Empirical Evidence for an SO(7) Fermion Dynamical Symmetry in Nuclei

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Empirical evidence for an SO(8)$\supset$SO(7) fermion dynamical symmetry in the Pd–Ru region is presented. This symmetry has the remarkable property that it can describe structural *variations* in a transition region between vibrator and γ-soft asymmetric rotor.

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The last several years have witnessed a rebirth of interest in symmetry ideas in nuclei. The primary emphasis has been on boson dynamical symmetries related to the interacting-boson-approximation (IBA) model.[1] These symmetries provide elegant idealized limiting cases as well as benchmarks for the treatment of nearby nuclei. Important as these may be, they as yet present an incomplete picture: Without a microscopic basis, the questions of why, and where in the nuclear chart, the symmetries exist remain essentially unanswerable at the phenomenological level.

This situation accounts for the intense effort[1,2] recently in the derivation of a microscopic foundation for the IBA. However, the repeated occurrence of the U(6) symmetries of the IBA suggests that they do not appear accidentally but rather reflect a fundamental behavior which should arise from the general shell structure and residual interactions in nuclei. It suggests that, perhaps, they stem from more basic *fermion* dynamical symmetries.

In recent years, the development of symmetry approaches to the shell model has indeed begun to bear fruit.[3-6] The approach of particular interest here is the Ginocchio model[5] which is entirely fermionic in origin and can have either SO(8) or Sp(6) symmetry. These parent groups have chain decompositions leading to subgroups displaying fermion dynamical symmetries (hereinafter called symmetries for short) SO(5) $\otimes$ SU(2), SO(6), and SO(7) for the former, and SO(3) $\otimes$ SU(2) and SU(3) for the latter. Of these five symmetries, SU(3) and SO(6) correspond to the IBA symmetries SU(3) and O(6) while, because of Pauli factors, the other three do not have exact counterparts [although their energy spectra are similar to U(5)]. While the empirical manifestation of boson symmetries in nuclei could, in principle, be taken as indirect evidence for the corresponding fermion symmetries, a more convincing result would be to identify a fermion symmetry exhibiting properties *different* from any of the boson symmetries, and to establish its empirical existence.

The SO(8)$\supset$SO(7) fermion symmetry cited above has exactly these properties, and it is the purpose of this Letter to show that it is indeed manifested empirically, in the Pd and Ru nuclei. Furthermore, it will be shown that it has particular relevance to certain classes of *transitional* nuclei.

The fermion model leading to this symmetry has been extensively[5,6] discussed. An essential step involves a rewriting of the fermion angular momenta, j, in terms of pseudo orbital and spin angular momenta k and i. Thus, $\mathbf{j}=\mathbf{k}+\mathbf{i}$. For particular choices of k or i, this decomposition describes fermion angular momenta starting from $j=\frac{1}{2}$ and increasing in unit increments up to some maximum value. For example, if $i=\frac{3}{2}$ then $k=2$ yields $j=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$. The advantage of this k-i pseudo-angular-momentum basis is that, with it, it is possible to construct a class of Hamiltonians in which fermion pairs coupled to 0 and 2 (S and D) are decoupled from the remainder of the shell model space. For $i=\frac{3}{2}$ (or $k=1$), these pair basis states span one representation of SO(8) [or Sp(6)] and can be mapped onto the states of the IBA-1 model.

Since the SO(8) or Sp(6) symmetries originate from the fundamental constituents of nuclei, nucleons, their link to the physical shell structure must be established before this model can describe real systems. This link has recently been made and an extended model, called the fermion-dynamical-symmetry model, developed.[6] This is an important step since, as a result of the appearance of the unique-parity orbit, the angular momenta in any given major shell in heavy nuclei involve a gap of two units at the upper end: For example, the 50–82 shell has the orbits $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$, and $h_{11/2}$. However, if we assume the dominance of S and D nucleon pairs in the low-energy region, there is a unique way[6] to reclassify the normal-parity states with the k-i basis so that either SO(8) or Sp(6) is associated with each major physical shell. The unique-parity orbital is thus decoupled[6] (see also Ref. 4) and forms its own symmetry, the excited states of which only occur when pairs of nucleons in this orbit are recoupled to nonzero angular momentum. Thus, the orbit does not directly affect the low-energy, low-spin region except that its occupation consumes a certain number of valence nucleons.

