String Propagation in a Tachyon Background

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The equation of motion of the tachyon field is obtained by requiring conformal invariance of the closed bosonic string propagator in a tachyon background. The cubic tachyon coupling is also obtained in this way. These equations are generalizations of the Kosterlitz-Thouless flow equations and involve nonperturbative contributions to the β functions.

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String theories[1] are good candidates for a unified theory of interactions including gravity. One of the most important problems in string theory is that of the determination of its vacuum structure.[2] In the past year, some progress has been made by the requirement of conformal invariance of the two-dimensional field theories describing string propagation in fixed backgrounds consisting of condensates of some of the string modes themselves.[2-4] For massless backgrounds this requirement yields precisely the equations of motion obtained from the low-energy limit of the string theory.[5]

In this Letter we shall study the propagation of a closed bosonic string in a background consisting of a condensate of its tachyonic mode. While our results corroborate the string-theory–σ-model connection, the route is rather subtle. In particular, β functions calculated by any truncation of the loop expansion give an entirely wrong result, and hide a crucial phase structure of the model. This is probably the reason why a treatment of the tachyon background problem is missing in the literature.

The world sheet action for a string with coordinates $X^\mu(\sigma_1,\sigma_2)$ in a background tachyon condensate $\Phi(x)$ is given in the orthonormal gauge by

$$S=\int d^2\sigma\{(1/4\pi\alpha')\partial_\alpha x^\mu\partial^\alpha x_\mu+g\Phi(x)\},\qquad(1)$$

where g is a coupling. In the following, we shall write $g=\lambda/a^2$, where λ is a dimensionless coupling and a is the lattice spacing on the world sheet. Furthermore, we shall write the model in terms of $y^\mu=x^\mu/(2\pi\alpha')^{1/2}$ to bring the kinetic piece into standard form. At the quantum level the relevant operator $\Phi(y)$ acquires an anomalous dimension. To compute this, we expand y^μ around a background $y_0^\mu(\sigma)$ in the standard fashion: $y^\mu(\sigma)=y_0^\mu(\sigma)+\xi^\mu(\sigma)$,

$$\Phi(y)=\Phi(y_0)+\sum_{n=1}^{\infty}\frac{1}{n!}\xi^{\mu_1}\cdots\xi^{\mu_n}\partial_{\mu_1}\cdots\partial_{\mu_n}\Phi(y_0),\qquad(2)$$

and the action becomes , for a $y_0(\sigma)$ satisfying classical equations of motion,

$$S[y_0+\xi]=S[y_0(\sigma)]+\int d^2\sigma\left\{\tfrac{1}{2}(\partial\xi)^2+m_0^2\xi^2+\frac{\lambda}{\alpha^2}\sum_{n=2}^{\infty}\frac{1}{n!}\xi^{\mu_1}\cdots\xi^{\mu_n}\partial_{\mu_1}\cdots\partial_{\mu_n}\phi(y_0)\right\}.\qquad(3)$$

In Eq. (3) we have introduced an infrared regulator mass m_0. Our final results are, of course, independent of m_0. An effective action for $y_0(\sigma_i)$ may be obtained by integrating out ξ. To linear order this may be obtained by summing over all tadpoles:

$$S_{\text{eff}}[y_0]=\int d^2\sigma\{\tfrac{1}{2}(\partial y_0)^2+\lambda(cm_0^2)\exp[-\delta\ln(cm_0^2\alpha^2)]\phi(y_0)\},\qquad(4)$$

where δ is the operator

$$\delta=(\nabla^2/8\pi+1),\qquad(5)$$

and $\nabla_\mu\equiv\partial/\partial y_0^\mu$. $c=\frac{1}{4}e^{2\gamma}$, where γ is Euler's constant. Thus, to this order, conformal invariance of the model given by Eq. (4) requires $\delta\phi(y)=0$. What this means is that the naive scaling dimension of the operator $\Phi(y_0)$ is canceled by its anomalous dimension—given by $(\nabla^2/4\pi)\Phi$. Reverting to the original string variables $x^\mu(\sigma)$, one has

$$[(\partial/\partial x^\mu)\,\partial/\partial x_\mu+4/\alpha']\phi=0,\qquad(6)$$

which is the mass-shell condition for the tachyon. The above facts are, in fact, extremely familiar: Summing over all tadpoles corresponds to normal ordering of the vertex operator which is a consistent vertex on mass shell.

