bound is $\gamma_c \simeq 0.27$ so that γ_c increases with Ω, at least in the range $\Omega = 0$–150. When we vary Ω with fixed γ, we find that the mass M grows nearly linearly with Ω, for those values of γ below the critical value $\gamma_c(\Omega)$, and in the range of values for Ω considered. In particular for $\gamma = 0.20$ which is fairly well below the value determined in Ref. 5, we must have $\Omega \geqslant 30$ corresponding to $\beta \geqslant 11$ in order to stabilize the soliton.

We display below some of the numerical results. For $\gamma = 0$, $\Omega = 0$,

$$M_N = 1348 \text{ MeV}, \quad M_\Delta = 1789 \text{ MeV},$$
$$\langle r^2 \rangle^{1/2}_{I=0} = 0.41 \text{ fm}, \quad \langle r^2 \rangle^{1/2}_{M,I=0} = 0.64 \text{ fm}.$$

For $\gamma = 0.05$, $\Omega = 0$,

$$M_N = 1281 \text{ MeV}, \quad M_\Delta = 1858 \text{ MeV},$$
$$\langle r^2 \rangle^{1/2}_{I=0} = 0.37 \text{ fm}, \quad \langle r^2 \rangle^{1/2}_{M,I=0} = 0.57 \text{ fm}.$$

For $\gamma = 0.10$, $\Omega = 0$,

$$M_N = 1221 \text{ MeV}, \quad M_\Delta = 2011 \text{ MeV},$$
$$\langle r^2 \rangle^{1/2}_{I=0} = 0.34 \text{ fm}, \quad \langle r^2 \rangle^{1/2}_{M,I=0} = 0.49 \text{ fm}.$$

For $\gamma = 0$, $\Omega = 75$,

$$M_N = 2241 \text{ MeV}, \quad M_\Delta = 2284 \text{ MeV},$$
$$\langle r^2 \rangle^{1/2}_{I=0} = 1.00 \text{ fm}, \quad \langle r^2 \rangle^{1/2}_{M,I=0} = 1.42 \text{ fm}.$$

For $\gamma = 0.10$, $\Omega = 75$,

$$M_N = 2154 \text{ MeV}, \quad M_\Delta = 2202 \text{ MeV},$$
$$\langle r^2 \rangle^{1/2}_{I=0} = 0.98 \text{ fm}, \quad \langle r^2 \rangle^{1/2}_{M,I=0} = 1.37 \text{ fm}.$$

For $\gamma = 0.27$, $\Omega = 75$,

$$M_N = 1993 \text{ MeV}, \quad M_\Delta = 2049 \text{ MeV},$$
$$\langle r^2 \rangle^{1/2}_{I=0} = 0.92 \text{ fm}, \quad \langle r^2 \rangle^{1/2}_{M,I=0} = 1.24 \text{ fm}.$$

These results are compared with the experimental values

$$M_N = 939 \text{ meV}, \quad M_\Delta = 1232 \text{ MeV},$$
$$\langle r^2 \rangle^{1/2}_{I=0} = 0.72 \text{ fm}, \quad \langle r^2 \rangle^{1/2}_{M,I=0} = 0.81 \text{ fm}.$$

The calculations were made with neglect of m_π, but we have checked that taking into account $m_\pi \neq 0$ does not modify the results substantially.

For the range of the values of γ and β determined above, the baryon mass is too large by a factor of 2. For $\gamma = 0.27$, $\Omega = 75$, we find $M_N = 1993$ MeV and $M_\Delta = 2049$ MeV. The Δ-N mass splitting is too small. If we lower the effective $\omega \to 3\pi$ squared coupling constant by 30%, the nucleon and Δ masses are lowered only by 10%, and therefore remain still too large.

It is interesting to note that while the presence of the non-Skyrme term plays an important role in the existence of a solution to the differential equation (3), the calculated values of the nucleon and Δ masses depend little on γ. For example, taking the hypothetical case $\gamma = 0$ with the same value of $\Omega = 75$, we have $M_N = 2242$ MeV and $M_\Delta = 2285$ MeV, which are not very different from the above situation. Our results also show that most of the nucleon mass comes from the minimal quadratic term $\mathscr{L}_0$. The ρ, ω terms, although important in stabilizing the soliton, make only a small contribution to the nucleon mass, of the order 300–400 MeV.

In conclusion, to the extent that the derivative expansion given by Eq. (1) is valid, the nucleon soliton mass is too large. It remains to be seen whether a linear σ model together with vector-meson contributions can change the situation.[11,12]

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