Lower Critical Dimension of Metallic Vector Spin-Glasses

A. J. Bray and M. A. Moore
Department of Theoretical Physics, The University, Manchester M13 9PL, United Kingdom

and

A. P. Young
Department of Physics, University of California, Santa Cruz, California 95064
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We argue that when a short-range spin-glass system is *below* its lower critical dimension d_l, which seems to be the case for isotropic vector spins in three dimensions, then the corresponding Ruderman-Kittel-Kasuya-Yosida (RKKY) system is in a different universality class and *at* its lower critical dimension. For dimensions greater than d_l, the RKKY and short-range systems have the same critical behavior. This appears to apply to Ising spins, and to anisotropic vector-spin models for which we discuss the dependence of T_c on anisotropy.

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The experimental evidence for a phase transition in spin-glasses is now fairly convincing.[1] In addition, the numerical studies of *Ising* spin-glasses also strongly suggest that a phase transition takes place in three-dimensional systems.[2] However, very similar numerical studies of *XY* and *Heisenberg* spin-glasses indicate that there is *no* transition in short-range vector systems in three dimensions,[3–5] and that three dimensions is *below* the lower critical dimensions, d_l, of short-range systems. Short-range interactions are appropriate for insulating spin-glasses such as $Eu_xSr_{1-x}S$. Since most of the best-studied spin-glasses are Heisenberg-type systems in which the anisotropy (whether it be Dzyaloshinskii-Moriya[6] or dipolar) is very small (e.g., *Cu*Mn, *Ag*Mn, $Eu_xSr_{1-x}S$), it is not immediately obvious how to reconcile the experimental work with the numerical results on vector spin-glasses. In this paper we demonstrate that for the Ruderman-Kittel-Kasuya-Yosida (RKKY) coupling pertinent to metallic spin-glasses[6] the spin interactions are sufficiently long-ranged that the *isotropic vector spin-glass in three dimensions is at its lower critical dimension, d^*, and so lies in a different universality class* from the case of short-range interactions. For the nonmetallic and probably also the metallic case, the dependence of T_c on the anisotropy D is such that as $D \to 0$, $T_c \to 0$, but so slowly that the experimentally observed values of T_c are compatible with our expressions for realistic values of the anisotropy. We also show that the critical exponents associated with the Ising-type transition in anisotropic systems are always those of the short-range Ising spin-glass.

An appropriate Hamiltonian for metallic spin-glasses is

$$\mathscr{H} = -\tfrac{1}{2}\textstyle\sum_{i,j} J_{ij}\xi_i\xi_j \mathbf{S}_i\cdot\mathbf{S}_j, \tag{1}$$

where the spin-spin coupling is of the RKKY form

$$J_{ij} = J\cos(2k_F R_{ij})/R_{ij}^3, \quad R_{ij} = |\mathbf{r}_i - \mathbf{r}_j|; \tag{2}$$

k_F is the Fermi wave vector of the conduction electrons, $\mathbf{r}_i$ is the position of the spin $\mathbf{S}_i$, which will be regarded here as a classical Heisenberg spin. The ξ_i are independent, quenched, random variables equal to 1 or 0 with probabilities c and $1-c$, respectively, with c the atomic concentration of the magnetic ions carrying the spin.[7] We shall study here also the general case of long-range interactions in a variety of dimensionalities d and generalize Eq. (2) to

$$J_{ij} = J\cos(2k_F R_{ij})/R_{ij}^{(d+\sigma)/2}. \tag{3}$$

Equation (2) is the special case of $d=3=\sigma$. It has been shown elsewhere[7,8] that after use of the well-known replica method to carry out the average over the site disorder, the following Landau-Ginzburg-Wilson Hamiltonian results:

$$-\beta\mathscr{H} = -\frac{\beta^2 J^2}{4}\sum_{\substack{\alpha,\beta\\a,b}}\sum_{k}(r + sk^2 + lk^\sigma)Q_{ab}^{\alpha\beta}(\mathbf{k})Q_{ab}^{\alpha\beta}(-\mathbf{k}) + \frac{w}{6}\sum_{\substack{\alpha,\beta,\gamma\\a,b,c}}\sum_{k_1}\sum_{k_2}Q_{ab}^{\alpha\beta}(\mathbf{k}_1)Q_{bc}^{\beta\gamma}(\mathbf{k}_2)Q_{ca}^{\gamma\alpha}(-\mathbf{k}_1-\mathbf{k}_2) + O(Q^4). \tag{4}$$

α, β, and γ are replica indices and run from 1 to n (which must be set equal to zero at the end of the calculation); a, b, and c label spin components and run from 1 to m; $m=3$ for Heisenberg spins. The term $sk^2Q_{ab}^{\alpha\beta}(\mathbf{k}) \times Q_{ab}^{\alpha\beta}(-\mathbf{k})$ is present both for long-range (LR) and short-range (SR) interactions whereas the term $lk^\sigma Q_{ab}^{\alpha\beta}(\mathbf{k})Q_{ab}^{\alpha\beta}(-\mathbf{k})$ only occurs with long-range forces.

