Geometrical Constraints and Equations of Motion in Extended Supergravity

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Consequences of geometrical constraints from integrability along all super lightlike lines are examined in N-extended supergravity theories. It is shown that these constraints give rise, through Bianchi identities, to equations of motion of the conformal type for all forms of physical fields, with $N > 4$. Our results point to an integrability program for $N > 4$ supergravity similar to that for $N > 2$ supersymmetric Yang-Mills fields.

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One of the most attractive candidates to unify gravity with other existing gauge interactions is N-extended supergravity (SUGRA) theory.[1] An attractive feature is that the superstring[2] theories reduce to extended SUGRA (and supersymmetric Yang-Mills) theories at energies well below the Planck scale.

On the other hand, there still remains a longstanding problem in particle physics: how to obtain the exact solutions for equations of motion in a nonlinear system, such as non-Abelian Yang-Mills (YM) or gravitational theories. There is no doubt that investigations into the integrability of nonlinear systems shed much light on the nonperturbative aspects of gauge theories and hopefully on the understanding of vacuum structures of gravitational theories.

Several years ago Witten,[3] in attempts to extend the twistor formulation for the full Yang-Mills fields,[4,5] pointed out that the second-order Yang-Mills field equation has a simple interpretation in the extended superspace as integrability conditions along spinorial lightlike lines, and has a corresponding supertwistor formulation. A set of lightlike lines is no longer a one-dimensional object for $N > 1$, and the integrability conditions become far from trivial. In fact, it can be shown[3,6,7] that for $N > 2$ the integrability conditions, which we refer to as geometrical constraints hereafter, imply Yang-Mills equations of motion via the Bianchi identities. It has been shown recently that all solutions to the $N = 3$ super YM theory must satisfy the constraint equations.[8] It has also been established recently that the geometrical constraints possess many of the integrability properties[9-11] as found in the self-dual Yang-Mills system,[12] a linear system with spectral parameters, infinite nonlocal conservation laws, parametric Bäcklund transformations, Riemann-Hilbert transformations, and infinite-dimensional affine algebra of Kac-Moody. These formulations quite possibly can provide a new route for the quantum supersymmetric fields.

In this paper we examine the consequences of geometrical constraints for four-dimensional-space extended SUGRA theories formulated in superspace, with arbitrary N. It is shown that for $N > 4$ these geometrical constraints lead, through Bianchi identities, to conformal-like equations of motion for all physical fields with $s \leqslant 2$, thus making the theories on shell. We therefore expect a similar integrability program for $N > 4$ SUGRA as for $N > 2$ supersymmetric Yang-Mills. In addition it is well known that some four-dimensional extended SUGRA has a natural correspondence[13] with SUGRA in high dimensions. Thus we expect similar conclusions for the SUGRA in higher dimensions. The details will be discussed in a separate paper.[14]

In the literature, the superspace formulation has been discussed for Poincaré and conformal SUGRA[15-17] with arbitrary N, in four dimensions. Especially Gates and Grimm[16] have found that certain constraints for torsions give rise to the equations of motion for $s = 1$, $\frac{3}{2}$, and 2 physical fields for $N > 4$. However, the constraints they adopted, which are called conventional constraints in the present paper, are different from the geometrical constraints and in general are more stringent than the geometrical constraints we used here.

It is interesting and important to find out the consequences of the geometrical constraints, since they naturally have the same nice integrability properties as those of the supersymmetric Yang-Mills fields. If the geometrical constraints lead to equations of motion, solving these constraints will lead to solutions to the full field equations. The geometrical constraints in $N = 1$ SUGRA had been discussed by Crispim-Romão, Ferber, and Freund,[18] but they did not result in equations of motion; thus it is unclear whether for higher N the geometrical constraints give equations of motion.

From differential-geometry formalism in superspace the covariant tensors, curvatures R and torsions T, appear in the graded commutator of two covariant derivatives defined as in the ordinary gauge theories, with flat indices,

$$\{\nabla_A, \nabla_B\} = R_{AB} - T_{AB}{}^C \nabla_C. \quad (1)$$

Among two kinds of Bianchi identities one becomes trivial,[19] once the following one is imposed:

$$I_{ABC}{}^{D}=\oint_{ABC}\{(R_{AB})_{C}{}^{D}-\nabla_{A}T_{BC}{}^{D}-T_{AB}{}^{E}T_{EC}{}^{D}\}=0, \tag{2}$$

where $\oint$ represents a graded cyclic sum with respect to A, B, and C.

