Role of String Excitations in the Last Stages of Black-Hole Evaporation

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We argue that the massive modes of the superstring can play an important role in the last stages of black-hole evaporation. If the Bekenstein-Hawking entropy is the true statistical entropy of an evaporating black hole, it becomes probable for a black hole to disappear by making a transition to an excited string state. This excited string state can then decay to massless radiation, avoiding the naked singularity of the semiclassical picture. We also construct the energy-volume phase diagram separating the three phases: pure radiation, black hole and radiation, and massive string modes and radiation.

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Superstring theories[1,2] are candidates for renormalizable and unitary quantum theories of gravity.[3] Renormalizability and unitarity, while extremely important, are not the only requirements confronting such a theory. At least as important are several puzzles which have been raised by the study of quantum fields in the presence of strong gravitational fields. Perhaps the most central of these puzzles is the question of what happens in the last stages of black-hole evaporation.

This puzzle arises because theory of black-hole evaporation, which is based on a semiclassical description involving quantum fields on classical space-time backgrounds, appears to require the breakdown of unitary deterministic evolution of pure quantum states.[4] Radiation coming out of a black hole is in a thermal (mixed) state as opposed to a pure quantum state.[5,6] This does not lead to any significant problems of interpretation during black-hole evaporation if one notes that a certain amount of information about the emitted radiation is hidden inside the event horizon. This information is contained in the correlations between photons absorbed by the black hole and those radiated to infinity. If we knew the quantum state inside the horizon we could combine it with the thermal state of outgoing radiation to make a pure state. Thus, as the black hole continues its process of evaporation, the information content inside it should increase with time. But the Bekenstein-Hawking (BH)[7,8] entropy of the black hole, given by $4\pi M^2$, decreases continuously as it evaporates. So it seems the black hole is losing information. The amount of information lost[9] is proportional to $4\pi(M_i^2 - M_f^2)$, where M_i is the original mass of the black hole and M_f is the final mass. Furthermore, if the black hole evaporates entirely the information contained in the phases of radiation inside the event horizon is completely lost.[10]

Another problem raised by the semiclassical description of black-hole evaporation is the naked singularity at the point in space-time at which the black hole finally disappears. It is not known how to fix boundary conditions for either classical or quantum fields at a naked singularity. Thus, the ability to predict the future is compromised to an even greater degree by the appearance of a naked singularity than the thermal nature of the Hawking radiation.

Now, it has often been speculated that some new dynamics provided by quantum gravity at short distances would provide an alternative to the breakdown of quantum mechanics which seems to be implied by these two problems. We argue that string theory might provide such an alternative.

Unfortunately string theory, as it has been formulated up to now, is not adequate to address these problems in detail. The reason is that string theory is presently understood only perturbatively. While it has been shown how to construct string theory perturbatively around curved backgrounds,[11] the resolution of these difficulties will involve large nonperturbative quantum gravitational effects which take us beyond the semiclassical expansion around a fixed background.

In spite of this there are some reasons to be hopeful that future developments of string theory may be able to shed light on these problems. One of these is that, from the point of view of string theory, Riemannian geometry seems to be embedded in some large structure. This is because each observable in Riemannian geometry is seen to be the first of an infinite set of observables corresponding to the higher excitations of strings. One might then speculate that a formulation of string theory which allows one to get away from the dependence on a fixed rigid background could be based on the geometry of an infinite-dimensional space, which might be the loop space of a Riemannian or super-Riemannian geometry. If this is the case then the ordinary Riemannian geometry would be only a low-energy approximation. In situations where the curvature approached the Planck dimensions, such as near singularities or in the last stages of black-hole evaporation, the more fundamental geometry would become the appropriate description. If this is the case the excitation of these higher string modes would signal the breakdown of Riemannian geometry.

Even without knowing the fundamental significance of the massive modes of the string we can be sure that their excitation takes us out of the perturbative regime in which semiclassical arguments are valid. The identification of the zero-slope string theory with a type of supergravity theory requires that $M_s/M_p \sim 1$. Therefore these higher string modes necessarily involve strong gravitational fields, and any state in which they are excited will involve nonperturbative quantum-gravitational effects.[12]

In the absence of a proper formulation of nonperturbative quantum string theory, we cannot give a rigorous account of black-hole evaporation in the context of string gravity. Yet thermodynamic studies of physical systems have yielded a host of useful information even when most details of their microphysics were unknown. With this in mind we study the statistical mechanics of configuration involving black holes, massive modes of strings, and massless radiation.