The occurrence of the SO(7) symmetry in the Hamiltonian can be understood as follows: When the fermion residual interaction is dominated by monopole pairing only, it leads to the SO(5) symmetry; when the quadrupole interaction dominates, it leads to SO(6); and when monopole and quadrupole terms are of *equal* strength, an SO(7) symmetry is generated. The corresponding IBA Hamiltonian has boson energy and quadrupole terms and is *intermediate between U(5) and O(6):* SO(7) is a symmetry *not* found in the IBA. Moreover, since these two terms in the Hamiltonian have different dependences on valence nucleon number, the SO(7) symmetry describes a structure which is *mass dependent.*

The particular pseudo-angular-momentum content of the SO(8) group, with $i=\frac{3}{2}$, automatically suggests several mass regions where SO(7) may be realized. One is the 28–50 shell which has normal-parity fermion orbits $p_{1/2}$, $p_{3/2}$, and $f_{5/2}$, and another is the 50–82 shell just described. A third region may be provided by the upper end of the 82–126 shell where the fermion orbits $p_{1/2}$, $p_{3/2}$, and $f_{5/2}$ are widely separated from the $f_{7/2}$ and $h_{9/2}$ orbits of the same parity. All in all, the best region seems to be the Pd and Ru isotopes from $A \approx 100$ to 110. Indeed, this is consistent with a recent IBA treatment[7] of these nuclei as intermediate between U(5) and O(6).

The SO(7) chain decomposition is $SO(8) \supset SO(7) \supset SO(5) \supset SO(3)$ and the eigenvalue expression for the excitation energies is given by[5]

$$E(N_1,\bar{\kappa},\tau,J) = -G_0\bar{\kappa}(\Omega-2N_1+\bar{\kappa}+5) + b_3\tau(\tau+3)+\tfrac{1}{5}(b_1-b_3)J(J+1). \quad (1)$$

In Eq. (1), $\Omega=\Omega_\pi+\Omega_\nu$ is the shell pair degeneracy of the normal-parity proton and neutron orbits (here $\Omega=16$) and N_1 is the effective fermion valence pair number defined by $N_1=\alpha N$, where N is half the total number of valence nucleons and α represents the fraction of these occupying the normal-parity levels. When $N_1 > \Omega/2$, the hole count $\Omega - N_1$ is used, thus yielding symmetry about the half-filled shell. The quantity $\bar{\kappa}$ is analogous to a phonon number while τ is similar to the O(5) boson quantum number of the IBA. The allowed values of $\bar{\kappa}$ and τ are given by $\bar{\kappa}=0,1,\ldots,N$ and $\tau=\bar{\kappa},\bar{\kappa}-2,\bar{\kappa}-3,\ldots,0$ or 1.

The energy-level spectrum of Eq. (1) consists of closely knit multiplets whose energies vary approximately with $\bar{\kappa}$ and whose spacing is controlled by G_0. Splittings within a multiplet arise from the b_3 and b_1-b_3 terms which originate from odd-multipole terms in the Hamiltonian. Extensive experience with shell-model calculations suggests that the first term, which stems from monopole and quadrupole terms, should dominate. Incidentally, this situation is in striking contrast to the (phenomenological) IBA-1 model where one has no *a priori* expectation of the relative importance of different terms.

The most interesting feature of Eq. (1) is the appearance of the fermion pair number N_1 on account of which the energies will *decrease* with increasing N_1, reaching a minimum at midshell. This inherent systematics is a reflection of the evolving structure of the wave functions. This same feature is evident directly in the intrinsic state[8] for SO(7) which is that of a weakly deformed, γ-soft vibrator whose effective deformation $\beta_{\rm int}$ [where $\beta_{\rm nucleus}\approx(2N/A)\beta_{\rm int}$], shown in Fig. 1, varies between 0 and 1 appropriate to the boson symmetries U(5) and O(6). As a consequence, for small b_1,b_3, observables such as the energy ratio $E_{4_1^+}/E_{2_1^+}$ increase (see Fig. 1) with N in the interval from 2.0 (vibrator) towards 2.5 (γ soft).