In Minkowski space, a lightlike vector in spinor notation takes the form $v^{\alpha\dot\alpha}=\lambda^{\alpha}\lambda^{*\dot\alpha}$, where λ is a commuting number and the asterisk denotes complex conjugate. In the extended superspace we can find translation operators along spinorial lightlike lines, $(1/\sqrt{2})(\lambda^{\alpha}D^{i}_{\alpha}+\lambda^{\dot\alpha}\bar D_{\dot\alpha i})$ where $i=1$ to N, whose square becomes the translation in the direction of v, $-iv^{\alpha\dot\alpha}\partial_{\alpha\dot\alpha}$ with $\partial_{\alpha\dot\alpha}\equiv(\sigma^{a})_{\alpha\dot\alpha}(\partial/\partial x^{a})$. D and $\bar D$ are spinorial differentiations. The integrability of connection fields along spinorial direction is nothing but the condition that the algebra by virtue of translation operators accompanied by λ does not change even if D and $\bar D$ are replaced by the corresponding covariant derivatives ∇ and $\bar\nabla$ (Ref. 3):

$$\{\nabla^{i}_{\alpha},\nabla^{j}_{\beta}\}+(\alpha\leftrightarrow\beta)=0,\quad \{\bar\nabla_{\dot\alpha i},\bar\nabla_{\dot\beta j}\}+(\dot\alpha\leftrightarrow\dot\beta)=0,\quad \{\nabla^{i}_{\alpha},\bar\nabla_{\dot\alpha j}\}=-2i\delta^{i}_{j}\partial_{\alpha\dot\alpha}. \tag{3}$$

These equations are our geometrical constraints. In addition, we also include the constraint $T_{ab}{}^{c}=0$, as in the ordinary gravitational theory, so that the spin connection $(\Omega_{m})_{b}{}^{a}$ can be expressed in terms of vierbeine $E_{m}{}^{a}$. As one of the remarkable features of geometrical constraints, in clear contrast to the case of conventional ones, curvatures associated with spinorial covariant derivatives get constraints:

$$(R^{ij}_{\alpha\beta})_{C}{}^{D}+(\alpha\leftrightarrow\beta)=(R_{\dot\alpha i\dot\beta j})_{C}{}^{D}+(\dot\alpha\leftrightarrow\dot\beta)=(R^{i}_{\alpha\dot\beta j})_{C}{}^{D}=0.$$

It means, from the theorem of Dragon,[19] that we have additional information about torsions. On the other hand, our torsion constraints themselves are much weaker than the conventional ones. In fact, from (1) and (3) several superfields can be defined as

$$(\bar\sigma_{c})^{\dot\gamma\gamma}T^{ijc}_{\alpha\beta}=\epsilon_{\alpha\beta}\bar A^{[ij]\dot\gamma\gamma};\quad T^{ij\gamma}_{\alpha\beta k}=\epsilon_{\alpha\beta}\bar B^{[ij]}{}_{k}{}^{\gamma};\quad T^{ij\dot\gamma k}_{\alpha\beta}=\epsilon_{\alpha\beta}\bar\Lambda^{[ij]k\dot\gamma}. \tag{4}$$

Note that $\bar A$ and $\bar B$ fields are absent and $\bar\Lambda$ is totally antisymmetric with respect to i, j, and k exchange in the conventional constraints.[16,17] In Eq. (4), brackets indicate the antisymmetric part.

Let us summarize below the consequences of Bianchi indentities, which are necessary for the derivation of equations of motion. For simplicity, we will work in the linearized level of the theory, i.e., ∇ or $\bar\nabla$ is replaced by D or $\bar D$ and all quadratic terms of dynamical torsion fields are neglected. For $I_{\underline{\alpha\beta\gamma}}{}^{d}=0$:

$$D^{i}{}_{\{\alpha}\bar A^{[jk]\dot\alpha}{}_{\beta\}}+(i\leftrightarrow j)=0,\quad (\bar\Lambda^{[jk]i\dot\alpha}-\tfrac{1}{8}iD^{i}_{\alpha}\bar A^{[jk]\dot\alpha\alpha})+(i\leftrightarrow j)=0. \tag{5}$$

For $I_{\underline{\alpha\beta\dot\gamma}}{}^{d}=0$:

$$\bar D_{\{\dot\alpha k}\bar A^{[ij]}{}_{\dot\beta\}\gamma}=[1/(N-1)][\delta^{i}_{k}\bar D_{\{\dot\alpha l}\bar A^{[lj]}{}_{\dot\beta\}\gamma}-(i\leftrightarrow j)]. \tag{6}$$

