Measurement of the Lense-Thirring Drag on High-Altitude, Laser-Ranged Artificial Satellites

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We describe a new method of measuring the Lense-Thirring relativistic nodal drag using LAGEOS together with another similar high-altitude, laser-ranged satellite with appropriately chosen orbital parameters. We propose, for this purpose, that a future satellite such as LAGEOS II have an inclination supplementary to that of LAGEOS. The experiment proposed here would provide a method for experimental verification of the general relativistic formulation of Mach's principle and measurement of the gravitomagnetic field.

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In special and general relativity there are several precession phenomena associated with the angular momentum vector of a body. If a test particle is orbiting a rotating central body, the plane of the orbit of the particle is dragged by the intrinsic angular momentum J of the central body, in agreement with the general relativistic formulation of Mach's principle.[1]

In the weak-field and slow-motion limit the nodal lines are dragged in the sense of rotation, at a rate given by[2]

$$\dot{\Omega} = [2/a^3(1-e^2)^{3/2}]J, \tag{1}$$

where a is the semimajor axis of the orbit, e is the eccentricity of the orbit, and geometrized units are used, i.e., $G=c=1$. This phenomenon is the Lense-Thirring effect, from the names of its discoverers in 1918.[2]

In addition to this there are other precession phenomena associated with the intrinsic angular momentum or spin $\mathbf{S}$ of an orbiting particle. In the weak-field and slow-motion limit the vector $\mathbf{S}$ precesses at a rate given by[1] $d\mathbf{S}/d\tau = \dot{\mathbf{\Omega}} \times \mathbf{S}$ where

$$\dot{\mathbf{\Omega}} \equiv -\tfrac{1}{2}\mathbf{v}\times\mathbf{a} + \tfrac{3}{2}\mathbf{v}\times\nabla U + \frac{1}{r^3}\left[-\mathbf{J} + \frac{3(\mathbf{J}\cdot\mathbf{r})\mathbf{r}}{r^2}\right], \tag{2}$$

where $\mathbf{v}$ is the particle velocity, $\mathbf{a} \equiv d\mathbf{v}/d\tau - \nabla U$ is its nongravitational acceleration, $\mathbf{r}$ is its position vector, τ is its proper time, and U is the Newtonian potential.

The first term of this equation is the Thomas precession.[3] It is a special relativistic effect due to the noncommutativity of nonaligned Lorentz transformations. It may also be viewed as a coupling between the particle velocity $\mathbf{v}$ and the nongravitational forces acting on it.

The second (de Sitter[4]–Fokker[5]) term is general relativistic, arising even for a nonrotating source, from the parallel transport of a direction defined by $\mathbf{S}$; it may be viewed as spin precession due to the coupling between the particle velocity $\mathbf{v}$ and the static —$g_{\alpha\beta,0}=0$ and $g_{i0}=0$—part of the space-time geometry.

The third (Schiff[6]) term gives the general relativistic precession of the particle spin $\mathbf{S}$ caused by the intrinsic angular momentum $\mathbf{J}$ of the central body—$g_{i0}\neq 0$.

We also mention the precession of the periapsis of an orbiting test particle due to the angular momentum of the central body. This tiny shift of the perihelion of Mercury due to the rotation of the Sun was calculated by de Sitter in 1916.[7]

All these effects are quite small for an artificial satellite orbiting the Earth.

We propose here to measure the Lense-Thirring dragging by measuring the nodal precession of laser-ranged Earth satellites. We shall show that two satellites would be required; we propose that LAGEOS[8-10] together with a second satellite LAGEOS X with opposite inclination (i.e., with $I^X = 180° - I$, where $I \simeq 109.94°$ is the orbital inclination of LAGEOS) would provide the needed accuracy.

The major part of the nodal precession of an Earth satellite is a classical effect due to deviations from spherical symmetry of the Earth's gravity field —quadrupole and higher mass moments.[11] These deviations from sphericity are measured by the expansion of the potential $U(r)$ in spherical harmonics. From this expansion of $U(r)$ follows[11] the formula for the classical precession of the nodal lines of an Earth satellite:

$$\dot{\Omega}_{\text{class}} \simeq -\frac{3}{2}n\left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{(1-e^2)^2}\left\{J_2 + J_4\left[\frac{5}{8}\left(\frac{R_\oplus}{a}\right)^2(7\sin^2 I - 4)\frac{1+\frac{3}{2}e^2}{(1-e^2)^2}\right] + \ldots\right\}, \tag{3}$$

where $R_{\oplus}$ is the mean equatorial radius, the J_{2n} are the even zonal harmonic coefficients of the gravitational field, $n \equiv 2\pi/P$ is the orbital mean motion, and P is the period.