In attempting to identify empirical sequences of nuclei displaying the SO(7) symmetry, it is essential to distinguish them from phase-transitional sequences towards a deformed rotor in which the energy levels will also systematically decrease. Thus, for example, the

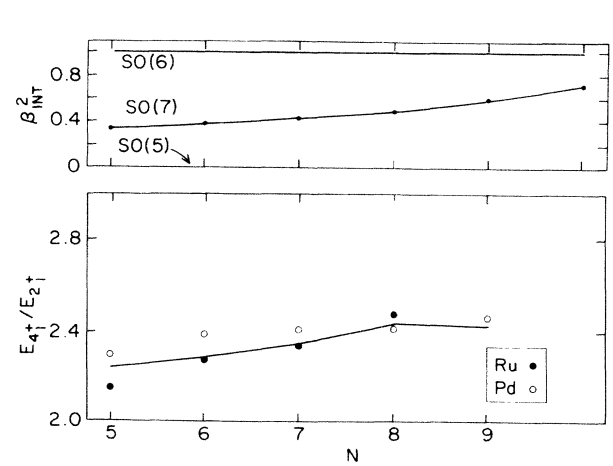

FIG. 1. β values for the intrinsic state (top). Empirical (Ref. 9) and calculated $E_{4_1^+}/E_{2_1^+}$ ratios (bottom); the Ru and Pd curves are nearly identical.

Ba–Gd nuclei near $A = 150$ are merely a prelude to the deformed rare-earth region. However, in such a region the $E_{4_1^+}/E_{2_1^+}$ ratio will increase towards 3.33. Also, a $B(E2)$ ratio such as $B(E2{:}2_2^+ \rightarrow 0_1^+)/B(E2{:}2_2^+ \rightarrow 2_1^+)$ approaches the Alaga-rule value (0.7) while the SO(7) [and O(6) and U(5)] value vanishes. In the $^{104-110}$Pd and $^{98-104}$Ru nuclei considered here as reflecting the SO(7) symmetry, this ratio is $\leqslant 0.05$. Similarly, the ratio $B(E2{:}3_1^+ \rightarrow 2_1^+)/B(E2{:}3_1^+ \rightarrow 4_1^+)$ approaches 2.5 in a transition to the rotor, but vanishes in SO(7): It is < 0.1 (0.2) in Pd (Ru).

The application of the SO(7) symmetry to Pd and Ru is shown in Figs. 1–3. A comment about the much-discussed[7,10] intruder states is perhaps relevant. Recent work[11] suggests that they are not of concern, for the levels shown here, except possibly for the 0_2^+ states in $^{106-108}$Ru. In any case, these latter nuclei will be seen to deviate from the SO(7) picture in other ways. The parameter values adopted for Ru (Pd) were $-G_0 = 45$ (47.5) keV, $\alpha = 0.96$ (0.91), $b_3 = 5.3$ keV, and $\frac{1}{5}(b_1 - b_3) = 7.2$ keV (the latter two for both Ru and Pd). Note that, as expected, $b_3, \frac{1}{5}(b_1 - b_3) << G_0$.

The agreement with experiment in both Figs. 1 and 2 is quite good. The $E_{4_1^+}/E_{2_1^+}$ ratio is closely reproduced as are the level patterns and systematics for each element. In this context, it is important to emphasize that the parameters, nearly identical in Ru and Pd, were held *constant* for each set of isotopes. The key distinguishing feature of the data, namely the smooth *decrease* in energies with mass, is excellently reproduced by the SO(7) scheme as an *automatic* result, and as the most characteristic signature, of the inherent structural evolution with N. Finally, even the slope changes predicted for ^{106}Ru and ^{112}Pd, which occur when $N_1 > \Omega/2$, are partly reflected in the data. Some deviations, of course, appear. The triplet levels, especially 0_2^+, are predicted slightly lower than observed, and the yrast levels in the heaviest isotopes shown drop well below the predictions. This latter tendency and the rise in $E_{0_2^+}$ signal a deviation from SO(7) towards a deformed character. In Ru the same evolution is indicated by the ratio $B(E2{:}2_2^+ \rightarrow 0_1^+)/B(E2{:}2_2^+ \rightarrow 2_1^+)$ which reaches 0.1 in ^{108}Ru. Moreover, the $E_{4_1^+}/E_{2_1^+}$ ratio in ^{106,108}Ru is 2.65 and 2.75 whereas for $^{104-110}$Pd and $^{98-104}$Ru it lies in the narrow interval from 2.14 to 2.48.

