Identification of Vortices in Superfluid ^{3}He-B

E. V. Thuneberg
Research Institute for Theoretical Physics, University of Helsinki, SF-00170 Helsinki, Finland
(Received 4 November 1985)

A numerical solution of the Ginzburg-Landau equations for superfluid ^{3}He-B vortices gives the following results: The low-pressure vortex is a new "double-core vortex" whose rotational symmetry around the vortex axis is broken. The high-pressure vortex is the v vortex. The theory gives a transition between these vortices which is in excellent agreement with experiments.

PACS numbers: 67.50.Fi

The experimentally observed transition in the vortex cores of rotating ^{3}He-B [1,2] has stimulated several theoretical attempts to identify the two stable vortex structures. The first vortex was calculated by Ohmi, Tsuneto, and Fujita[3]; it is called the o vortex. The second vortex—the v vortex—was discovered by Salomaa and Volovik.[4] They identified it as the low-pressure vortex but could not find the high-pressure vortex. The same result was also reported by Passfogel, Tewordt, and Schopohl.[5] By use of an approximate approach, Fetter and Theodorakis[6,7] found a transition between the o and the v vortices. In this Letter I describe a third vortex which has a double core and a broken rotational symmetry around the vortex axis. A numerical solution of the Ginzburg-Landau (GL) equations identifies this new vortex as the low-pressure vortex and gives a transition to the v vortex near the tricritical point in excellent agreement with experiment.

Because there are no coreless vortices in the B phase, any equilibrium vortex must have the minimum circulation. There is no reason to break the translation symmetry along the vortex axis. It follows that the GL differential equations can be reduced to the form ($\delta = x,y,z$)[8]

$$\begin{aligned}
&(\gamma\partial_x^2+\partial_y^2)A_{\delta x}+(\gamma-1)\partial_x\partial_y A_{\delta y}-[-\mathbf{A}+\beta_1\mathbf{A}^*\mathrm{Tr}(\mathbf{A}\tilde{\mathbf{A}})+\beta_2\mathbf{A}\,\mathrm{Tr}(\mathbf{A}\tilde{\mathbf{A}}^*)\\
&\qquad\qquad+\beta_3\mathbf{A}\tilde{\mathbf{A}}\mathbf{A}^*+\beta_4\mathbf{A}\tilde{\mathbf{A}}^*\mathbf{A}+\beta_5\mathbf{A}^*\tilde{\mathbf{A}}\mathbf{A}]_{\delta x}=0,\\
&(\partial_x^2+\gamma\partial_y^2)A_{\delta y}+(\gamma-1)\partial_x\partial_y A_{\delta x}-[-\mathbf{A}+\beta_1\mathbf{A}^*\mathrm{Tr}(\mathbf{A}\tilde{\mathbf{A}})+\beta_2\mathbf{A}\,\mathrm{Tr}(\mathbf{A}\tilde{\mathbf{A}}^*)\\
&\qquad\qquad+\beta_3\mathbf{A}\tilde{\mathbf{A}}\mathbf{A}^*+\beta_4\mathbf{A}\tilde{\mathbf{A}}^*\mathbf{A}+\beta_5\mathbf{A}^*\tilde{\mathbf{A}}\mathbf{A}]_{\delta y}=0,\\
&(\partial_x^2+\partial_y^2)A_{\delta z}-[-\mathbf{A}+\beta_1\mathbf{A}^*\mathrm{Tr}(\mathbf{A}\tilde{\mathbf{A}})+\beta_2\mathbf{A}\,\mathrm{Tr}(\mathbf{A}\tilde{\mathbf{A}}^*)+\beta_3\mathbf{A}\tilde{\mathbf{A}}\mathbf{A}^*+\beta_4\mathbf{A}\tilde{\mathbf{A}}^*\mathbf{A}+\beta_5\mathbf{A}^*\tilde{\mathbf{A}}\mathbf{A}]_{\delta z}=0.
\end{aligned} \tag{1a}$$

