Limitations of Heterotic-Superstring Phenomenology

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The low-energy effective theory arising from compactification of the $E_8 \otimes E_8'$ heterotic superstring on $M_4 \times K_6$ is discussed. Symmetry between the observed and shadow world will occur only for manifolds (of which no examples are yet known) with special values of $\mathrm{Tr}(R \wedge R)$. Other scenarios are described in which no new gauge bosons or chiral fermions appear.

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In order to arrive at chiral fermions in four-dimensional Minkowski space-time (M_4) after compactification of a superstring theory constructed in M_{10} it is essential that there be a Yang-Mills gauge symmetry in M_{10}. Of the two anomaly-free choices O(32) and $E_8 \otimes E_8'$ the latter is more promising phenomenologically[1] and hence its implications have been studied more. By making assumptions about how the $E_8 \otimes E_8'$ heterotic string[2] reduces to an effective theory in M_4, attempts have been made[3-5] to identify features, especially possible new undiscovered "light" particles predicted by the superstring theory. One obvious example is the supersymmetric partners of the known gauge bosons, quarks, and leptons; these were already predicted by the older supersymmetry and supergravity schemes. Two predictions specific to superstrings have led to phenomenological analyses of (i) an additional neutral gauge boson[6,7] beyond those of $SU(3) \otimes SU(2) \otimes U(1)$ and (ii) additional chiral fermions[8] beyond those of the observed three families, as expected in, e.g., 27-plets of E_6. Here we emphasize that none of these extra states are *necessary* in successful superstring phenomenologies[9] and discuss the specifics of gauge-symmetry–breaking schemes that contain no such extra states, as well as discussing some other phenomenologically interesting alternatives.

The principal step in making contact with physics in M_4 is in compactification of six spatial dimensions to a size too small to have been yet observed ($\lesssim 10^{-17}$ cm) and arguably[10] not much larger than the Planck scale, 10^{-33} cm. Unlike the manifolds first discussed[1] it is now believed[3,11] that this six-dimensional manifold must be Ricci flat and Kähler; that is, it is a Calabi-Yau space (K) with SU(3) holonomy. The number of chiral families is $n_f = \frac{1}{2}|\chi(K)|$, where $\chi(K)$ is the Euler characteristic of the simply connected Calabi-Yau manifold, and the gauge group SU(3) of $E_8 \supset SU(3) \otimes E_6$ is identified with the SU(3) spin connection of K. This gives an unacceptably large n_f, and it is necessary[3] to reduce $\chi(K)$ by considering a multiply connected quotient manifold K/G obtained by factoring out a discrete symmetry group G that acts freely on K; the resulting χ is $\chi = \chi(K)/\dim(G)$. In this way, an acceptable n_f such as 3 or 4 can be obtained.[12] In the more general cases that we analyze below, the role of the holonomy group of K will be played by the structure group $SU(N)$ ($N = 3,4,5$) of a stable holomorphic bundle. For $N=3$ this just reduces to the holonomy group of K. In the cases $N=4$ or 5 the calculation of the number of families is generalized to $n_f = \frac{1}{2}[c_3(K/G)]$, where $[c_3(K/G)]$ is the third Chern number of the bundle over the manifold and $[c_3(K/G)] = [c_3(K)]/\dim(G)$. The unbroken gauge group at this stage will be at most E_6 and depends on vacuum expectation values for matter fields in flat directions for the cases $N=4$ and 5.

At the same time, and very attractively, these theories allow gauge-symmetry breaking by arranging that noncontractable Wilson loops on the multiply connected manifold have nonzero vacuum values. That is,

$$U = \mathscr{P}\exp\left(\int_c iA_m \, dx^m\right) \qquad (1)$$

is not equal to 1, although F_{mn} vanishes everywhere ($\mathscr{P}$ denotes path ordering). For an Abelian discrete symmetry this corresponds to Hosotani's symmetry-breaking scheme[13] for Kaluza-Klein theories; it is group-theoretically equivalent to Higgs scalars in the adjoint representation and hence rank preserving. For a non-Abelian discrete symmetry, the rank can be reduced.