Clearly the orbital parameters n, a, e, and I must be very accurately known to identify the classical contribution to the precession. We investigate these points below, but the difficulty of measurement of the Lense-Thirring dragging is apparent from the following facts: LAGEOS has a classical precession of $\sim 126°$ per year, while its Lense-Thirring precession is only 31 arc msec/yr. Our principal difficulty arises from poor knowledge of certain moments of the potential of the Earth.

A way of overcoming this problem would be to measure accurately $J_2, J_4, J_6, \ldots,$ by orbiting several high-altitude, laser-ranged satellites, plus LAGEOS to measure the Lense-Thirring effect. A better solution would be to orbit polar satellites. In 1976, Van Patten and Everitt[12] proposed the measurement of the Lense-Thirring effect with two counter-rotating drag-free satellites in polar orbit, i.e., counter-rotating in the same plane perpendicular to the equator. Because the classical precession is proportional to $\cos(I)$, which vanishes for polar orbits with $I = \pi/2$, the contribution to the nodal precession due to the quadrupole and higher moments of the Earth is zero for polar satellites. However, in order to measure the inclination with the technology then available to sufficient accuracy to extract the Lense-Thirring precession, it would have been necessary to use two counter-rotating drag-free polar satellites, measuring their distance with satellite-to-satellite Doppler ranging while the satellites were near the poles.

Today, however, the tracking accuracy of satellites has improved dramatically. The knowledge of the orbits of some artificial satellites using ground-based pulsed laser ranging systems will soon be sufficient to measure general relativistic perturbations.[13,14] Consequently, a general method[15,16] to measure the Lense-Thirring effect would be to have two laser-ranged satellites with any arbitrary pair of opposite (i.e., I and $180° - I$) inclinations to allow the separation of the classical and the relativistic precessions.

In particular LAGEOS, launched in 1976, has the best known orbit of any satellite because of its high altitude—semimajor axis = 12 270 km—and its design to minimize air drag and radiation[8-10] pressure. For this satellite the Lense-Thirring effect, Eq. (1), due to the Earth's intrinsic angular momentum $\mathbf{J}_{\oplus} \simeq 5.9 \times 10^{40}$ gm cm^2/sec, is $\dot{\Omega}_{\rm LT} \simeq 31$ arc msec/yr, eastward. For an artificial satellite the measurable quantity is $\dot{\Omega} - \omega_e$, where ω_e is the Earth's time-dependent rotation rate. The estimates of UT1—in effect the Earth's rotation rate—with the use of lunar laser ranging (LLR) and very long-baseline interferometry (VLBI) have an accuracy of fractions of a millisecond[17] of time corresponding to an accuracy of a few arc milliseconds of rotation. Consequently, the nodal precession is measurable[18] with an accuracy of 1 or 2 arc msec/yr. Therefore, the relativistic precession is quite detectable, if other contributions can be accurately accounted for.

In addition to the Lense-Thirring effect it is also necessary to consider the Sun's de Sitter–Fokker effect, i.e., the spin precession of the system "Earth plus an orbiting satellite" considered as a gyroscope, due to the static field of the Sun. The average geodesic precession projected into the Earth's equatorial plane is, for LAGEOS,[19] $\dot{\Omega}_{\rm DF} \simeq 17$ arc msec/yr, eastward. The relativistic node shift is dominated by the Lense-Thirring effect, and is in an eastward direction.

The orbital parameters n, a, and e appearing in Eq. (3) are measured with sufficient accuracy for our purpose via the LAGEOS laser ranging. However, the uncertainty in the inclination and thus in the classical part of the nodal precession may well be the dominant source of uncertainty in determining the relativistic part of the precession. Currently, a four-year span of LAGEOS orbital inclination residuals (from a dynamically consistent reference orbit) when fitted with a simple model of periods 1050, 560, and 280 days leads to residuals about this curve on the order of 2 arc msec.[18] These residuals are determined from independent 15-d intervals of laser tracking data. The Lense-Thirring effect is a secular effect, while most classical perturbations are of a periodic nature. Hence, it is the average (over a sufficiently long time) of the inclination error that concerns us. Under visual inspection the residuals appear to be Gaussian distributed. In one year, there are 24 such data points, so that after one year one expects $\sim 2/\sqrt{24} \sim 0.4$ arc msec of random error in the estimate of I for the span of that year, implying from Eq. (3) a contribution to the nodal precession error of ~ 3 arc msec/yr. This random error presumably can be reduced somewhat by an even longer period of data, though a careful study of systematic errors and of the statistics will be required to estimate the ultimate accuracy. Secondly, there are some correlations with our proposed second satellite which will lead to an improvement in the model, and a reduction in the residuals. For example, the effects of uncertainties in tidal parameters can be reduced by adjusting their values by use of data from the two satellites; changes in the inclination due to indeterminacy in the values of the J_{2n} are equal and opposite for the two satellites,[11] thereby, from Eq. (3), corresponding to equal and opposite nodal precessions. Any change in the inclination due to polar motion is already solved for in the experimental value of the inclination $I(t)$, and errors introduced should be relatively short period. A recent paper[20] shows 2-arc-msec residuals between VLBI and LAGEOS polar motion data, based on 5-d data intervals. This supports our expectation of average pole accuracy over longer periods at the 0.5-arc-msec level or better in the near future, and thus relatively small contributions to the uncertainty in the inclination.