In order to apply statistical arguments to such a problem we need to make some assumption concerning the rates of dynamical processes. We assume that in a quantum theory of gravity there will be a space of states $H(E,J)$ which describes the possible quantum states of the gravitational field which have energy E and angular momentum J. [In order that E and J be well defined we must assume that all of the states in $H(E,J)$ are in some sense asymptotically flat so that there is an asymptotic region from which we may measure E and J.] We will further assume that there is in $H(E,J)$ a region $R(E,J)_{\rm bh}$ which contains the quantum states which correspond to a black hole, and a region $R(E,J)_{\rm strings}$ which contains the corresponding excited states of the strings. The logarithm of the number of states in each of these regions will be assumed to be proportional to the relevant entropy. In particular, we will assume that the BH entropy measures correctly the number of degenerate black-hole quantum states in $H(E,J)$. Finally, we assume that the rates for transitions between states in the two regions are on the order of the rates for transitions between states within the regions, and that all of these rates are on the order of $1/E$.

Given all of these assumptions we may assume that the relative entropies of states in the two regions give us the relative probabilities that the system will be found in them, independently of the initial conditions.

We now assume that we begin with a black hole in empty space, with an initial mass $M >> M_p$. It evaporates according to the Hawking process until it has a mass E, which is much less than M, but still greater than M_p. As E decreases we consider the probability for it to make a transition to one of the massive string states in the region $R(E,J)_{\rm strings}$.

In order to simplify the calculation we assume that $\gamma = M_p\sqrt{\alpha'} = M_p/M_s$ is greater than 1. When the zero-slope limit of a type-2 superstring model is taken, the Newton constant G and slope parameter α' are related by $8\pi G = g^4(\alpha')^{D/2-1}/2\pi^4$, where D is the dimension of space-time, and g is a dimensionless string coupling constant.[3] For $D=10$ this implies that $(M_p/M_s)^2 = 2\pi^{5/4}/g$. For weak coupling $\gamma = M_p/M_s$ can be large. So our approximation is not unrealistic. The entropy of a black hole of energy E is $S_b = 4\pi GE^2$, and the entropy of a massive string configuration[13] of energy E is $S_s = -a\ln(E) + bE$, where $a=10$ for closed and heterotic superstrings, and $b = \pi(2+\sqrt{2})\sqrt{\alpha'}$ for the heterotic string and $b=\sqrt{8}\pi\sqrt{\alpha'}$ for type-2 superstrings. We see that when the energy E goes below $\sim \gamma M_p$, $S_b(E) < S_s(E)$ (see Fig. 1). Thus an evaporating black hole can increase its entropy by making a quantum transition to a state corresponding to a bunch of massive string excitations, when its mass is less than γM_p. Given the assumptions outlined above, such a transition then becomes overwhelmingly probable.[14]

Now as shown previously[13] the massive closed string excitations have negative specific heat and, like the black hole,[8] cannot be in equilibrium with an infinite radiation bath. Thus, in the infinite-volume limit, these states evaporate into massless modes of the string and disappear.

Thus, we conclude that it is entropically favorable for the black hole to make a transition to one of the massive modes of the string during the last stages of its evaporation. These in turn evaporate leaving zero-mass thermal radiation. The result is that the evaporating black hole leaves no remnant. Among the massive string excitations are states which correspond to linearized excitations of the degrees of freedom of the fundamental (infinite-dimensional) geometry of the string theory (in the same sense that among the massless modes are states which correspond to the linearized excitations of the Riemannian geometry). By exciting these degrees of freedom it appears that the black hole can evaporate completely, avoiding the singularity that is inevitable in the semiclassical picture.