Figure 3 shows the $B(E2{:}2_1^+ \rightarrow 0_1^+)$ values and the ratio $R_0 = B(E2{:}0_2^+ \rightarrow 2_1^+)/B(E2{:}2_1^+ \rightarrow 0_1^+)$. The SO(7) predictions here involve no new parameters (except for a normalization of the former quantity at

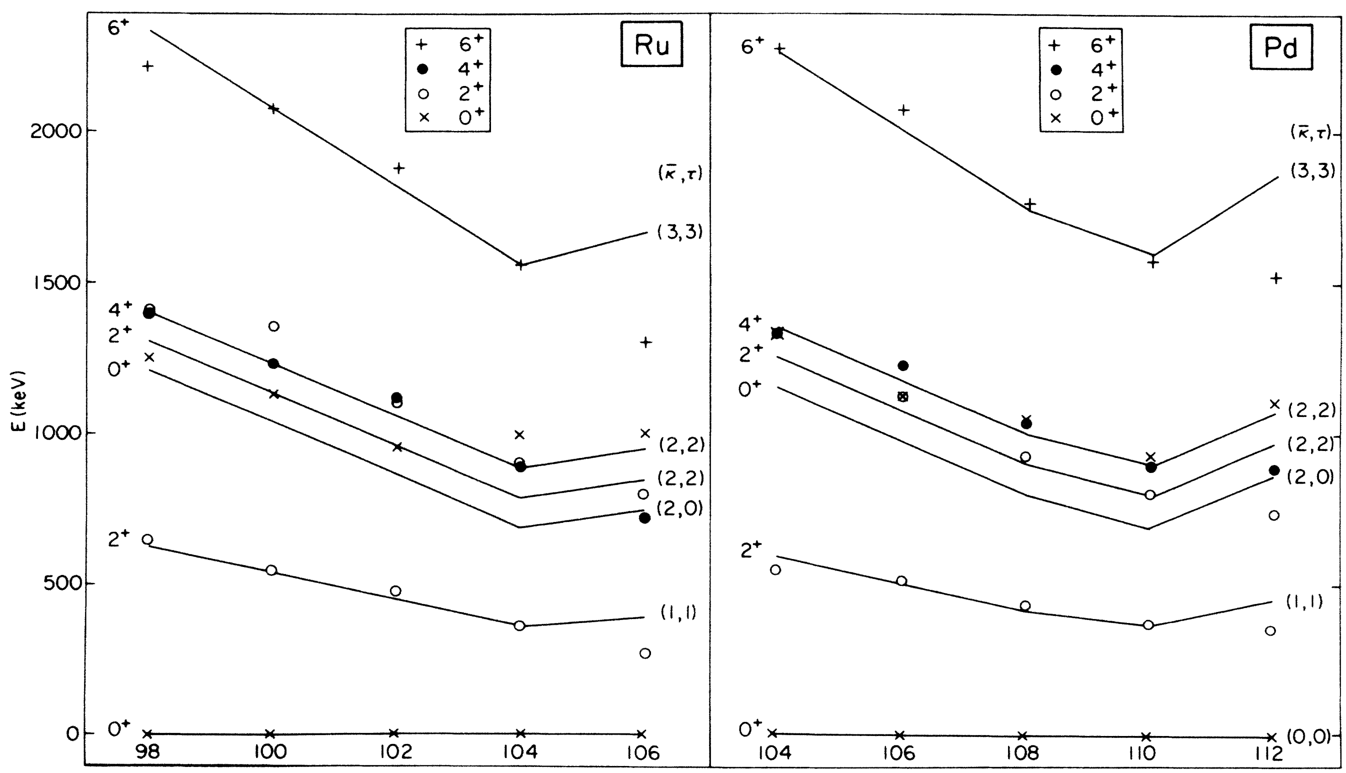

FIG. 2. Predicted and empirical (Refs. 7, 9–11) energy levels for Ru and Pd. The parameters, given in the text, are held *constant* for each set of isotopes.

