Nonlinear Reflection and Refraction of Planar Ion-Acoustic Plasma Solitons

T. Nagasawa[(a)] and Y. Nishida

Department of Electrical Engineering, Utsunomiya University, Utsunomiya, Tochigi 321, Japan
(Received 22 April 1986)

Experimental observations on the reflection and refraction of a planar ion-acoustic soliton from a metallic mesh electrode are performed in a uniform double-plasma device. Reflection and refraction angles are observed to depend on the incident wave amplitude, showing a nonlinear Snell's law.

PACS numbers: 52.35.Sb, 43.25.+y, 52.35.Fp, 52.35.Mw

Experimental and theoretical studies of the reflection of linear and nonlinear ion-acoustic waves in a plasma with a sharp density gradient have been performed by many researchers.[1,2] Dahiya, John, and Saxena[3] have shown clear evidence of the reflection of solitary waves (solitons) by the density gradient. They claimed that the reflection occurs from the sharp density gradient, the scale size of which is of the same order as or smaller than the width of the initial soliton. Nishida[1] has pointed out that no appreciable dependence of the reflection coefficient on the incident wave amplitude and on the sheath thickness is observed in the case of disk reflectors, while the reflection coefficient varies with these parameters in the case of a mesh reflector. He also predicted that the reflection angle might change with the reflection rate and wave amplitude. Raychaudhuri and co-workers[2] also have shown the existence of the reflected soliton from reflectors with various shapes. Self-refraction has also been studied for nonlinear ion-acoustic waves in a uniform plasma.[4]

In this Letter we wish to show the first experimental observation of the reflection and refraction angles which depend on the incident amplitude of the soliton at the boundary. In the usual systems, such as for an acoustic wave or an electromagnetic wave in a uniform medium, it is popularly known that the reflection and/or refraction angle should be the same as an incident angle, if the medium is uniform enough. This is because in most of the cases the waves considered have small amplitudes and the phenomena should be linear. When large-amplitude waves are considered, however, the situation would be changed and nonlinear effects could be expected. Here in the present investigation, we are interested in a large-amplitude ion-acoustic wave, i.e., soliton. It is shown for the first time, to our knowledge, that the reflection angle, and the refraction angle as well, changes with the wave amplitude at the constant incident angle. We call this phenomenon a nonlinear Snell's law for convenience. This phenomenon would be expected to occur universally for a large-amplitude wave in many other fields.

The experiments were performed in a double plasma device covered with many permanent magnets (see Fig. 1),[1] in which a plane separation grid is fixed but the metallic mesh reflector (30×35 cm; 6 mesh/cm) can be adjusted to the desired angle to the incident planar ion-acoustic soliton. The reflector electrode can be biased from about -23 V (floating potential) to -500 V with respect to the grounded target chamber. The argon plasma parameters are typically electron density $n_e \simeq (2\text{–}5)\times10^8$ cm^{-3}, electron temperature $T_e \simeq 2.0\text{–}2.5$ eV, and ion temperature $T_i \simeq T_e/(8\text{–}10)$ in a neutral gas of pressure $P \simeq 3\times10^{-4}$ torr with base pressure 7×10^{-7} torr. The waves were picked up by a tiny (0.1 mm in diameter by 1.0 mm in length) cylindrical Langmuir probe biased slightly above the plasma potential in order to detect perturbations in the electron saturation current, and hence in electron density. The signals were displayed on an oscilloscope screen or analyzed with a boxcar integrator and displayed on an XY recorder.

Typical examples of the wave form of the incident (I), reflected (R), and transmitted (T) waves are demonstrated in Fig. 2 at three different bias voltages on the reflector. There are two types of reflected waves with different polarities. In Fig. 2(a) for the reflector potential floating, we can clearly see that the wave with negative polarity is reflected back propagating to the x direction (z=const see Fig. 1). It is shown in Fig. 2(b) for a negatively biased reflector with bias voltage -150 V that both positive- and negative-polarity reflected waves exist at the same time, but their amplitudes are quite small compared with those in Fig. 2(a); thus reflections are hardly ob-

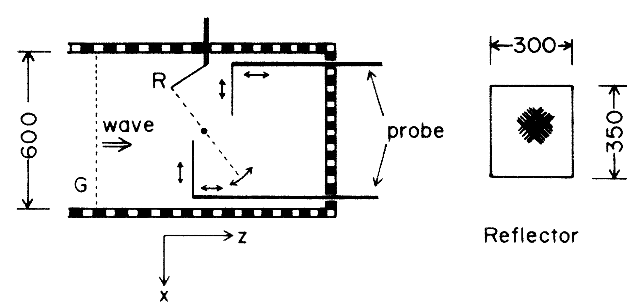


FIG. 1. Schematics of the experimental apparatus. G is a separation grid for exciting solitons and R is a metallic mesh reflector (6 mesh/cm).