If one tries to compute the effective action to higher orders in ϕ naively, one encounters a puzzle. Normal ordering seems to be all there is to the model: After normal ordering of the vertices all individual diagrams involving $\phi(y_0)$ insertions with internal ξ loops are finite. Since there are no new divergences of order ϕ^2, ϕ^3, etc., this implies that Eq. (6) is the only condition that is required for conformal invariance. From the string point of view this is clearly wrong: A low-energy expansion of the string-field-theory action have, e.g., ϕ^3 terms, since tachyons have a three-point coupling![6]

To clarify the situation, consider the simplified case in which $\Phi(y)$ has a single momentum mode in one of the directions and is of the form $\phi(y)=\cos\beta y$. The world-sheet model (1) is now simply a sine-Gordon model. The above scenario then amounts to saying that all infinities in the theory may be removed by normal ordering of the coupling[7]:

$$\lambda/a^2 \to \lambda(cm_0^2)\exp[(\beta^2/8\pi-1)\ln(cm_0^2a^2)]. \quad (7)$$

The sine-Gordon model may, however, be treated directly by renormalization-group (RG) methods[8,9] which shows that the parameter β as well as λ flows under an RG transformation. It is well known that this model is equivalent to a Coulomb gas—the nontrivial flows of β and λ are instrumental in giving rise to the Kosterlitz-Thouless[10] transition from a conducting to insulating phase.

The crucial point is that as $\beta^2 \to 8\pi$ the operator $\cos\beta y$ becomes a marginal operator as is clear from Eq. (5) and the theory ceases to be super-renormalizable. While normal ordering makes each individual graph finite in the $a\to 0$ limit, the sum of all graphs diverges when $\beta^2>8\pi$, and the naive picture is wrong. This accounts for the fact[7] that the vacuum of the normal-ordered theory becomes unstable for $\beta^2 \geq 8\pi$ and the theory ceases to make sense. $\beta^2=8\pi$ is the position of the Kosterlitz-Thouless transition and the apparent vacuum instability simply signifies a transition to a different vacuum. In fact, it turns out that one can define a well-behaved continuum theory for *all* β as a double expansion in λ and $\delta_\beta=\beta^2/8\pi-1$, in which the renormalized λ and δ_β are held fixed (rather than λ/a^2).[11] The various β functions have been calculated to be

$$\beta_\alpha=2\alpha\delta+A\alpha^3, \quad \beta_\delta=\tfrac{1}{32}\alpha^2+B\delta\alpha^2, \quad (8)$$

where A and B are nonuniversal coefficients. However, the quantity $C=2A+B$, is universal.[9] These β functions lead to an RG flow diagram similar to that obtained from Kosterlitz renormalization group.

Returning to the general case, it is now clear that the loop expansion lies. To get the correct physics one has to sum over all loops. This may be done in a way similar to that of Ref. 9. We shall present the main results. The general strategy will be to expand terms of the form $(a^2)^{-D}$ as $1-D\ln a^2+(D^2/2!)\ln^2 a^2$ in a region where D is small and to concentrate on the leading logarithmic divergence. This is done in the spirit of the ϵ expansion, i.e., it is expected that in the final expression all the logarithms from all the various diagrams are resummed by the renormalization group and conformal invariance is tantamount to the requirement that the β functions vanish. There is an implicit assumption that this series defines a (nonperturbatively) renormalizable (in the generalized sense of Friedan[12]) theory. D is, in general, a combination of δ_1, δ_2, and δ_{12}. Requiring that D acting on $\phi(y_1)\phi(y_2)\ldots$ be small has the following interpretation: If $L[\phi(y)]$ is the Lagrangean, concentrate on the Fourier mode, $\int d^py\, e^{ip_1y}L[\phi(y)]$ with $p_1^2\sim 0$, and similarly on $\int d^py\, e^{ip_2y}\delta L[\phi(y)]/\delta\phi$ with $p_2^2/8\pi\sim 1$ (this corresponds to the field, that is varied in the action, being an on-shell tachyon with $p^2/8\pi\sim +1$). Thus our derivation of the tachyon Lagrangean is valid only in a certain region in momentum space. We need not require, however, that each of the fields in the Lagrangean have a definite momentum dependence.