We must emphasize that this picture of the final stage of black-hole evaporation depends crucially on

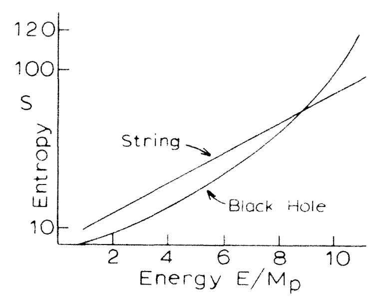

FIG. 1. The entropy of black holes vs strings as a function of energy for the heterotic string and $M_s = 0.1M_p$.

the assumption that the BH entropy is a measure of the number of quantum states of a black hole, regardless of its previous history. The truth of this assumption is, in turn, dependent on the assumption that loss of information really has taken place. For if all of the information needed to restore the thermal radiation to a pure state were still present inside the black hole then the number of possible microstates of the black hole would have to be much larger than the $\exp[4\pi A M_p^2]$ allowed by the BH entropy. It would have to be larger by a factor of $\exp[(M/M_p)^2]$, where M is the original mass of the black hole.

Indeed, under the assumption that information loss does not take place we see that it is extremely unlikely that the black hole will make a transition to a state involving massive string modes at any stage of the evaporation process. This is because the number of accessible microstates of the black hole will then at late times be at least $\exp[4\pi(M/M_p)^2]$, where M is again its original mass. This is enormously larger than the number of states accessible to the excited modes of the string, which is on the order of $E^{-10}\exp[bE]$, where E is now much less than M. String theory can perhaps resolve the difficulty of the naked singularity, but it appears unlikely to be able to resolve the problem of the breakdown of unitary evolution of pure quantum states during black-hole evaporation processes.

In order to understand better the thermodynamic processes involving strings and black holes we next study configurations involving black holes (bh), massive string modes (s), and radiation (r) at fixed energy and volume. Such studies have been made for black holes in equilibrium with radiation[8] and for massive string modes in equilibrium with radiation.[13] First, we show that all three phases cannot be simultaneously in equilibrium. Let $\Omega_i(E_i)$, $i=1,2,3$, be the densities of states of three systems with energies E_i, and $E=E_1+E_2+E_3$ the fixed total energy of the combined system. The heat capacity of each system is given by $C=-[T^2 d^2\ln(\Omega)/dE^2]^{-1}$ and $\ln(\Omega)$ is the entropy. The variation of the total entropy of the combined system when the E_i are changed by ΔE_i, keeping the total energy fixed, is equal to

$$(1/T_1-1/T_3)\Delta E_1+(1/T_2-1/T_3)\Delta E_2.$$

For stationary variations $T_i=T$ for all i. Equilibrium requires that the entropy is a maximum and therefore the second-order variation has to be negative. This implies

$$\frac{1}{T^2}\left[\frac{\Delta E_1^2}{C_1}+\frac{\Delta E_2^2}{C_2}+\frac{(\Delta E_1+\Delta E_2)^2}{C_3}\right]>0$$

and so $1/C_i+1/C_j$ must be positive for all distinct i and j. Therefore equilibrium is impossible if any two components of the system have negative specific heat. Thus if we have all three phases in a box, the equilibrium configuration will be bh and r, or s and r, or r alone depending on the total energy density and volume.

First we compute the critical volume V_c, above which only r can be in thermal equilibrium, for the two systems bh and r, and s and r. The conditions given above imply $E_r < M/4$ for the bh and r system, where M is the black-hole mass. For the s and r system the condition is $E_r < E_s^2/4aT$. These inequalities impose restrictions on V. For the black hole and radiation $\sigma V_c = 2^{20}\pi^4 E^5/5^5$ and for strings and radiation

$$\sigma V_c=\left(E+\frac{3a}{2b}-D\right)b^4\left(D-\frac{5a}{2b}\right)^4\left(D-\frac{3a}{2b}\right)^4,$$

where $D=[4Ea/b+(3a/2b)^2]^{1/2}$.