Before discussing symmetry breaking, let us first recall the important consistency condition[3]

$$\mathrm{tr} F \wedge F = 30\, \mathrm{tr} R \wedge R \tag{2}$$

which restricts the embedding of the SU(3) holonomy in the gauge group $E_8 \otimes E_8'$. Initially we shall ignore E_8' and assume that it remains exact (we shall later relax this assumption). Then for the embedding

$$E_8 \supset \mathrm{SU}(3) \otimes E_6, \tag{3a}$$

$$\mathbf{248} = (\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{78}) \oplus (\mathbf{3},\mathbf{27}) \oplus (\mathbf{3}^*,\mathbf{27}^*), \tag{3b}$$

one satisfies Eq. (2) because the quadratic Casimir operator of the **8** is 3 times that of ($\mathbf{3} \oplus \mathbf{3}^*$) and hence the **248** counts as $30(\mathbf{3} \oplus \mathbf{3}^*)$ or $30(\mathbf{6})$ of O(6). One then arrives at n_f 27-plet families of E_6 and these contain chiral fermions—particularly a ($\mathbf{5} \oplus \mathbf{5}^*$) of the SU(5) subgroup—beyond the observed ($\mathbf{10} \oplus \mathbf{5}^*$). If we assume that the nontrivial Wilson lines, Eq. (1), are in singlets of the SU(3) holonomy, i.e., in the ($\mathbf{1},\mathbf{78}$) of Eq. (3b), then the symmetry breaking of E_6 by a non-Abelian discrete group G cannot reduce the rank below 5 while keeping the correct weak hypercharges of the quarks and leptons. Writing $E_6 \supset \mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ one finds that with the SU(2) of weak isospin in $\mathrm{SU}(3)_L$ the weak hypercharge $\mathrm{U}(1)_Y$ lies in both $\mathrm{SU}(3)_L$ and $\mathrm{SU}(3)_R$ and hence an orthogonal U(1) must be left unbroken.[3,6,7,14]

Let us reexamine the consistency condition, Eq. (2), and make an observation which becomes important when we look at more general cases of a theory with an SU(N) structure group ($N \geqslant 3$), and a nonsimply connected Calabi-Yau space with non-Abelian discrete symmetry G. Consider the regular maximal subalgebras of E_8 other than (3) above:

$$E_8 \supset \mathrm{SU}(5) \otimes \mathrm{SU}(5), \tag{4a}$$

$$\mathbf{248} = (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus (\mathbf{5},\mathbf{10}^*) \oplus (\mathbf{5}^*,\mathbf{10}) \oplus (\mathbf{10},\mathbf{5}) \oplus (\mathbf{10}^*), \tag{4b}$$

$$E_8 \supset \mathrm{SU}(9), \tag{5a}$$

$$\mathbf{248} = \mathbf{80} \oplus \mathbf{84} \oplus \mathbf{84}^*, \tag{5b}$$

$$E_8 \supset \mathrm{O}(16), \tag{6a}$$

$$\mathbf{248} = \mathbf{120}(\text{vector}) \oplus \mathbf{128}(\text{spinor}), \tag{6b}$$

$$E_8 \supset \mathrm{SU}(2) \otimes E_7, \tag{7a}$$

$$\mathbf{248} = (\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{133}) \oplus (\mathbf{2},\mathbf{56}). \tag{7b}$$

This completes the list of five regular maximal subalgebras.

Consider the $N=3$ case first, where SU(3) is the holonomy group of K. Now in (4) put the SU(3) holonomy in the first SU(5) so that

$$\mathbf{5} = \mathbf{3} \oplus 2(\mathbf{1}), \quad \mathbf{10} = 2(\mathbf{3}^*) \oplus \mathbf{3} \oplus \mathbf{1},$$

$$\mathbf{24} = \mathbf{8} \oplus 2(\mathbf{3} \oplus \mathbf{3}^*) \oplus 4(\mathbf{1}).$$