We propose an experiment that is easier and less costly to implement than polar satellites would be. We propose that the forthcoming[21] satellite LAGEOS II (or some other future satellite LAGEOS X) be chosen to have the following orbital parameters:

$$a_{\text{LAGEOS }X} = a_{\text{LAGEOS}} \pm a_\epsilon,$$
$$I_{\text{LAGEOS }X} = \pi - I_{\text{LAGEOS}} \pm I_\epsilon, \qquad (4)$$
$$e_{\text{LAGEOS }X} = e_{\text{LAGEOS}} \pm e_\epsilon,$$

where a_ϵ, I_ϵ, and e_ϵ are small. In other words, the second satellite must have almost the same semimajor axis, period, and eccentricity as LAGEOS but an almost supplementary inclination. Because the classical precession (3) is linearly dependent on $\cos(I)$, with this particular choice of orbital parameters the classical nodal precessions of LAGEOS and LAGEOS X will be nearly equal and opposite. On the other hand the Lense-Thirring and the geodesic precession—which are independent of I—will be the same for both satellites so that, apart from other perturbations, the sum of the two measured total precessions will be twice the relativistic precession of LAGEOS. The philosophy of this two-satellite proposal is that one uses external data to fix the even zonal harmonics of the Earth's potential. One must conservatively take an external estimate of the error in these terms. Unfortunately, the present uncertainty δJ_{2n} in the numerical value of the even zonal harmonic coefficients J_{2n} is of the order[18,22] $\delta J_{2n} \simeq 10^{-6} J_2$. Since the Lense-Thirring precession $\dot{\Omega}_{\text{LT}}$ is $\dot{\Omega}_{\text{LT}} \simeq 6.9\times10^{-8}\dot{\Omega}_{\text{class}}$, the indeterminacy in the theoretical value of the classical nodal precession is more than 10 times the relativistic effect to be measured. The errors in these quantities limit the acceptable differences a_ϵ, I_ϵ, and e_ϵ between the two satellite orbits.

In order to have less than 3% individual contributions to the experimental uncertainty (based on expected improvements in the uncertainties $\delta J_{2n}/J_2$ to $\sim 3\times10^{-7}$) the differences a_ϵ, I_ϵ, and e_ϵ between the orbital parameters of LAGEOS and LAGEOS X must be less than $a_\epsilon \simeq \pm 16$ km, $I_\epsilon \simeq \pm 0.13°$, and $e_\epsilon \simeq \pm 0.04$. Moreover, in order to calculate the total precession $\dot{\Omega}_{\text{total}}$ it is also necessary to consider nonlinear contributions from even zonal harmonics; contributions from nonzonal spherical harmonics; lunar, solar, and planetary gravitational perturbations; atmospheric drag, radiation pressure; and earth tides. All these disturbing forces cause periodic and secular orbital perturbations that are very small compared to perturbations due to the Earth's quadrupole moment. The nonlinear contributions from the J_{2n} coefficients are equal and opposite for LAGEOS and LAGEOS X[18] and anyway, since the precession associated with $(J_2)^2$ is of the order $\dot{\Omega}_{(J_2)^2} \simeq 10^{-4}\dot{\Omega}_{\text{class}}$, these contributions could be calculated with sufficient accuracy to allow the determination of the Lense-Thirring effect. Similarly the secular nodal precessions associated with the nonzonal spherical harmonics and lunar, solar, and planetary perturbations can be calculated with sufficient accuracy to allow their extraction.[18] Concerning the nodal precession associated with radiation pressure and earth tides, information on these perturbations is extracted from the time variation of $\dot{\Omega}$ when measured over a long enough period of time.[18,23] Because LAGEOS is a spherical satellite and has a very small orbital eccentricity, the error in the calculated secular rate of nodal precession due to direct solar radiation pressure is small[19] compared to $\dot{\Omega}_{\text{LT}}$. Regarding the direct solar radiation pressure, the main effect for a nearly symmetric satellite is due to eclipses of part of the orbit. The eclipse seasons for LAGEOS last roughly 90 days and are separated by periods about twice that long. When the Sun is at equal distances from the satellite orbital plane, but in opposite regions, that is, before and after the Sun crosses the orbital plane, the out-of-plane components of the solar radiation pressure force are nearly equal and opposite; the contributions of the satellite eclipse to the nodal precession are nearly equal and opposite. For this reason the error, over one year, in the calculated secular rate of nodal precession due to the satellite eclipses is small[19] compared to $\dot{\Omega}_{\text{LT}}$. Systematic effects due to the Earth's albedo and to small differences in the satellite albedo for changes in the direction of the solar radiation with respect to the satellite rotation axis have to be considered, particularly in terms of interaction with eclipse-season effects. However, the prospects for modeling the radiation pressure and the atmospheric drag effects on the node with small errors are good.[19] A detailed analysis of various perturbing forces will be the subject of a following paper.[19]