This is a partial differential equation in two variables for the order parameter $\mathbf{A}$, which is complex 3×3 matrix. Equation (1a) has to be solved together with the boundary condition

$$\lim_{r\to\infty}\mathbf{A}(r,\phi)=\mathbf{1}\exp(i\phi), \tag{1b}$$

where r and ϕ are the polar coordinates and $\mathbf{1}$ is the unit matrix. The distances are measured in units of the temperature-dependent coherence length $\xi(T)$ and the order parameter is normalized to the unit matrix in the bulk B phase.[9]

The problem (1) contains the parameters β_i and γ, which are functions of the pressure. I use the β coefficients tabulated by Sauls and Serene.[10] This means that whenever a pressure is quoted in this paper it means the pressure according to these coefficients. This pressure is only roughly equal to the real pressure; the tricritical point, for example, is at 28.5 bars on the Sauls-Serene scale, the real pressure being 21 bars. Zero pressure corresponds here to the weak-coupling values of β_i. The constant γ is assumed to be equal to its weak-coupling value 3 to all pressures.

The problem (1) was solved numerically. The space was discretized to a square lattice and at each iteration an increment was added to $\mathbf{A}(x_n,y_m)$ that was proportional to the left-hand side of (1a).[11] This proved to be very effective if not too small a lattice constant was used. Computing times of some minutes were enough to obtain convergence on a moderately fast computer. It is remarkable that all the qualitative results and to a large extent also the quantitative results presented in this Letter can be produced with a relatively large lattice constant of $\xi(T)$. With this choice the proportionality constant mentioned above can be taken as 0.1, and 1000 iterations yields high accuracy.

In order to classify the various solutions it is useful to consider the symmetries of the problem (1).[4] They are (a) rotations around the vortex axis by an arbitrary angle α combined with phase multiplication $[\exp(i\alpha)C_\alpha^z]$, (b) reflection in the x-y plane (σ^z), (c) reflection in the x-z plane combined with complex conjugation ($T\sigma^y$), and (d) combinations thereof. The solution of the problem that has all these symmetries is the o vortex,[3] and its order parameter is shown in Fig. 1(a). The solution that breaks the σ^z symmetry is the v vortex, which is shown in Fig. 1(b).

A third solution was found as follows. For simplicity of presentation consider the order parameter on a single axis (x axis); for the v vortex this is not any restriction because the rotational symmetry then determines $\mathbf{A}$ everywhere. An outstanding property of the v vortex is the long tail of the components A_{zx} and A_{xz}. They decay with distance like $1/r$.[5,12] This behavior follows from the fact that replacement of the unit matrix of the bulk B phase by an arbitrary rotation matrix does not change the energy. Such a rotation is called the spin-orbit rotation and it may heal slowly. In the v vortex, the tail is caused by a rotation around the y direction. This pure spin-orbit rotation cannot, however, continue to the center of the vortex, because the rotational symmetry of the v vortex requires there that

$$A_{zy} = iA_{zx}, \quad A_{yz} = iA_{xz}. \tag{2}$$

(These conditions are easily verified by application of $iC^z_{\pi/2}$, for example.) This means that the components A_{zy} and A_{yz} have to be pulled up near the vortex center, which costs energy. This constitutes a reason to break the rotational symmetry.

Consider moving along the x axis. Discarding the rotational symmetry one could continue the spin-orbit rotation of the order parameter through the center of the vortex to the negative x axis. Indeed, on the x axis a vortex could look like a pure spin-orbit rotation (around the y axis) for two of the three components of the bulk B phase; only the component parallel to the rotation axis (A_{yy}) has to go to zero at the vortex center. Exactly this happens in the new vortex shown in Fig. 2.