The $O(\phi^2)$ contribution to the effective action may be written as

$$S_{\text{eff}}^{(2)}=(\lambda cm_0)^2\int d\sigma_1\, d\sigma_2(cm_0^2a^2)^{-(\delta_1+\delta_2)}D(\sigma_{12})\phi(y(\sigma_1))\phi(y(\sigma_2)), \quad (9)$$

where

$$\delta_{1,2}=\nabla_{1,2}^2/8\pi+1, \quad \delta_{12}=-\nabla_1\nabla_2/8\pi+1, \quad (10)$$

$$D(\sigma_{12})=e^{\Delta_{12}}-\Delta_{12}-1, \quad (11)$$

$$\Delta_{12}=-(4\pi)^{-1}\ln[cm_0^2(\sigma_{12}^2+a^2)]\nabla_1^\mu\nabla_2^\mu(\sigma_{12}\equiv\sigma_2-\sigma_1). \quad (12)$$

The subscripts 1 and 2 mean that the corresponding operator acts only on $\phi(y(\sigma_1))$ or $\phi(y(\sigma_2))$, respectively. It is convenient to go over to momentum space in the world sheet:

$$S_{\text{eff}}^{(2)}=2(\lambda cm_0^2)\int d^2q\,(cm_0^2a^2)^{(\delta_1+\delta_2)}D(q)\phi_1(q)\phi_2(-q), \quad (13)$$

where

$$D(q)=\int d^2\sigma_{12}e^{iq\cdot\sigma_{12}}D(\sigma_{12}),\quad \phi(y(\sigma))=\int d^2q e^{iq\cdot\sigma}\phi(q). \tag{14}$$

The divergences in Eq. (13) come from two sources: (a) from the q independent part of the operator $D(q)$ and (b) from the q^2 part of $D(q)$. In the following we shall retain terms up to $O(\delta\phi^2)$. The divergences of the first kind may be easily extracted for backgrounds with

$$(\delta_1+\delta_2-2\delta_{12}+1)\phi(y(\sigma_1))\phi(y(\sigma_2))$$

small which corresponds to $(p_1+p_2)^2/8\pi\sim +1$. This infinite piece contributes a term to the action

$$-\tfrac{1}{2}\lambda^2(cm_0^2)\int d\sigma\left[\left\{\frac{\delta_1+\delta_2}{2\delta_{12}-1}+1\right\}\phi(y(\sigma_1))\phi(y(\sigma_2))\right]\ln(cm_0^2a^2)\Big|_{\sigma_1=\sigma_2=\sigma}. \tag{15}$$

If both $\phi(y_1)$ and $\phi(y_2)$ are on shell we can neglect $(\delta_1+\delta_2)/(2\delta_{12}-1)$, and obtain the cubic term calculated in string theory[6] (see below). This combines with Eq. (4) above to give a coupling constant renormalization. The divergence from the q^2 part may be extracted easily in the region where $(\delta_1+\delta_2-2\delta_{12})\phi(y_1)\phi(y_2)$ is small which means $(p_1+p_2)^2\sim 0$. This is

$$-\tfrac{1}{2}\lambda^2\pi\{\int d^2q\, q^2(1+2\delta_{12})\phi_1(q)\phi_2(q)\}\ln(cm_0^2a^2), \tag{16}$$

which reads, in position space,

$$-\tfrac{1}{2}\lambda^2\pi\int_\sigma d^2\sigma\{\partial_\alpha y_0^\mu\,\partial^\alpha y_0^\nu\,[\partial_\mu\phi\,\partial_\nu\phi]+2\delta_{12}\,\partial_\mu\phi_1\,\partial_\nu\phi_2\}\ln cm_0^2a^2, \tag{17}$$

which is a counterterm to the kinetic piece of the effective action. The $\delta_{12}\phi_1\phi_2$ piece can be neglected if we are only interested in on-shell tachyons. The coefficient of this piece is not universal[9] so this is just as well.