Thus we have $\mathbf{8} + 27(\mathbf{3} \oplus \mathbf{3}^*)$, hence satisfying Eq. (2). In (5) write $\mathbf{9} = \mathbf{3} \oplus 6(\mathbf{1})$, whereupon

$$\mathbf{80} = \mathbf{8} \oplus 6(\mathbf{3} \oplus \mathbf{3}^*) \oplus \text{ singlets},$$

$$\mathbf{84} \oplus \mathbf{84}^* = 21(\mathbf{3} \oplus \mathbf{3}^*) \oplus \text{ singlets},$$

and Eq. (2) is satisfied. In (6) put $\mathbf{16} = \mathbf{3} \oplus \mathbf{3}^* \oplus 10(\mathbf{1})$, giving

$$\mathbf{120} = \mathbf{8} \oplus 11(\mathbf{3} \oplus \mathbf{3}^*) \oplus \text{ singlets},$$

$$\mathbf{128} = 16(\mathbf{3} \oplus \mathbf{3}^*) \oplus \text{ singlets},$$

to see that Eq. (2) is consistent. Finally even (7) is consistent for an SU(2) holonomy—as in breaking[15] from M_{10} to $M_6 \times K_3$—since the triplet of SU(2) counts as four doublets. Thus the consistency condition Eq. (2) is satisfied for any of the regular maximal subalgebras of E_8. More generally if we embedded SU(N) in an SU(5) of (4a) with $\mathbf{5} = (\mathbf{N}) \oplus (5-N)(\mathbf{1})$ then either there results $\mathbf{15} \oplus 10(\mathbf{6}) + 11(\mathbf{4} \oplus \mathbf{4}^*)$ of an SU(4) structure group which is equivalent to $30(\mathbf{4}+\mathbf{4}^*)$'s, or there results $\mathbf{24} \oplus 5(\mathbf{10} \oplus \mathbf{10}^*) \oplus 10(\mathbf{5} \oplus \mathbf{5}^*)$ which is equivalent to $30(\mathbf{5} \oplus \mathbf{5}^*)$'s of the SU(5) structure. Both cases obviously satisfy Eq. (2). Likewise with the embedding $\mathbf{9} + \mathbf{N} \oplus (9-N)(\mathbf{1})$ of SU(N) in SU(9) or with $\mathbf{16} = (\mathbf{N} \oplus \mathbf{N}^*) + (16-2N)(\mathbf{1})$ of O(16) we find Eq. (2) satisfied. I.e., we have the equivalent of $30(\mathbf{N} \oplus \mathbf{N}^*)$'s of SU($N$) for all the above embedding for $N=3$, 4, or 5; and the unbroken gauge group is arranged to be E_6, O(10), or SU(5), respectively. None of this is too surprising since these can be seen to be gauge-equivalent rearrangements of the "regular" embedding of SU(N) in E_8. The "irregular" embeddings of SU(N) in E_8 corresponding to various special maximal subgroups of E_8 are quite a different story as we will now show.

Consider, for example, the maximal $G_2 \otimes F_4$ subalgebra of E_8. If we place the SU(3) holonomy in G_2, then under SU(3) $\otimes$ F_4 the **248** of E_8 is

$$(\mathbf{8},\mathbf{1}) \oplus (\mathbf{3} \oplus \mathbf{3}^*,\mathbf{1}) \oplus (\mathbf{1},\mathbf{26}) \oplus (\mathbf{1},\mathbf{52}) \oplus (\mathbf{3} \oplus \mathbf{3}^*,\mathbf{26}),$$

again equivalent to $30(\mathbf{3} \oplus \mathbf{3}^*)$'s. Now for $F_4 \supset \mathrm{SU}_A(3) \otimes \mathrm{SU}_B(3)$ with the **26** of F_4 reducing as $\mathbf{26} \to (\mathbf{8},\mathbf{1}) \oplus (\mathbf{3},\mathbf{3}) \oplus (\mathbf{3}^*,\mathbf{3}^*)$, identifying the SU(3) holonomy with $\mathrm{SU}_B(3)$ satisfies Eq. (2). However, identifying the holonomy with $\mathrm{SU}_A(3)$ does not satisfy Eq. (2) since one finds the equivalent of $60(\mathbf{3} \oplus \mathbf{3}^*)$'s. Similar cautionary remarks hold for other irregular embeddings of the SU(N) structure group in the special maximal E_8 subalgebras. It is amusing to note (though this point seems moot for phenomenologically acceptable models) that such embeddings always given an integral multiple of $30(\mathbf{N} \oplus \mathbf{N}^*)$'s and never a half odd integer. This in turn seems to preclude the interesting possibility of embedding the structure group half in E_8 and half in E_8' in order to maintain complete symmetry between our world and the shadow world until manifolds are found (see Sec. 6 of Ref. 9) that have $\mathrm{Tr}(R \wedge R)$ differing by an integer [2 in the case of $30(\mathbf{3} \oplus \mathbf{3}^*)$ per E_8; 4 for the example of this paragraph, etc.] from its value on Calabi-Yau spaces of the polynomial in $CP(N)$ type. We will return below to this interesting idea and provide a way to at least partially maintain this symmetry.

