Charge-Symmetry Breaking in Neutron-Proton Elastic Scattering

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The effects of spin-dependent charge-symmetry–breaking forces can be measured in elastic n-p scattering. We find that a term in the one-pion exchange potential arising from the neutron-proton mass difference is dominant. This, together with single-photon exchange, gives a result in agreement with a recent precise measurement made at TRIUMF. Other shorter-ranged effects associated with rho exchange, meson mixing, two-pion exchanges, and quark interactions give smaller contributions.

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An accurate experimental comparison of the neutron (n) and proton (p) analyzing powers in n-p elastic scattering at a laboratory energy of 477 MeV has recently been completed at TRIUMF.[1] Any difference between the two analyzing powers provides direct evidence for the violation of charge symmetry.[2] The effect is expected to be small, of the order of the fine-structure constant, and so the measurement involves only the changes in the angle at which the proton's analyzing power goes through zero (near $\theta_{\text{c.m.}}=70°$).

Charge symmetry holds if the Hamiltonian is invariant under rotations of 180° about the y axis in isospin space, if the z axis corresponds to the charge axis. It is a less rigorous constraint than charge independence which requires invariance under all rotations about any axis. For more information regarding the significance of charge symmetry we refer the reader to a number of reviews.[2–4]

Since the breaking of charge symmetry had never been established unambiguously[2–4] (i.e., in the absence of the Coulomb force), the TRIUMF result that $\Delta A=[37\pm17(\text{stat.})\pm8(\text{syst.})]\times10^{-4}$ is significant. We quote the quantity $\Delta A\equiv A_n-A_p$ at the (neutron) zero-crossing angle. This term is equivalent to the change in the zero-crossing angle, but is easier to use.

Our aim here is to clarify the physics behind this nonzero result. We shall see that the dominant effect is associated with a piece of the one-pion exchange potential (OPEP), which arises when the n-p mass difference is carefully included.[5,6] The conventional OPEP is well known, but in this process one isolates a spin-transition matrix element that has never been measured before. In this sense we are engaged in a new test of the meson exchange theory of nuclear forces.

Only a charge-symmetry-breaking (CSB) potential of class IV (according to the classification of Henley and Miller[2]) can contribute to ΔA. That is, the spin and isospin operators must both be odd under exchange. The simplest forces of this kind are

$$V_1=(\tau_1-\tau_2)_z(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{L}v(r) \qquad (1)$$

and

$$V_2=(\tau_1\times\tau_2)_z(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{L}w(r). \qquad (2)$$

(Here $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ is the internucleon separation and $\mathbf{L}$ is the orbital angular momentum operator in the c.m. system.) Because the spin operators in Eqs. (1) and (2) mix spin singlet and triplet states, only states with total angular momentum (J) equal to the orbital angular momentum can contribute.

Using the potentials of (1) and (2) it is straightforward to evaluate the CSB amplitude $f(\theta)(\boldsymbol{\sigma}_n-\boldsymbol{\sigma}_p)\cdot\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is a unit vector normal to the scattering plane. Within the formalism of Gersten,[5,6] $f(\theta)$ is determined by a set of singlet-triplet mixing angles $\bar{\gamma}_J$ as

$$f(\theta)=(i/k)\sum_J(2J+1)\bar{\gamma}_J e^{i(\bar{\delta}_J+\bar{\delta}_{JJ})}d^J_{10}(\theta). \qquad (3)$$

In Eq. (3), k is the c.m. momentum, $\bar{\delta}$ are the relevant bar phase shifts,[7] and d^J_{10} are the Wigner functions. The effects of the strong nuclear force are included through the solutions $R(r)$ of the Schrödinger equation for the Reid soft-core potential,[8] which adequately describes the relevant ($J=L$) experimental phase shifts. For the class-IV force given in Eq. (1) we find

$$\bar{\gamma}_J=-Mk4[J(J+1)]^{1/2}\times\int_0^\infty dr\, r^2 R_J(r)v(r)R_{JJ}(r), \qquad (4)$$

where M is the average nucleon mass. If the force is given by Eq. (2) there is an additional factor of $(-1)^J$ and $v(r)$ is replaced by $w(r)$.

The difference in analyzing power, ΔA, is computed from the interference between $f(\theta)$ and the charge-symmetric amplitudes obtained from Ref. 7. We show in Table I various contributions to ΔA at the angle

TABLE I. $10^4\Delta A$ as function of energy. The separate contributions of electromagnetic (EM), OPEP, TPEP, ρ, and $\rho\omega$ interactions are given.

E_{lab} (MeV)	$\theta_p(0)$	EM	OPEP	TPEP	ρ	$\rho\omega$	Total	Expt.
477	70	6	34	−0.4	9 ±3	5 ±1.2	54 ±4	37 ±17 ±8
350	72	3	35	0.1	6.5 ±3	1.6 ±0.6	46 ±4	
188	96	10	6	−0.5	−1.2 ±0.4	3.0 ±0.8	17 ±1	

where θ_p goes to zero. (ΔA is essentially independent of angle in that region.) These results are displayed at three laboratory energies, 477, 350, and 188 MeV, which are respectively the energies of the recently completed TRIUMF measurement, a future TRIUMF measurement, and an experiment proceeding at the Indiana University Cyclotron Facility. We now discuss these contributions, beginning with those of longest range.