For $I_{\underline{\alpha\beta\gamma}}{}^{\dot\delta}=0$:

$$D^{i}_{\alpha}\bar\Lambda^{[jk]l\dot\alpha}+(i\leftrightarrow j)=0. \tag{7}$$

For $I_{\underline{\alpha\beta\dot\gamma}}{}^{\dot\delta}=0$:

$$\bar D_{\{\dot\alpha l}(\bar\Lambda^{[ij]k}{}_{\dot\beta\}}-\tfrac{1}{8}iD^{k}_{\alpha}\bar A^{[ij]}{}_{\dot\beta\}}{}^{\alpha})=-2i[\delta^{i}_{l}(\bar f^{[jk]}_{\{\dot\alpha\dot\beta\}}-\tfrac{1}{8}i\partial_{\alpha\{\dot\alpha}\bar A^{[jk]}{}_{\dot\beta\}}{}^{\alpha})+\text{cyclic sum of } i,j,k\,], \tag{8}$$

where $(R^{ij}_{\alpha\beta})^{cd}(\bar\sigma_{cd})_{\dot\alpha\dot\beta}\equiv4i\epsilon_{\alpha\beta}\bar f^{[ij]}_{\{\dot\alpha\dot\beta\}}$, and $\underline{\alpha}={}^{i}_{\alpha}$, $\underline{\dot\alpha}=\dot\alpha i$; braces denote the symmetric part.

The general property of the on-shell supermultiplet is well known[20]; each physical particle is described by a field with totally symmetric spinor indices and totally antisymmetric internal indices. For example, the lowest $(\theta=\bar\theta=0)$ component of the superfield $\bar F^{[ij]}_{\{\dot\alpha\dot\beta\}}$ defined below has indices with such properties. It is, in fact, the spin-1 field strength of our theory. Once the physical spin-1 field is obtained, the $s=\frac{3}{2}$ and 2 fields can be constructed by picking up the lowest components of superfields, $\bar\Sigma^{i}{}_{\{\dot\gamma\dot\beta\dot\alpha\}}\equiv\bar D_{\{\dot\gamma j}\bar F^{[ji]}_{\dot\beta\dot\alpha\}}$, $\bar V_{\{\dot\delta\dot\gamma\dot\beta\dot\alpha\}}\equiv\bar D_{\{\dot\delta i}\bar\Sigma^{i}{}_{\dot\gamma\dot\beta\dot\alpha\}}$, where the spinor indices are totally symmetrized.

We will first derive the equation of motion for the $s=1$ physical field, by showing the $\bar F$ superfield equation. Although the equation of motion essentially stems from (7) and (8), the presence of the $\bar A$ field potentially may ruin the derivation of the equation of motion. Fortunately, however, we can find a suitable redefined superfield,

$$\bar\lambda^{[ijk]}{}_{\dot\alpha}\equiv\bar\Lambda^{[ij]k}{}_{\dot\alpha}+\tfrac{1}{8}i[D^{i}_{\alpha}\bar A^{[jk]}{}_{\dot\alpha}{}^{\alpha}-(i\leftrightarrow j)]. \tag{9}$$

It is obvious from the second equation of (5) that the so defined $\bar\lambda$ field is totally antisymmetric under i, j, and k exchange. Furthermore, from the first equation of (5) and Eqs. (6), (7), and (8), we can show the following important relations:

$$D^l_\alpha \bar{\lambda}^{[ijk]}{}_{\dot\alpha} + (l \leftrightarrow i) = 0, \tag{10}$$

$$\bar{D}_{\dot\alpha l} \bar{\lambda}^{[ijk]}{}_{\dot\beta} = \tfrac{1}{2}\epsilon_{\dot\alpha\dot\beta} X_l{}^{[ijk]} - 2i(\delta^i_l \bar{F}^{[jk]}_{\{\dot\alpha\dot\beta\}} + \text{cyclic sum of } i,j,k). \tag{11}$$