Let us focus for illustration on the maximal subalgebra $SU(5) \otimes SU(5)$. Study of the regular maximal subalgebras leads to similar conclusions.

We now need to consider the action of the non-Abelian discrete group G. Suppose G has an irreducible representation (irrep) of the same dimension m as the defining representation of a given Lie algebra L and that to every element of G there corresponds a nontrivial Wilson line. By Schur's lemma the only matrix which commutes with all of G is the unit $m \times m$ matrix; in particular, no generator of the Lie algebra is left fixed by G. This action on the generators of the algebra may be extended to an arbitrary representation of the algebra (though this need no longer be an irrep of G). Thus, such Wilson lines completely break the symmetry L.

To illustrate this, we shall assume that there are Calabi-Yau spaces whose freely acting non-Abelian discrete group is either the symmetric group S_n or its normal subgroup A_n, the alternating group of $\frac{1}{2}n!$ even permutations. Aside from one-dimensional representations we note that S_3 has a two-dimensional irrep, A_4 has a three-dimensional irrep, and A_5 has three-, four-, and five-dimensional irreps. In general, both S_{m+1} and A_{m+1} $(m > 3)$ have an m-dimensional irrep. By the observation given above, this action can completely break $SU(m)$. It is easy to picture this geometrically: (A_{m+1}) S_{m+1} are the (proper) discrete symmetries of the regular polytope[16] α_{m+1}, the $(m+1)$ simplex in R^m (for example, α_4 is a tetrahedron). Breaking all of these symmetries breaks the $SO(m)$ rotational invariance of the special maximal subalgebra $SO(m)$ in $SU(m)$.

For example, the choice $G = S_3$ and a two-dimensional representation can break SU(2) completely, or SU(3) to U(1), SU(4) to $SU(2) \otimes U(1)$ and so on; in the E_6 scenario, $G = S_3$ can break to the minimal rank-5 symmetry $SU(3) \otimes SU(2) \otimes U(1) \otimes U(1)$. (See the second paper of Ref. 3.)

Consider now the more interesting case of $G = A_5$ (sixty elements). With nontrivial Wilson lines in all elements, and using the five-dimensional representation, we may break completely one SU(5). Likewise, Witten's method[9] of identifying an SU(5) structure group with an SU(5) subgroup of E_8 also eliminates an SU(5). The SU(3) holonomy supplemented with vacuum expectation values in the **27** and **27*** along flat directions fill out the SU(5) while leaving $N = 1$ supersymmetry intact.

To recapitulate, we can eliminate SU(5) factors from $E_8 \otimes E_8'$ in two ways: first by nontrivial Wilson lines in the five-dimensional irrep of $G = A_5$ (or S_5) for a Calabi-Yau manifold K/G with $\pi_1(K/G) = G$, and second by the mechanism of Ref. 9. We will call these the *non-Abelian* and *structure-group mechanisms*. Further rank-preserving symmetry breaking is provided by nontrivial Wilson loops in Abelian subgroups of G (*Abelian mechanism*).

A partial solution to the problem of keeping the symmetry between our world and the shadow world is now at hand. We eliminate (i.e., break) an SU(5) in E_8 by the structure-group mechanism, and an SU(5) from E_8' by the non-Abelian mechanism.[17] Then the remaining SU(5) and SU(5)′ can be broken to $SU(3) \otimes SU(2) \otimes U(1)$ and a shadow $[SU(3) \otimes SU(2) \otimes U(1)]'$, respectively, by the Abelian mechanism. We should note that even though the gauge groups are the same, the matter multiplets are not equivalent in the two worlds unless $n_f = 0$. Thus, the Georgi-Quinn-Weinberg analyses will differ in the two cases and we will find, for instance, $\Lambda_{QCD} \neq \Lambda'_{QCD}$.

It is also possible to break the SU(5)′ completely by a second application of the non-Abelian mechanism (rather than using the Abelian mechanism). Then the shadow E_8' is completely broken, and so below the Planck scale we can have "standard" SU(5) (supersymmetric) grand unification. Similar considerations are easily worked out for the SU(4) structure group.