The Hamiltonian of Eq. (4) has been extensively studied already[8–10] so we will just outline the results here. The upper critical dimension for $\sigma > 2$ is 6 (the conventional short-range result).[11] For $\sigma < 2$, the upper critical dimension is 3σ. Where T_c is finite the boundary between LR and SR regions is given[8, 10] by the condition $\sigma = 2 - \eta_{SR}$, where η_{SR} is the exponent η associated with the short-range fixed point and is negative in $6-\epsilon$ dimensions.[11] To check that this is true even when η_{SR} is negative we have studied the correlation function at $T = T_c$, which, in the non-mean-field LR region ($d < 3\sigma$), may be written

$$G(k) \equiv \langle Q_{ab}^{\alpha\beta}(\mathbf{k}) Q_{ab}^{\alpha\beta}(-\mathbf{k}) \rangle = \frac{1}{k^{\sigma}} f(\alpha_+ k^{-\lambda_+}, \alpha_- k^{-\lambda_-}), \tag{5}$$

where the correction-to-scaling exponents λ_+ and λ_- are negative but with the larger, λ_+, changing sign when $\sigma = 2 - \eta_{SR}$.[10] α_1 and α_2 are the amplitudes of the eigenvectors corresponding to λ_+ and λ_-, and are linear combinations of $s - s^*$ and $w - w^*$, where s^* and w^* are the fixed-point values of s and w. Using the equations of Chang and Sak,[10] we have shown that the crossover function f remains finite as $k \to 0$ and hence that the critical exponent η_{LR} is $2-\sigma$; in other words, $G(k) \approx k^{-\sigma}$ as $k \to 0$ even when $\sigma > 2$ (but $\sigma < 2 - \eta_{SR}$). Notice that the bare propagator $g(k)$ at $r = 0$ takes the form $g^{-1}(k) = \beta^2 J^2 (sk^2 + lk^{\sigma})$ and is dominated by the nonscaling term sk^2 as $k \to 0$ when $\sigma > 2$. In the LR *mean-field* region ($d > 3\sigma$, $\sigma < 2$) the exponent η_{LR} will be $2 - \sigma$ but in the SR mean-field region ($d > 6, \sigma > 2$) η_{SR} will be zero. The boundary between the two mean-field regions is at $\sigma = 2$. In the LR mean-field region, the exponents are $\nu = 1/\sigma$, $\gamma = 1$, $\beta - 1$, etc., but in the SR mean-field region, $\nu = \frac{1}{2}$, $\gamma = 1$, $\beta = 1$.

We now turn to the question of the lower critical dimension. The numerical work of Refs. 3 and 4 suggests that its value for isotropic vector spin-glasses with short-range forces is close to 4, and this is the value which was adopted in the preparation of Fig. 1. Unfortunately, we know of no convincing analytic argument for determining the lower critical dimension for short-range forces. However, we shall now argue that the lower critical dimension can be obtained analytically where long-range forces dominate.

To proceed, we discuss the case of a zero-temperature transition. The exponent η, which governs the decay of spatial correlations, is defined by $G(\mathbf{r}) = r^{-d+2-\eta} f(r/\xi)$, as $r \to \infty$ with r/ξ fixed. Since at $T = 0$ the correlations cease to decay in space (provided that the ground state is nondegenerate), it follows that

$$2 - \eta = d. \tag{6}$$

This is a general result for η at a zero-temperature transition which applied both for long-range and short-range forces. The lower critical dimension d^* for long-range forces is now straightforwardly obtained, provided that one can assume that T_c approaches zero