In summary we have

$$\dot{\Omega}^{\text{LAGEOS}}_{\text{exp}} = \dot{\Omega}^{\text{LAGEOS}}_{\text{class}} + \dot{\Omega}^{\text{LAGEOS}}_{\text{rel}} + \dot{\Omega}^{\text{LAGEOS}}_{\text{other}}, \qquad (5)$$

where $\dot{\Omega}_{\text{exp}}$ is the experimentally measured value of the rate of nodal precession and $\dot{\Omega}_{\text{other}}$ is the small rate of advance, computable with sufficient accuracy, due to classical disturbing forces other than those caused by the even zonal spherical harmonics.

For the second satellite, we have

$$\dot{\Omega}^{\mathrm{LAGEOS}\ X}_{\mathrm{class}} = -\dot{\Omega}^{\mathrm{LAGEOS}}_{\mathrm{class}} + \Delta\dot{\Omega}_{\mathrm{class}}; \qquad (6)$$

$$\Delta\dot{\Omega}_{\mathrm{class}} = \frac{3}{2} n \left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{(1-e^2)^2} \left[\left(-\frac{7}{2}\frac{a_\epsilon}{a} - I_\epsilon \tan I + \frac{4ee_\epsilon}{1-e^2}\right) J_2 + \ldots\right], \qquad (6\ )$$

and the total nodal precession is

$$\dot{\Omega}^{\mathrm{LAGEOS}\ X}_{\mathrm{exp}} = \dot{\Omega}^{\mathrm{LAGEOS}\ X}_{\mathrm{class}} + \dot{\Omega}^{\mathrm{LAGEOS}\ X}_{\mathrm{rel}} + \dot{\Omega}^{\mathrm{LAGEOS}\ X}_{\mathrm{other}} = -\dot{\Omega}^{\mathrm{LAGEOS}}_{\mathrm{class}} + \Delta\dot{\Omega}_{\mathrm{class}} + \dot{\Omega}^{\mathrm{LAGEOS}}_{\mathrm{rel}} + \dot{\Omega}^{\mathrm{LAGEOS}\ X}_{\mathrm{other}}. \qquad (7)$$

Finally, from (5) and (7), we have

$$2\dot{\Omega}^{\mathrm{LAGEOS}}_{\mathrm{rel}} = \dot{\Omega}^{\mathrm{LAGEOS}}_{\mathrm{exp}} + \dot{\Omega}^{\mathrm{LAGEOS}\ X}_{\mathrm{exp}} - \dot{\Omega}^{\mathrm{LAGEOS}}_{\mathrm{other}} - \dot{\Omega}^{\mathrm{LAGEOS}\ X}_{\mathrm{other}} - \Delta\dot{\Omega}_{\mathrm{class}}.$$

In conclusion we stress that because of the increased tracking accuracy of the orbits of artificial satellites using laser ranging systems, the Lense-Thirring and de Sitter–Fokker relativistic nodal precessions are now significant and should be taken into account in the theoretical determination of orbital perturbations of high-altitude satellites such as LAGEOS. These relativistic nodal precessions should also be taken into account in geodesy calculations for a more accurate determination of the irregular gravitational field of the Earth, i.e., quadrupole and higher mass moments.

The experiment described above will provide an alternative verification, together with the Everitt-Fairbank experiment,[24] of Mach's principle and will give at last a measurement of the Earth's gyrogravitational or magnetogravitational field. Finally, we observe that orbiting a satellite with an inclination opposite to that of LAGEOS will also give a better determination of UT1.[18,19] For these reasons I propose that the forthcoming high-altitude laser-ranged satellite LAGEOS II or a similar future satellite have an inclination I of 70.06°.

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