Equation (11) actually defines our physical $s=1$ field $\bar{F}$ and an auxiliary field X in terms of $\bar{\lambda}$. It should be stressed here that Eqs. (10) and (11) are the same in their form as those analyzed in the paper of Gates and Grimm[16] for their $\bar{\Lambda}^{[ijk]}{}_{\dot\alpha}$ field, which corresponds to our $\bar{\Lambda}^{[ij]k}{}_{\dot\alpha}$ field in the absence of $\bar{A}$. In fact, if the $\bar{A}$ field does not exist the second equation of (5) implies that our $\bar{\Lambda}$ field has totally antisymmetric internal indices and $\bar{\Lambda} \equiv \bar{\lambda}$. The $\bar{F}$ field equation is thus obtained by the same prescription as the one adopted in Ref. 16, which we will sketch very briefly here. We note from (11) the following two relations:

$$\bar{D}_{\dot\beta j} X_k{}^{[kjl]} = -4i(N-2)\bar{D}^{\dot\alpha}{}_j \bar{F}^{[jl]}_{\{\dot\alpha\dot\beta\}}, \quad (N-2)\bar{D}^{\dot\beta}{}_l \bar{D}^{\dot\alpha}{}_j \bar{F}^{[jl]}_{\{\dot\alpha\dot\beta\}} = 0. \tag{12}$$

We next use two fundamental Eqs. (10) and (11) to get

$$D^i_\beta \bar{F}^{[jk]}_{\{\dot\alpha\dot\beta\}} = \partial_{\beta\{\dot\alpha} \bar{\lambda}^{[ijk]}_{\dot\beta\}}. \tag{13}$$

Applications of $\bar{D}$ twice on (13) yield with the help of (12)

$$iD^i_\beta \bar{D}^{\dot\beta}{}_j \bar{F}^{[jk]}_{\{\dot\alpha\dot\beta\}} = -(N-4)\partial_\beta{}^{\dot\beta} \bar{F}^{[jk]}_{\{\dot\alpha\dot\beta\}} + \tfrac{3}{4} i \partial_{\beta\dot\alpha} X_j{}^{[jik]}, \tag{14}$$

$$i(N-2)(N-3)\partial_\alpha{}^{\dot\alpha} \bar{D}^{\dot\beta}{}_j \bar{F}^{[jk]}_{\{\dot\alpha\dot\beta\}} = 0. \tag{15}$$

Finally multiplying (14) by $(N-2)(N-3)\partial_\alpha{}^{\dot\alpha}$ we obtain from (15)

$$(N-2)(N-3)(N-4)\partial_\alpha{}^{\dot\alpha}\partial_\beta{}^{\dot\beta} \bar{F}^{[ij]}_{\{\dot\alpha\dot\beta\}} = 0, \tag{16}$$

which is the equation of motion for the $s=1$ field. Note that the equations of motion are second order. Therefore they are of the conformal type. Then equations of motion for $s=\frac{3}{2}$ and $s=2$ fields follow from Eqs. (12), (15), and (16):

$$(N-2)(N-3)(N-4)\partial_\alpha{}^{\dot\alpha}\partial_\beta{}^{\dot\beta} \bar{\Sigma}^i_{\{\dot\gamma\dot\beta\dot\alpha\}} = 0, \quad (N-2)(N-3)(N-4)\partial_\alpha{}^{\dot\alpha}\partial_\beta{}^{\dot\beta} \bar{V}_{\dot\delta\dot\gamma\dot\beta\dot\alpha} = 0. \tag{17}$$

Up to here we have not argued the equations of motion for the $s=\frac{1}{2}$ $\bar{\lambda}$ field itself and physical scalar field, say $\bar{\phi}^{[ijkl]}$, which has not appeared in the Bianchi identities at all. However, we have a superfield defined [through (10)] by $\bar{P}^{[ijkl]}_{\alpha\dot\alpha} \equiv D^i_\alpha \bar{\Lambda}^{[jkl]}{}_{\dot\alpha}$, which can be interpreted as the field strength of the scalar "potential" field $\bar{\phi}$.[15] Both $\bar{\lambda}$ and $\bar{P}$ turn out to satisfy their own field equations. Namely the D^k_γ operation and the successive D^l_β operation to (16) yield, through (13), the following two equations:

$$(N-4)\,\Box\,\partial_\alpha{}^{\dot\alpha} \bar{\lambda}^{[ijk]}{}_{\dot\alpha} = (N-4)\,\Box\,\partial_\alpha{}^{\dot\alpha} \bar{P}^{[ijkl]}_{\beta\dot\alpha} = 0, \tag{18}$$

where we have omitted the factor $(N-2)(N-3)$.