The new vortex has reflection symmetry in the x-z and in the y-z planes ($T\sigma^y$ and $-T\sigma^x$). As a conse-

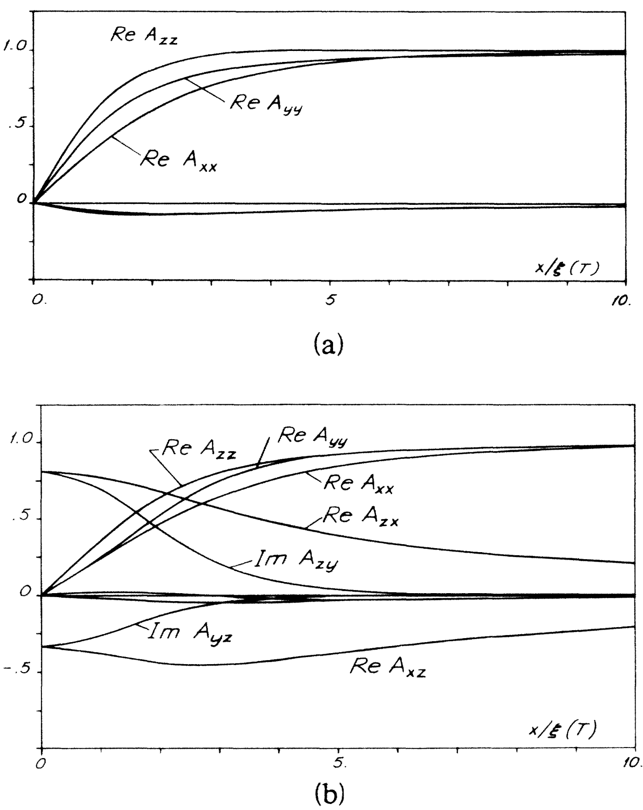

FIG. 1. The order parameter of (a) the o vortex and (b) the v vortex for weak coupling ($p \approx 0$). $\mathbf{A}$ is shown in rectangular coordinates on the x axis. Because of the rotational symmetry $[\exp(i\alpha)C^z_\alpha]$ this defines $\mathbf{A}$ everywhere. The imaginary parts of A_{xy} and A_{yx} are small but nonzero.

