Energy-Level Statistics of Integrable Quantum Systems

Recently Casati, Chirikov, and Guarneri[1] pointed out that the statistical behavior of the spectrum of a rectangular well $E_{mn}=m^2+\alpha n^2$, $m,n=1,2,\ldots,$ with irrational α did not coincide with the expectation put forward by Berry and Tabor.[2] They discovered deviations from the random behavior of the nearest-neighbor spacing distribution in the form of excessive fluctuations about the expected exponential distribution and confirmed the excessive stiffness of the spectrum at the scale of many level spacings as reported by ourselves and Zirnbauer.[3] Feingold[4] commented on these facts, pointing to a slow and irregular decay of the excessive fluctuations when millions of states are included. Also, he points to the increase of the linear range of the Δ_3 statistic with increasing energy. Actually the latter point bears no further discussion as it was cleared up completely in a recent paper by Berry.[5] He explained the data for the rectangular well by calculating the semiclassical limit of the Δ_3 statistic. The excessive long-range stiffness is also present in other systems such as Hamiltonians with a homogeneous polynomial as potential.[6,7]

The strong fluctuations in the nearest-neighbor spacing distribution, on the other hand, occur in what Berry denotes as the universal regime, i.e., for distances in the spectrum of the order of one level spacing. Although they may not survive for very large quantum numbers, the result of Casati, Chirikov, and Guarneri[1] casts severe doubts on numerical work that is necessarily limited to a finite and often small number of levels. Therefore, we ask ourselves whether this very slow convergence is generic to integrable systems or whether it is a particular (probably number-theoretic) feature of the rectangular well. For this purpose we constructed spectra of 99011 levels starting from the ground state for fourth- and sixth-order separable homogeneous polynomial potentials. We checked that variational and semiclassical calculations give the same results and used the latter in the form of

$$E_{mn}=m^p+\alpha n^p, \quad m,n=1,2,\ldots$$

$[p=2q/(q+1)$ when the order of the polynomial is $2q]$. For the value of $\alpha=\pi/3$ the nearest-neighbor spacing distribution agrees with the Poisson distribution with a confidence level of 50% $(q=3)$ and 80% $(q=2)$ compared with essentially zero for the rectangular well. To illustrate this we plot in Fig. 1 a histogram of the exponential of the spacings measured in units of the local-average level spacing for the rectangular well [Fig. 1(a)] and the fourth-order potential [Fig. 1(b)]. We also found that on variation of the parameter α in the case of the rectangular well the confidence level fluctuates by orders of magnitude

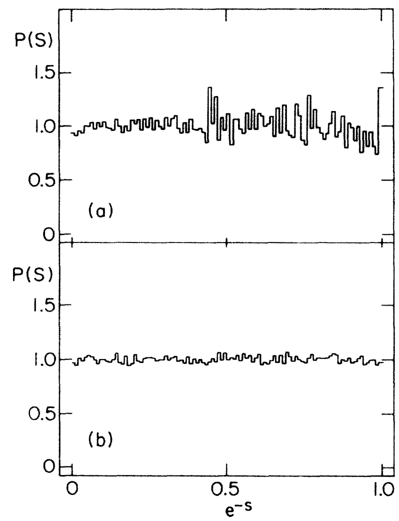

FIG. 1. The histogram of 99010 exponentiated spacings $\exp(-S_i)$ measured in units of the local-average level distance for (a) a rectangular well and (b) an x^4 potential. The bin size is equal to 0.01. The values of the normalized χ^2 are equal to 12.7 and 0.9, respectively.

(although it always remains negligibly small).

We conclude that the unexpected fluctuations in the nearest-neighbor spacing distribution for the rectangular well are nongeneric. They are a particular feature of the square well and possibly of some other billiards.

The authors are grateful to G. Casati for suggesting this test. This work was supported in part by National Science Foundation Grant No. PHY84-15064 at the University of Illinois at Urbana-Champaign.

T. H. Seligman
Instituto de Fisica
University of Mexico in Cuernavaca
Mexico, D.F., Mexico

J. J. M. Verbaarschot
Department of Physics
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

Received 10 February 1986
PACS numbers: 03.65.Ge, 05.45.+b

[1]G. Casati, B. V. Chirikov, and I. Guarneri, Phys. Rev. Lett. **54**, 1350 (1985).

[2]M. V. Berry and M. Tabor, Proc. Roy. Soc. London, Ser. A **356**, 375 (1977).

[3]T. H. Seligman, J. J. M. Verbaarschot, and M. R. Zirnbauer, Phys. Rev. Lett. **53**, 215 (1984).

[4]M. Feingold, Phys. Rev. Lett. **55**, 2626 (1985).

[5]M. V. Berry, Proc. Roy. Soc. London, Ser. A **400**, 229 (1985).

[6]T. H. Seligman, J. J. M. Verbaarschot, and M. R. Zirnbauer, J. Phys. A **18**, 2751 (1985).

[7]T. H. Seligman and J. J. M. Verbaarschot, to be published.