Now we can construct a phase diagram for the three phases: (1) bh and r, (2) s and r, and (3) pure r. If the total energy is E and the volume V, phase 1 has energy $E=\sigma VT^4+1/8\pi T$ and entropy $S_{\mathrm{bh+r}}=\frac{4}{3}\sigma VT^3+1/16\pi T^2$. For phase 2 $E=\sigma VT^4+aT/(bT-1)$ and

$$S_{\mathrm{s+r}}=\tfrac{4}{3}(\sigma VT^3)+baT/(bT-1)-(bT-1)/T.$$

Solving these equations numerically we have found the entropies $S_{\mathrm{bh+r}}$ and $S_{\mathrm{s+r}}$ as functions of E and V. For a given E and V, the system with higher S will be preferred. This is plotted in the phase diagram of Fig. 2. The two critical volume curves intersect at $E=7M_p$ and $V=1.1\times10^5$ Planck units. Below this energy V_c for phase 2 is higher than V_c for phase 1.

Using this phase diagram we can study the quasistatic evaporation of an order–Planck-mass black hole. Consider a black hole and radiation system with total energy $5M_p$. Starting at a point P in the phase diagram, we can increase the volume slowly. At point Q, the system will cross the phase boundary and enter the string and radiation phase. If we continue increasing the volume, the massive strings will evaporate at point R into pure radiation. This is a possible mechanism

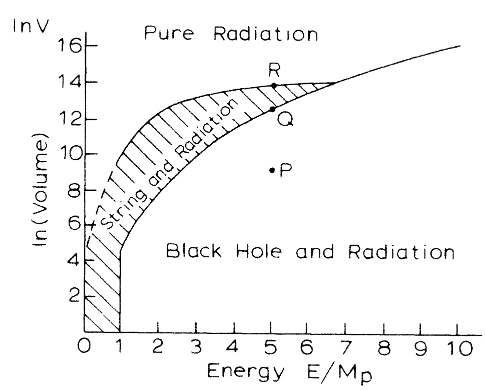

FIG. 2. Volume-energy phase diagram for the heterotic string and $M_s=0.1M_p$.

for black-hole evaporation.

Before closing we note that the Bekenstein bound[15] for the entropy of a system of energy E and size R still holds. In the case of the rigidly rotating string described earlier $R = A\pi = 4E\alpha'$ so that $2\pi ER = 8\pi E^2\alpha' >$ (entropy of string) $\sim bE\sqrt{\alpha'} - a \ln E$.

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[11]C. Callan, D. Friedan, E. Martinec, and M. Perry, to be published.

[12]It is even possible that for certain values of the string tension, the higher string modes themselves form black holes [see G. T. Horowitz, Lectures given at the International School of Cosmology and Gravitation, Course 9, Ettore Majorana Center for Scientific Culture, Erice, Italy, 12–22 May 1985 (to be published)]. Note that the classical string configuration $t = \tau$, $x = A(\sigma - \pi/2)\cos(\omega\tau)$, $y = A(\sigma - \pi/2)\sin(\omega\tau)$, is a solution [see C. Rebbi, Phys. Rep. **12**, 1 (1974)] of the classical string equation of motion provided $A\omega\pi/2 = 1$. This represents a rigidly rotating string in the x,y plane. The mass M and angular momentum J of this configuration are $M = A\pi/4\alpha'$, $J = A^2\pi^2/16\alpha'$ so that $J = \alpha' M^2$. First we note that the size $A\pi$ of the string is smaller than its Schwarzschild radius $2MG$ iff $1 < G/2\alpha'$. When we use the result $G = 1/M_p^2$ and let $\alpha' = 1/M_s^2$ the above inequality reduces to $M_s > \sqrt{2}M_p$. But the string carries angular momentum. J. Therefore it should be described by the Kerr metric. If $J > GM^2$ the Kerr black hole has a naked singularity. This condition amounts to $\alpha' > G$ or $M_s < M_p$. So if $M_s >> M_p$ the higher excitations of the string undergo gravitational collapse and become Kerr black holes. Fortunately they do not form naked singularities.

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[14]Note that here we are using the entropies of a four-dimensional black hole and the ten-dimensional string. This is the correct thing to do, because the black hole is assumed to start out having a mass much greater than M_s, so that the only degrees of freedom of the gravitational field that are involved are the four-dimensional ones. However, when it becomes probable to make a transition to a state in which the massive modes of the string are excited the black hole has a radius on the order of the compactification scale, so that all of the degrees of freedom of the string may be excited.

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