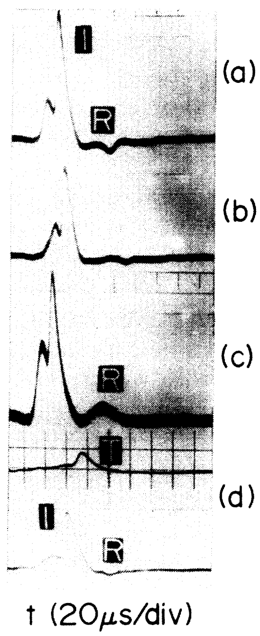


FIG. 2. Typical examples of the wave form. (a) Reflection of negative pulse: $V_{bias} = -25$ V (floating), (b) no reflections: $V_{bias} = -150$ V, (c) reflection of positive pulse: $V_{bias} = -500$ V, and (d) transmitted wave: $V_{bias} = -50$ V.

served. This situation corresponds to the "matched terminal condition" in electric circuits. However, as is shown in Fig. 2(c), the positive reflected wave appears, propagating to the x direction, with a deep negative bias voltage of about -500 V on the reflector, and the wave with negative polarity disappears. In Fig. 2(d), an example of the transmitted wave is shown, and the transmission rate of the incident wave is about 30% in this example. In most of the cases for $\delta n/n \lesssim 25\%$, the transmission rate is constant at about 30% regardless of the wave amplitude, where $\delta n/n$ is the ratio of the perturbed to unperturbed density. However, the transmission rate strongly depends on the bias voltage of the reflector. In all of these examples, the incident wave amplitude is kept almost constant.

Examples of reflection angles for the incident wave with amplitude $\delta n/n = 7\%$ and 17% are shown in Fig. 3, in which the reflection angles are different between the two waves. Namely, the wave with $\delta n/n = 7\%$ has reflection angle 56°, while the wave with $\delta n/n = 17\%$ has 51° measured from the normal to the reflector plane, for a constant incident angle of 67°. It should be pointed out that the reflection angle is a function of the initial soliton amplitude for a constant incident angle (θ_I). The reflection angle (θ_R) becomes smaller as the initial soliton amplitude increases as shown in Fig. 4. In Fig. 4(a) the reflected wave has negative polarity. The characteristics of this negative wave show that the velocity is subsonic and its width is almost the

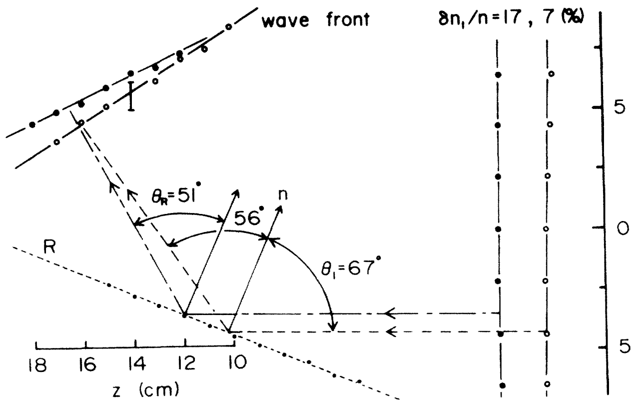


FIG. 3. Typical examples of the wave front of incident solitons and reflected waves. The incident soliton amplitude refers to $\delta n/n = 7\%$ (open circles) and 17% (filled circles) and both of the incident angles are 67°. R is a reflector.

same as that of the planar soliton. It has been confirmed that the negative wave is a hole[5] excited by a small amount of ion beams ejected from sheath areas. It is also found that the reflection angle of the positive wave changes with the amplitude of the initial soliton [see Fig. 4(b)]. The reflected wave in this case has supersonic velocity and its characteristics have been confirmed as showing the soliton natures.