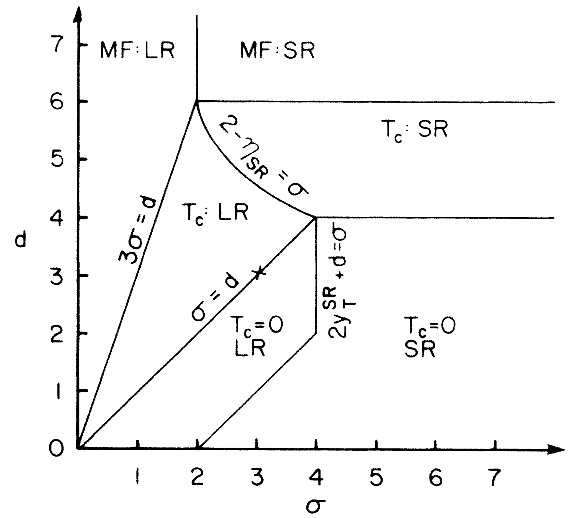

FIG. 1. Phases of a vector spin-glass in the d-σ plane. MF denotes a mean-field transition. T_c:LR stands for a finite-temperature phase with exponents in the long-range universality class, $T_c = 0$ LR denotes the region in which a zero-temperature phase transition occurs in the long-range universality class, etc. The cross marks the physical point $d = \sigma = 3$ pertinent to the RKKY interaction. The precise shape of the boundary between the $T_c = 0$ SR and $T_c = 0$ LR regions depends on the form assumed for y_T^{SR}. Here we took $y_T^{SR} = (4-d)/2$ for $4 \geq d \geq 2$ and $y_T^{SR} = 1$ for $d \leq 2$.

as $d \to d^*$. Then the exponents associated with the transition at T_c should smoothly join onto those of the zero-temperature region. Now $\eta_{LR} = 2 - \sigma$ at the finite-temperature transition, and η_{LR} at zero-temperature transition is $2 - d$. These join smoothly only if

$$d^* = \sigma, \tag{7}$$

which shows that RKKY systems are at their lower critical dimension if the long-range fixed point is stable.

Further support for this result comes from consideration of y_T^{LR}, the zero-temperature exponent which determines how the long-range forces between block spins of linear dimension b scale with b:

$$J'_{LR}(b) = b^{-y_T^{LR}} J_{LR}$$

[from Eq. (4), one can identify $J^2 l$ as J_{LR}^2]. The physical significance of y_T^{LR} is that it determines the behavior of the correlation length as $T \to 0$ for LR spin-glasses for $d < d^* = \sigma$, i.e.,

$$\xi \approx (J/T)^{\nu_{LR}} \tag{8}$$

as $T \to 0$, with $\nu^{LR} = 1/y_T^{LR}$. If $d < d^*$, then $y_T^{LR} > 0$, but if $y_T^{LR} < 0$ there is a phase transition at nonzero temperature, since the effective coupling is an increasing function of the length scale b. The dimension at which $y_T^{LR} = 0$ is d^*, the lower critical dimension for long-range interactions. Analogous results hold for short-range interactions. Under the usual type of renormalization-group transformation $k=k^*/b$, $Q_{ab}^{\alpha\beta}(\mathbf{k}) \rightarrow b^{1-\eta/2}Q_{ab}^{\alpha\beta}(\mathbf{k}')$, one deduces that

$$J_{\rm LR}'^2(b) = b^{2-\eta-\sigma}J_{\rm LR}^2 \qquad (9)$$

since the coefficient $J_{\rm LR}^2$ is *only* changed by these rescaling factors, as at the finite-temperature fixed point. Hence

$$1/\nu^{\rm LR} = y_T^{\rm LR} = (\sigma - d)/2, \qquad (10)$$

with use of Eq. (6) for η. Observe that $y_T^{\rm LR}=0$ at $d=d^*=\sigma$, as expected.

We now determine the boundary in the d-σ plane which separates the long-range zero-temperature behavior specified by $y_T^{\rm LR}$ from the short-range zero-temperature behavior specified by $y_T^{\rm SR}$. The scaling expression of Eq. (8) for $J_{\rm LR}$ holds in both regions, but the short-range forces will dominate when $y_T^{\rm SR} < y_T^{\rm LR}$ since the long-range forces will be negligible compared to the short-range forces on sufficiently large length scales. Thus the boundary between the two types of behavior is at $y_T^{\rm SR} = y_T^{\rm LR}$. For XY spins, $m=2$, it was suggested in Ref. 4 that $y_T^{\rm SR} = (4-d)/2$, if $4 \geq d \geq 2$, but that $y_T^{\rm SR}=1$ for $d<2$. It is unclear if this holds for general values of $m>1$, but for convenience they have been used in Fig. 1. We now have all the necessary information to obtain the phase diagram in Fig. 1. Note that *the "physical point" $d=\sigma=3$ lies on the boundary between long-range zero-temperature behavior and long-range finite-temperature behavior.* This observation depends on the fact that the lower critical dimension for short-range isotropic vector spin-glasses is greater than 3.[3-5] An analogous figure can be drawn for Ising spin-glasses (and indeed for any system).[12] However, since the lower critical dimension for short-range Ising systems is thought to be less than 3,[2] the point $d=\sigma=3$ lies in the short-range, finite-T_c region of the diagram. Thus, Ising spin-glasses with RKKY interactions lie in the same universality class as short-range Ising systems. By contrast, for isotropic Heisenberg models, the RKKY and short-range systems lie in different universality classes and have different lower critical dimensions.