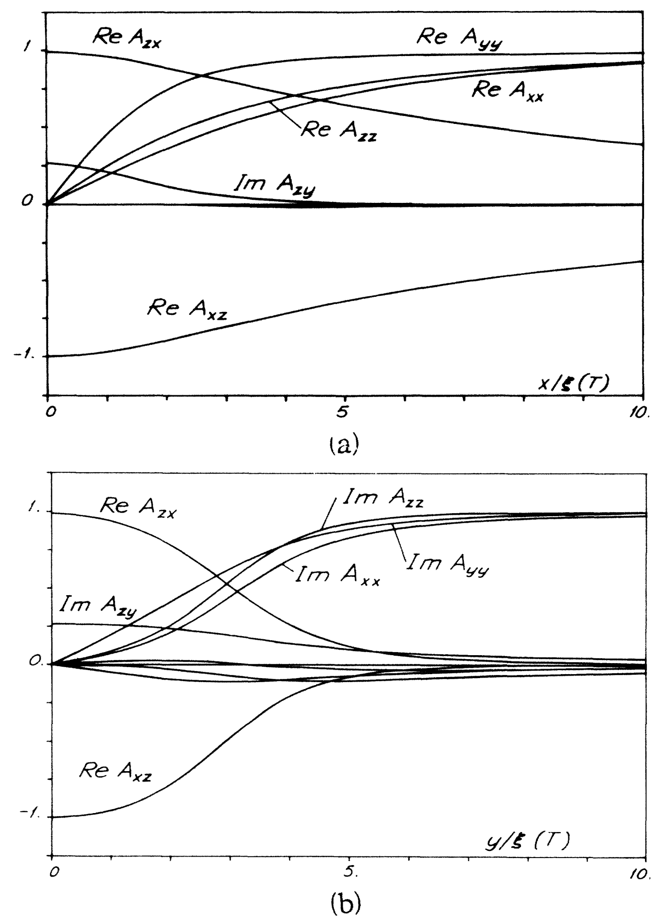

FIG. 2. The order parameter of the new vortex (a) on the x axis and (b) on the y axis for weak coupling. In (a) the imaginary parts of A_{xy}, A_{yx}, and A_{yz} are small but nonzero. In (b) the real parts of A_{xy} and A_{yx} and the imaginary part of A_{yz} are small but nonzero. In intermediate directions all the eighteen degrees of freedom are nonzero. The rotational-symmetry breaking is strong: Rotational symmetry would require, for example, that the component ReA_{zx} in (a) be equal to the vanishingly small ImA_{yz} in (b).

quence it also has a discrete rotational symmetry by 180° (C_π^z), but the continuous rotational symmetry is strongly broken. These symmetries were not utilized in the solution of (1) in the first place, but they always turned out to be satisfied in the end. It follows from the point symmetries that the order parameter consists of nine real functions on the x and the y axes, but elsewhere all the eighteen real degress of freedom of **A** are nonzero. Symmetries allow four nonzero components in the vortex center, but actually the center is dominated by the planar phase. On the x axis the order parameter rotates slowly around the y direction by the total angle of 180°.[13] On the y axis the changes are more rapid: The planar phase of the center is out of phase with the bulk B phase and there is an interface between them. The local energy density has a maximum at these points [$x=0$, $y=\pm 2.7\xi(T)$], which I call the cores of the vortex. The trapping potential for ions is a double-well potential that has minima at these points.[14] More specifically, if the energy of the ion in the normal state is taken to be zero and in bulk superfluid it is unity, the energies in the vortex center and in the core are 0.69 and 0.47, respectively (weak coupling). Although the shape of this potential is very different from the rotationally symmetric potential of the v vortex, the magnitudes of the potentials are almost equal.

Figure 3 displays the energies of the three vortices as functions of pressure. At all pressures the energy of the o vortex is clearly higher than the energies of the other two. At low pressures the new vortex has the lowest energy. It has been shown previously[4,5] that the o vortex has no local stability, but is a saddle point of the energy. All my numerical results indicate that the same is true for the v vortex at zero pressure (i.e., at weak-coupling values of the β coefficients). In fact, in the weak coupling I was not able to find the v vortex at all unless I constrained the order parameter to have the rotational symmetry. The new vortex did not show any sign of further symmetry breaking. With an increase of the pressure, the energy difference between the new and the v vortex decreases and the energies cross at roughly 3 bars below the tricritical pressure. The identity of all the vortices continues beyond this point and even through the whole A-phase region up to the melting pressure (supposing that the B phase can superheat so far). This means that the v vortex becomes locally stable at higher pressures, and that the transition to the v vortex is of first order. It is worth noting that symmetries alone could have allowed a second-order transition because the v vortex has all the symmetries of the new vortex. Knowing the order parameters of the vortices, it is simple to calculate the behavior of various quantities at the transition. Especially interesting are the susceptibility anisotropy and the spontaneous magnetization because

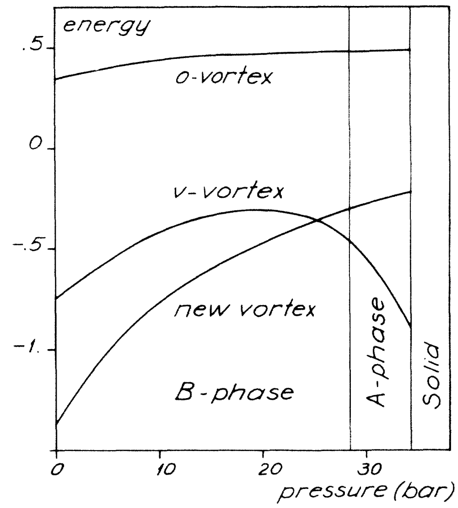

FIG. 3. The energies of the three vortices as a function of pressure. The energy is expressed in units $f_c^B\xi(T)^2$, where f_c^B is the condensation energy density of the bulk B phase, and the logarithmic energy contribution $4\pi(\gamma+2)\ln[R/\xi(T)]/3$ has been subtracted.

the measured quantities λ and κ[15] are proportional to these, respectively. The present theory gives a 20% jump upwards for λ (at $T=0.9T_c$) and a 50% jump also upwards for κ at the transition from the new vortex to the v vortex.