If the string is coupled to a background gravitational field $g_{\mu\nu}$ there is an additional contribution to the kinetic piece,[3,4,12,13]

$$-(8\pi)^{-1}\int d^2\sigma\, R_{\mu\nu}\,\partial_\alpha y_0^\mu\,\partial^\alpha y_0^\nu\,\ln(cm_0^2a^2). \tag{18}$$

Collecting the various divergent, lattice-spacing-dependent terms appearing in the effective action from Eqs. (4), (15), (16), and (18), one has, to this order,

$$S_{\text{eff}}^{(\text{div})}(y_0)=-\Bigg\{(8\pi)^{-1}\int d^2\sigma\,\partial_\alpha y_0^\mu\,\partial^\alpha y_0^\nu\,[R_{\mu\nu}+4\lambda^2\pi^2(\partial_\mu\phi\,\partial_\nu\phi)+2\delta_{12}\,\partial_\mu\phi_1\,\partial_\nu\phi_2]$$
$$-\lambda(cm_0^2)\int d^2\sigma\left[\left\{\frac{\nabla^2}{8\pi}+1\right\}\phi+\frac{\lambda}{2}\phi^2+\frac{\lambda}{2}\frac{\delta_1+\delta_2}{2\delta_{12}-1}\phi(y_1)\phi(y_2)\right]\Bigg\}\ln(cm_0^2a^2). \tag{19}$$

Thus, for conformal invariance to hold one must have

$$R_{\mu\nu}=-4\lambda^2\pi^2[\partial_\mu\phi\,\partial_\nu\phi+O(\delta\phi^2)], \tag{20a}$$

$$(\nabla^2/8\pi+1)\phi+\tfrac{1}{2}\lambda\phi^2=O(\phi^3). \tag{20b}$$

Equation (20b) may be seen to be the equation of motion for the tachyon field—clearly exhibiting a ϕ^3 interaction.

Further insight may be gained by our treating of the conformal mode[14] of the world-sheet metric properly. In the conformal gauge the world-sheet metric is $\gamma_{\alpha\beta}(\sigma)=e^{2\rho(\sigma)}\eta_{\alpha\beta}$. The term representing the interaction with the tachyonic field has an extra factor of $e^{2\rho(\sigma)}$. The effect of retaining the $\rho(\sigma)$ is equivalent to replacement of a^2 by $a^2e^{-2\rho(\sigma)}$ in the coincident propagators for the ξ fields. In this framework, the condition for conformal invariance is simply the requirement that the effective action is independent of $\rho(\sigma)$. One now has an extra divergent piece in the effective action which is proportional to the world-sheet curvature $\sqrt{\gamma}R^{(2)}\approx 2\partial_\alpha\,\partial^\alpha\rho$, i.e., a renormalization of a possible dilaton coupling[3,15] to the string. Noting that there is a contribution to this term at the zero loop[14] level—equal to $(D-26)/24\pi$—one has [to $O(\delta\phi^2)$]

$$S_{\text{eff}}^{\text{dil}}=\int d^2\sigma\left\{\frac{D-26}{24\pi}+\frac{1}{16\pi^2}\left(-R+4\pi^2\lambda_0^2\left[-\frac{(\nabla\phi)^2}{8\pi}+\phi^2\right]\right)\right\}2\partial_\alpha\,\partial^\alpha\rho(\sigma)[\ln(cm_0^2a^2)-2\rho(\sigma)].$$

Variation with respect to $\rho(\sigma)$ leads to

$$\frac{D-26}{24\pi}+\frac{1}{16\pi^2}\left[-R+4\pi^2\lambda_0^2\left(-\frac{(\nabla\phi)^2}{8\pi}+\phi^2\right)\right]=0. \tag{20c}$$

It may be easily checked that linear combinations of Eqs. (20a)-(20c) are simply equations of motion following from an action containing the Einstein term for the metric and a standard action of a tachyonic scalar field with a ϕ^3 interaction, and a cosmological constant proportional to $26-D$.

One can now look for solutions of these equations. Other than the trivial solution $\phi=R=0$, there are solutions of the form $\phi=\text{const}=\phi_0$:

$$(D-26)/24\pi-V(\phi_0)=0, \quad V'(\phi_0)=0. \tag{21}$$

Since $V(\phi_0)<0$, this requires $D<26$ (in contrast to the situation studied by Nemeschansky and Yankielowicz and by Jain, Shanker, and Wadia[16]). Although this is outside the realm of validity of the derivation of these equations it suggests that the idea[17] that some ten-dimensional string theories correspond to stable vacua of the bosonic string might have some dynamical basis. It is also possible that on including the dilaton field one might find stable tachyon-free vacua without altering the critical dimension.[3,18]

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