We now discuss the effect of anisotropy which, following Ref. 4, we will assume can be written as

$$\mathscr{H}_D = -D\sum_{\substack{i,j\\a,b}} K_{ij}^{ab}S_i^aS_j^b, \text{ with } \sum_{i,a}K_{ii}^{aa}=0. \qquad (11)$$

Both Dzyaloshinskii-Moriya anisotropy and dipolar coupling can be put into this form. The overall strength of the anisotropy is specified by D. The transition will now be Ising-type[7] so that RKKY and short-range systems will be in the same universality class, but they will have a different dependence of T_c on anisotropy as we shall now show.

We must first extend Eq. (9) to finite temperatures when the rescaling of $Q_{ab}^{\alpha\beta}(\mathbf{k})$ is now no longer simply $b^{1-\eta/2}$. Setting $b=e^l$, we shall assume that Eq. (9) generalizes for $T \ll J_{\rm LR}(l)$ to

$$dJ_{\rm LR}(l)/dl = -y_T^{\rm LR}J_{\rm LR}(l) - AT^2/J_{\rm LR}(l), \qquad (12)$$

where A is a constant expected to be of order unity. A $T^2/J_{\rm LR}(l)$ term [rather than $J_{\rm LR}(T/J_{\rm LR})^\theta$, $\theta \neq 2$] is written in for the sake of definiteness. McMillan[13] argued that $\theta=2$ was a consequence of the symmetry $P(J_{ij}) = P(-J_{ij})$ expected for the zero-temperature bond distribution at the fixed point. [The value of θ only affects the power of the logarithm in Eq. (13) below.] For $d=\sigma=3$, $y_T^{\rm LR}=0$. The analogous rescaling equation for D is $dD(l)/dl = y_D D(l)$ where y_D was argued to equal $d/2$.[4] Solving for $J(l)$ and $D(l)$ with $y_T^{\rm LR}=0$, and estimating T_c from $J_{\rm LR}(l^*) = D(l^*) \approx T_c$, we find

$$T_c = \frac{J}{[1+(2A/y_D)\ln(T_c/D)]^{1/2}}$$
$$\approx \frac{J}{[\ln(J/D)]^{1/2}} \text{ for } D \ll J. \qquad (13)$$

Equation (13) shows that as $D\rightarrow 0$, $T_c \rightarrow 0$, but that for realistic values of $J/D(\approx 100)$, the transition temperature $T_c \approx J$.

At the present time we do not believe that it is possible to refine our estimate of T_c to the point where meaningful comparisons with experimental data such as that in the work of Vier and Schultz[14] can be made. This is partly because the spin interactions in real systems are of greater complexity[15] than envisaged in the RKKY expression of Eq. (2). Furthermore, at large separations, the spin interaction may be influenced by the mean-field path, λ. If this gives rise to an exponential factor $\exp(-R/\lambda)$, the interaction J_{ij} will technically be short range. If $\lambda/a \gg T_c/D$, where a is the average spacing between spins, the role played by λ is unimportant and Eq. (13) for T_c will still apply. However, in the opposite limit, the spin interactions are effectively short-ranged and the formula given for T_c in Ref. 4 will apply, i.e., $T_c/J \approx J(D/J)^x$ with $x\approx \frac{1}{4}$, which again implies a weak dependence of T_c on anisotropy, but not as insensitive as in the long-range limit. This last result should also hold for insulating spin-glasses.

It is also conceivable that isotropic RKKY systems might have a finite T_c, analogous to the "marginal" transition discussed by Anderson and Yuval[16] for a long-range one-dimensional Ising ferromagnet, but it is more likely that $T_c \rightarrow 0$ as $D\rightarrow 0$. In addition, we note that Dzyaloshinskii and Volovik[17] have obtained the result $d_l=4$ for short-range isotropic vector spin-glasses and $d^*=3$ for RKKY systems by assuming that spin-glasses can be represented by a gauge theory.

The validity of this assumption is hard to assess and is not made in our work, which proceeds along completely different lines.

To conclude, all three-dimensional experimental spin-glass systems should have a phase transition in the universality class of the short-range Ising spin-glass model. The dependence of T_c on anisotropy is only logarithmic for metallic spin-glasses with RKKY interactions. An experimental test of this prediction would be welcome, but such a check will be complicated by mean-free-path effects and lack of detailed knowledge of the precise form of the spin-spin interaction.

After this work was submitted, we became aware of Monte Carlo simulations[18] of an isotropic RKKY Heisenberg spin-glass model which seem to indicate a lower critical dimension greater than three. However, the sizes studied may be too small to see the marginal behavior predicted here. Also, our Eq. (10) has been independently derived.[19]

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