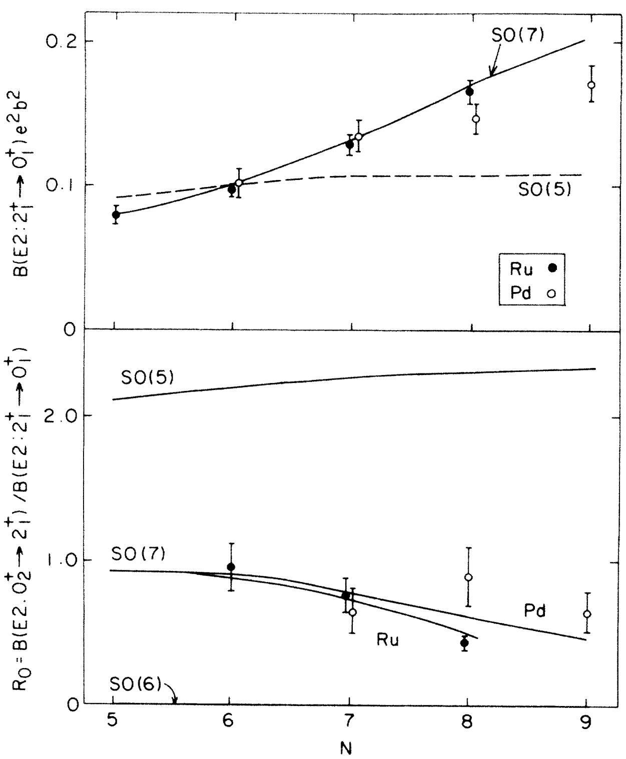

FIG. 3. Predicted and experimental (Refs. 12–17) $B(E2)$ values. Separate curves are shown for Ru and Pd where they differ significantly.

$N=6$). The predicted $B(E2{:}2_1^+ \to 0_1^+)$ values are in excellent agreement with the data for $N \leq 8$. R_0 is a crucial indicator since the character of the 0_2^+ level is one of the distinguishing marks of a vibrator, where it has allowed $E2$ transitions to the one-phonon 2_1^+ level, and of O(6), where it has $\tau=3$ and decays only to the 2_2^+ level. Once again the SO(7) predictions reflect the data very well and the transition from SO(5)-like towards SO(6)-like. Other observables such as $R_4 = B(E2{:}4_1^+ \to 2_1^+)/B(E2{:}2_1^+ \to 0_1^+)$ are in good agreement with the SO(7) values. The only notable discrepancy occurs for the ratio $R_2 = B(E2{:}2_2^+ \to 2_1^+)/B(E2{:}2_1^+ \to 0_1^+)$ which, empirically, is below all the SO(5), SO(6), and SO(7) predictions and only slightly closer to SO(7) than to the SU(3) value of zero.

To summarize, empirical evidence has been presented suggesting that the Pd and Ru nuclei are good empirical realizations of the SO(7) symmetry. This has three important ramifications. It is the first disclosure of a *fermion* dynamical symmetry in heavy nuclei that is not simply analogous to an established boson symmetry, and thus suggests that the symmetries observed in nuclei do have a foundation in the underlying shell structure. Second, it is the first observed example of a symmetry that incorporates *variable* structure and which is capable of describing *transition* regions. This immediately expands the realm of application of nuclear symmetry ideas. Third, it suggests that further study of the fermion-dynamical-symmetry model[5,6] and other fermion symmetry schemes[4] as a microscopic foundation for the IBA would be worthwhile.

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