We thus obtain equations of motion for $s=1$, $\frac{3}{2}$, and 2 fields, which are second order in space-time differentiation, and of the conformal type.[16] This is because of the many auxiliary fields. For example, if we eliminate the X field Eq. (14) tells us [with the help of (12)] that the $\bar{F}$ field satisfies a first-order differential equation, which appears in the on-shell Poincaré-extended SUGRA.[15] However, so far we do not see the geometrical origin for such elimination of the X field. Also, the fact that our constraints are weaker is manifested in the existence of additional "nonphysical" fields such as $\bar{A}$ and $\bar{B}$, which make the derivation of field equations much more complicated. A detailed discussion on the full field content of the theory will be given in a future publication.

Finally, let us comment on another sequence of physical fields with spins up to $N/2$, suggested to exist in the extended conformal SUGRA.[21] We can confirm the existence of such fields in our theory and learn that these fields satisfy equations of motion for $N>2$, as the result of geometrical constraints. Though in (6) we have presented only one equation, the same Bianchi identity has another independent consequence with respect to the $\bar{A}$ and $\bar{B}$ fields. The combination of such an additional equation with (6) and $I_{\underline{\alpha\dot\beta\gamma}}{}^{\underline{\delta}} = 0$, and also a combination of the first equation of (5) with $I_{\underline{\alpha\beta}c}{}^d = 0$, yield crucial equations, respectively

$$\bar{D}_{\dot\alpha j} C^i_\alpha = (\delta^i_j/N)\bar{D}_{\dot\alpha l} C^l_\alpha, \tag{19}$$

$$D^i{}_{\{\alpha} C^j_{\beta\}} + (i \leftrightarrow j) = 0, \tag{20}$$

where

$$C^i_\alpha \equiv (N-2)(N-3)\bar{B}^{[il]}{}_{l\alpha} + (N-2)(i/4)\bar{D}^{\dot\alpha}{}_l \bar{A}^{[li]}{}_{\dot\alpha\alpha}.$$

We use these two equations to get

$$D^i{}_{(\alpha}\bar{D}_{\dot{\alpha}j}C^j_{\beta)} = -2Ni\partial_{(\alpha\dot{\alpha}}C^i_{\beta)}. \qquad (21)$$

The application of $\bar{D}^{\dot{\alpha}}{}_i$ to (21) gives $\partial_{(\alpha}{}^{\dot{\alpha}}\bar{D}_{\dot{\alpha}i}C^i_{\beta)}=0$, which implies the existence of a scalar "potential" field; $\bar{D}_{\dot{\alpha}i}C^i_{\alpha}=\partial_{\alpha\dot{\alpha}}\Omega$. In terms of C and Ω, (21) can be written as $\partial_{(\alpha\dot{\alpha}}\tilde{C}^i_{\beta)}=0$, where $\tilde{C}^i_{\alpha}\equiv C^i_{\alpha}-(i/2N)D^i_{\alpha}\Omega$. Carrying out another space-time differentiation we get an equation of motion, $\Box\tilde{C}^i_{\alpha}=0$ (simultaneously $\tilde{C}^i_{\alpha}$ satisfy another condition $\partial_{\alpha}\bar{D}_{\alpha i}\tilde{C}^i_{\alpha}=0$, which implies that $\tilde{C}^i_{\alpha}$ are "almost-chiral" superfields). The $\tilde{C}$ field is nothing but a superfield, which contains the sequence of physical fields we are interested in as its components: The lowest components of $D^i{}_{(\alpha}D^j_{\beta}\cdots D^k_{\gamma}\tilde{C}^l_{\delta)}$ [with totally symmetric spinor indices and therefore totally antisymmetric internal indices, as is clear from (20)], $\tilde{C}^i_{\alpha}$ itself, and $\bar{D}_{\dot{\alpha}i}\tilde{C}^i_{\alpha}$ are physical fields (or a field strength of scalar potential), with $1\leqslant s\leqslant N/2$, $s=\frac{1}{2}$, and $s=0$, respectively. It is now clear that these physical fields satisfy equations of motion,

$$\Box D^i{}_{(\alpha}D^j_{\beta}\cdots D^k_{\gamma}\tilde{C}^l_{\delta)}=0, \qquad (22)$$

where the number of D's runs from zero to $N-1$. As the C superfield survives only for $N>2$, (22) shows that the component fields are on shell for $N>2$.

The results mentioned in this Letter thus lead to the conclusion that the geometrical constraints imply equations of motion for all forms of expected physical fields (including those with spin up to $N/2$, which is greater than 2 for $N>4$) in the $N>4$ extended supergravities. Our result puts extended SUGRA in four-dimensional space with $N>4$ on the same footing as supersymmetric Yang-Mills fields in four-dimensional space with $N>2$ for developing similar integrability programs. We expect similar results for their corresponding theories formulated in higher dimensions.

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