The refraction angles of the transmitted wave are also measured as a function of the incident wave amplitude; an example is shown in Fig. 5. It should be recognized that the refraction angle also becomes

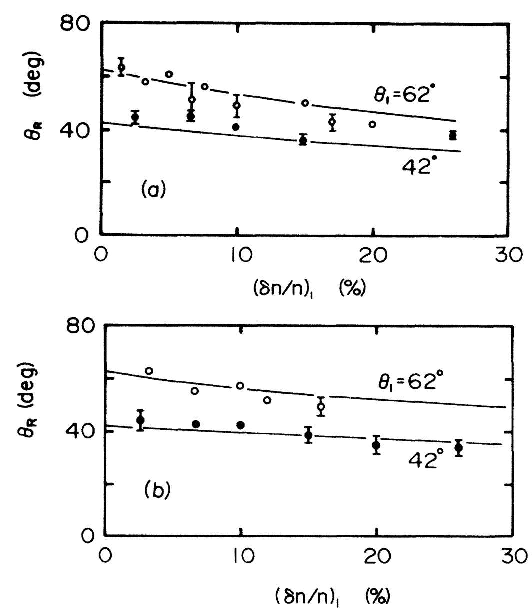


FIG. 4. Reflection angles θ_R vs incident wave amplitude for constant incident angle θ_I. The reflected wave has negative polarity in (a) and positive polarity in (b). Solid lines are calculated from Eq. (2).

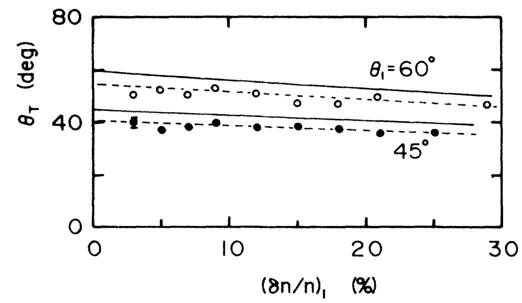


FIG. 5. Refraction angle θ_T vs incident wave amplitude for constant incident angle θ_I. Solid lines are calculated from Eq. (2).

smaller with the increase of the incident wave amplitude, but its dependence is even weaker than that for the reflected wave. Furthermore, in the limit of zero amplitude of the incident wave, there is a difference in angle between the incident and reflected waves, which has not been observed in the wave reflection.

The ion-acoustic soliton has been well established to have the velocity given by[6]

$$v/C_s = 1 + \alpha(\delta n/n), \qquad (1)$$

where v and C_s are respectively the velocities of the soliton and of the ion sound wave and α is constant equal to about 0.5 in most of our experiments (theoretically $\alpha = \frac{1}{3}$). When we recall Snell's law, the relation between an incident angle θ_I and a reflection (or refraction) angle $\theta_R\,(\theta_T)$ is given by

$$\frac{\sin\theta_{R(T)}}{\sin\theta_I} = \frac{v_{R(T)}}{v_I} = \frac{1+\alpha(\delta n/n)_{R(T)}}{1+\alpha(\delta n/n)_I}, \qquad (2)$$

where θ is measured from the normal axis to the reflector plane and suffixes stand for the reflected (R), transmitted (T), and incident (I) wave, respectively. By considering the experimental values, we can calculate the reflection and refraction angles as a function of the incident wave amplitude, after measuring the reflection coefficient and the transmission rate. The calculated results are shown in Figs. 4 and 5 by solid lines. Here, in the case of a negative reflected pulse, we also applied Eq. (2) with $\alpha(R)$ negative as is observed experimentally. The results show reasonably good agreement with the experimental observations except for the angle difference at $(\delta n/n)_T \simeq 0$ in the wave refraction. This difference is qualitatively interpreted as follows. When the wave comes through the mesh electrode, the incident wave of about 60%–70% in its amplitude is absorbed. Thus, this situation corresponds equivalently to the wave propagating through from a medium I to II with different index of refraction, although the actual plasma is quite uniform except for the narrow sheath region where the strong wave absorption occurs. Therefore, we may expect refraction at the sheath area of the mesh electrode even in the small-amplitude limit. The refraction angle also depends on the bias voltage to the reflector but it is essentially determined by the ratio of the amplitude of the incident to transmitted wave within our experimental errors.