For experimental reasons the measurements[15] have not been performed just below the superfluid transition temperature, but the extrapolated vortex-core transition line is in very good agreement with the present theoretical results. The experiments cannot as yet resolve the jump of κ in the GL region and the 50% jump remains as a prediction. (This jump is opposite to that observed at low temperatures.) In contrast, the jump of λ in the GL region can be measured, and it agrees very well with the present theoretical result.

Similar work has been done by Salomaa and Volovik.[16]

I acknowledge having benefitted from the work of Dr. M. Salomaa and Dr. G. Volovik, which gave the original impulse to the present investigation.

[1] P. J. Hakonen, O. T. Ikkala, S. T. Islander, O. V. Lounasmaa, and G. E. Volovik, J. Low Temp. Phys. **53**, 423 (1983).

[2] P. J. Hakonen, M. Krusius, M. M. Salomaa, J. T. Simola, Yu. M. Bunkov, V. P. Mineev, and G. E. Volovik, Phys. Rev. Lett. **51**, 1362 (1983).

[3] T. Ohmi, T. Tsuneto, and T. Fujita, Prog. Theor. Phys. **70**, 647 (1983).

[4]M. M. Salomaa and G. E. Volovik, Phys. Rev. Lett. **51**, 2040 (1983), and Phys. Rev. B **31**, 203 (1985).

[5]T. Passfogel, L. Tewordt, and N. Schopohl, J. Low Temp. Phys. **56**, 383 (1984).

[6]A. L. Fetter and S. Theodorakis, Phys. Rev. Lett. **52**, 2007 (1984).

[7]A. L. Fetter, Phys. Rev. B **31**, 7012 (1985).

[8]Tr denotes the trace, $\tilde{\mathbf{A}}$ the transpose of $\mathbf{A}$, and $\mathbf{A}^*$ the complex conjugate of $\mathbf{A}$.

[9]This means that the global spin-orbit rotation is set equal to zero; the general solution can be obtained by multiplying the solution of (1) by a constant rotation matrix. The normalization to unity means that the coefficients β_i are normalized: $3\beta_1+3\beta_2+\beta_3+\beta_4+\beta_5=1$.

[10]J. A. Sauls and J. W. Serene, Phys. Rev. B **24**, 183 (1981).

[11]E. V. Thuneberg, University of Helsinki Report No. HU-TFT-85-19, 1985 (to be published).

[12]This tail is not contained in the *Ansatz* of Refs. 6 and 7, which probably explains why those results differ from all the others.

[13]It is natural that the rotation is around the y direction because the order parameter is weakest in this direction as a result of the supercurrent of the vortex. In addition, A_{xx} has to be one of the rotated components because its gradient costs more energy than those of the others. This applies also to the rotationally symmetric vortices and explains at least partly why the v vortex has been found stable but the u and the w vortices not. (The latter two allow rotations only around the z and the x directions, respectively.)

[14]The trapping potential can easily be calculated by use of formula (3.2) of D. Rainer and M. Vuorio, J. Phys. C **10**, 3093 (1977).

[15]P. J. Hakonen, M. Krusius, G. Mamniashvili, and J. T. Simola, in *Proceedings of the Seventeenth International Conference on Low Temperature Physics (LT-17),* edited by U. Eckern, A. Schmid, W. Weber, and H. Wühl (North-Holland, Amsterdam, 1984), p. 1366.

[16]M. M. Salomaa and G. E. Volovik, following Letter [Phys. Rev. Lett. **56**, 363 (1986)].