In summary, the limitations we have reached make it seem premature to propose definitive phenomenological tests for superstrings, particularly those requiring new light gauge bosons or chiral fermions, until superstring dynamics have been shown to select the topology of the compact manifold. This seems out of reach at present.

Compactification can leave $N = 1$ supersymmetry unbroken but this must be broken at some energy scale; this supersymmetry-breaking mechanism is not well understood at present. If we assume that supersymmetry is broken at a *very* high energy[18] near the Planck mass, then the result could be nonsupersymmetric SU(5) grand unification which has phenomenological difficulty with the lower limit on the proton lifetime; this can be patched up in several ways.[19] With supersymmetry broken at low energies[18] one will need to investigate in more detail the phenomenological consequences. Our purpose here is only to emphasize the scenarios which avoid any new particles near the weak scale.

Although we have been discussing Calabi-Yau manifolds, the results of Grisaru, van de Ven, and Zanon[20] suggest that the superstring equations of motion are satisfied only in an orbifold[21] limit.

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[1]E. Witten, Phys. Lett. **149B**, 351 (1984); P. H. Frampton, H. van Dam, and K. Yamamoto, Phys. Rev. Lett. **54**, 1114 (1985).

[2]D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985), and Nucl. Phys. **B256**, 253 (1985), and to be published.

[3]P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985); E. Witten, Nucl. Phys. **B258**, 75 (1985).

[4]J. Breit, B. A. Ovrut, and G. Segre, Phys. Lett. **158B**, 33 (1985).

[5]M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, Nucl. Phys. **B259**, 549 (1985).

[6]S. M. Barr, Phys. Rev. Lett. **55**, 2778 (1985).

[7]P. Binetruy, S. Dawson, I. Hinchliffe, and M. Sher, Lawrence Berkeley Laboratory Report No. LBL-20317 (to be published); G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, Phys. Rev. Lett. **56**, 432 (1986).

[8]V. Barger, N. Deshpande, R. J. N. Phillips, and K. Whisnant, Phys. Rev. D **33**, 1912 (1986).

[9]This result is stated by E. Witten, Nucl. Phys. **B268**, 79 (1986).

[10]V. S. Kaplunovsky, Phys. Rev. Lett. **55**, 1036 (1985).

[11]C. G. Callan, D. Friedan, E. J. Martinec, and M. J. Perry, Nucl. Phys. **B262**, 593 (1985).

[12]S.-T. Yau, in *Symposium on Anomalies, Geometry and Topology,* edited by W. A. Bardeen and A. R. White (World Scientific, Singapore, 1985), p. 395.

[13]Y. Hosotani, Phys. Lett. **129B**, 193 (1983).

[14]R. Holman and D. B. Reiss, Phys. Lett. **166B**, 305 (1986).

[15]M. B. Green, J. H. Schwarz, and P. C. West, Nucl. Phys. **B254**, 327 (1985); P. K. Townsend, Phys. Lett. **139B**, 283 (1984).

[16]See, e.g., H. S. M. Coxeter, *Regular Polytopes* (MacMillan, New York, 1963), 2nd ed.

[17]Nonsimply connected Calabi-Yau spaces with non-Abelian homotopy groups larger than S_3 are not yet known; S.-T. Yau, private communication.

[18]When supersymmetry is broken at very high energy near the Planck scale it is very difficult to avoid a cosmological constant which is over a hundred orders of magnitude too large; breaking nearer to the weak scale gives a cosmological constant which may be only a few tens of orders of magnitude too large and has the possible advantage of alleviating the gauge hierarchy problem.

[19]P. H. Frampton and S. L. Glashow, Phys. Lett. **131B**, 340 (1983), and **135B**, 515 (1984). See also T. W. Kephart and N. Nakagawa, Phys. Lett. **141B**, 329 (1984), and Phys. Rev. D **30**, 1978 (1984); S. Nandi, Phys. Lett. **142B**, 375 (1984).

[20]M. T. Grisaru, A. E. M. van de Ven, and D. Zanon, to be published.

[21]See L. Dixon, J. Harvey, C. Vafer, and E. Witten, Nucl. Phys. **B261**, 678 (1985), and to be published; Witten, Ref. 9.