The reflection rate changes also with the bias voltage on the reflector.[1] In the present experiment, the amplitudes of the reflected waves with negative and positive polarities are observed separately. The results show that an originally large-amplitude negative pulse, which is identified as a hole at the floating potential on the reflector, diminishes with the increase of the bias voltage on the negative side, while the positive pulse, which is identified as a soliton, increases in amplitude with negative bias voltage. At the bias voltage about -200 V under some conditions, the positive and negative pulses have the same amplitude to cancel out each other, and no reflection has been observed. In analogy, the negative-pulse reflection corresponds to short-circuit termination in an electric circuit, while the positive-pulse reflection to open-circuit termination and no reflection to matched termination.

We also found that the reflection layer is located even farther from the reflector surface in the case of a positive pulse (soliton), while the origin of the negative pulse (hole) is closer to the electrode. The spatial difference is about 0.2–0.5 cm which is several tens of Debye length. Therefore, the hole is excited well within a sheath around the reflector.

In conclusion, the first experimental observations on the reflection and refraction angles of the large-amplitude ion-acoustic wave have been performed, showing both the reflection and refraction angles to be determined by the incident wave amplitude: nonlinear Snell's law. By taking into account the soliton nature in the wave velocity, the Snell's law interprets the experimental results of dependence of reflection and refraction angles on the incident wave amplitude. However, the refraction angle is always smaller than the incident angle even in the limit of zero amplitude. This fact implies that the absorption of the wave in a sheath region around the reflector is essential to the refraction phenomena.

The authors are indebted to Professor N. Yajima for his valuable discussions and comments. A part of the present work is supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture, Japan.

[a]Permanent address: Department of General Education, Ashikaga Institute of Technology, Ashikaga, Tochigi 326, Japan.

[1]Y. Nishida, Phys. Fluids **27**, 2176 (1984), and references therein.

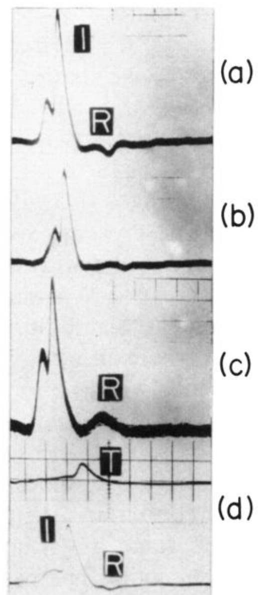
[2]H. H. Kuehl, Phys. Fluids **26**, 1577 (1983); S. Raychaudhuri, Hong-Yong Chang, E. K. Tsikis, and K. E. Lonngren, Phys. Fluids **28**, 2125 (1985); S. Raychaudhuri, Be Trieu, E. K. Tsikis, and K. E. Lonngren, private communication.

[3]R. P. Dahiya, P. I. John, and Y. C. Saxena, Phys. Lett. **65A**, 323 (1978).

[4]L. A. Ostrovskii and V. I. Shrira, Zh. Eksp. Teor. Fiz. **71**, 1412 (1976) [Sov. Phys. JETP **44**, 738 (1976)]; V. I. Shrira, Zh. Eksp. Teor. Fiz. **79**, 87 (1980) [Sov. Phys. JETP **52**, 44 (1980)]; E. Okutsu and Y. Nakamura, J. Phys. Soc. Jpn. **51**, 3022 (192).

[5]K. Saeki, P. Michelsen, H. L. Pécseli, and J. J. Rasmussen, Phys. Rev. Lett. **42**, 501 (1979).

[6]See, for example, H. Ikezi, in *Soliton in Action*, edited by K. E. Lonngren and A. Scott (Academic, New York, 1978), p. 153; Y. Nishida, Buturi **37**, 396 (1982) (in Japanese); Y. Nakamura, IEEE Trans. Plasma Sci. **10**, 180 (1982); K. E. Lonngren, Plasma Phys. **25**, 943 (1983).



t (20μs/div)

FIG. 2. Typical examples of the wave form. (a) Reflection of negative pulse: $V_{\text{bias}} = -25$ V (floating), (b) no reflections: $V_{\text{bias}} = -150$ V, (c) reflection of positive pulse: $V_{\text{bias}} = -500$ V, and (d) transmitted wave: $V_{\text{bias}} = -50$ V.