(1) Electromagnetic spin-orbit force.—The class-IV part of electromagnetic spin-orbit force arises from terms involving the current (γ_μ) coupling at one vertex and the anomalous magnetic ($\sigma_{\mu\nu}q_\nu$) coupling at the other [see Fig. 1(a)].[9] The term, which is of the form of Eq. (1), has a fairly small effect at the cross-over angle, provided that realistic form factors and distortions are included. In large part, this is the result of a tendency for cancellation between $J=1$ and $J=2$ in the dominant real part of $f(\theta)$. This cancellation occurs because the strong phase shifts are repulsive for $J=1$ and attractive for $J=2$. As a consequence, operators of the form of Eq. (2) produce bigger effects on ΔA than those of Eq. (1).

(2) One-pion exchange.—The n-p mass difference induces a piece of the OPEP of the form (2), with[6,10]

$$w(r) = -\frac{g_\pi^2}{4\pi}\frac{M^2}{M^2+k^2}\frac{M_n-M_p}{4M^2} \times \frac{1}{r}\frac{d}{dr}\left(\frac{e^{-m_\pi r}-e^{-\Lambda r}}{r}\right). \quad (5)$$

The factor $M^2/(M^2+k^2)$ is a relativistic correction, and we use $\Lambda = 600$ MeV/c, corresponding to a dipole form factor of mass 1200 MeV/c.[6]

To understand the nuclear contributions to the class-IV forces it is worthwhile to examine them as a function of r. At 477 MeV, changes in ΔA are approximately proportional to a linear combination of changes in γ_J ($\Delta\gamma_J$): $0.051\Delta\gamma_1 - 0.047\Delta\gamma_2$; so this linear combination of integrands is displayed in Fig. 2. The OPEP contribution peaks at about 1.2 fm. The effects of centrifugal and short-range repulsions suppress contributions from regions where r is very small, while for $J=2$, the nucleon-nucleon attraction gives an enhancement in the region between 1 and 2 fm. One may therefore expect terms of shorter range than the OPEP to be suppressed.

(3) ρ-meson and mixed ρ-ω exchanges.—CSB in ρ exchange arises from the n-p mass difference and is of the form (2), whereas the exchange of a mixed state of a ρ and ω is of the form (1).[2,9] As shown in Table I and Fig. 2, the ρ exchange has a much bigger effect than that of the ρ-ω exchange term, but the OPEP term is the biggest. There is considerable leeway in the heavy-meson exchange terms because of the high sensitivity to the input coupling constants, especially the ratio of tensor to vector rho-meson–nucleon coupling constant, 6.1 ±0.6.[11] In addition, the ρ-ω mixing Hamiltonian has an uncertainty of at least 30%.[2] Here we use empirical coupling constants for the rho[11] and omega[12] mesons. The rho and omega-nucleon form factors are taken from the Bonn potential.[13] Estimates of the variation in ΔA caused by these uncertainties are given in Table I. The freedom to use different sets of radial wave functions causes some additional variation. By comparing our results with those of Ge and Svenne[14] who use the Paris potential,[15] we find ~10% differences. (This sensitivity to the potential is small-

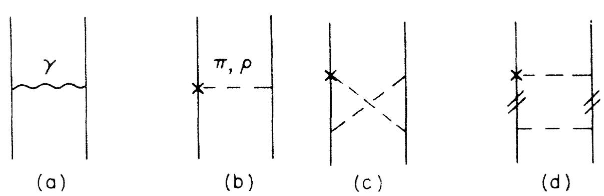

FIG. 1. Diagrams for CSB (a) photon, (b) pion, and (c),(d) two-pion exchange. The cross-hatching refers to the usual subtraction procedure. The cross indicates the CSB vertex function.

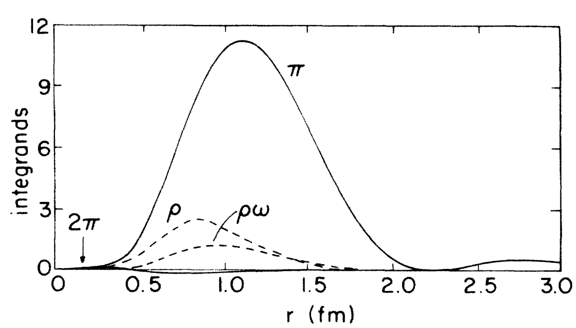

FIG. 2. Coordinate-space contributions at 477 MeV. See text for the definitions of the curves.

er for the pion and photon terms.)

(4) Two-pion exchange (TPEP).—The significant effects of TPEP are well known. However, the charge dependence of this term is weaker than expected since the contributions from exchanges of crossed and uncrossed pions tend to cancel.[16] In our case, the n-p mass difference again gives a CSB term. The TPEP terms have been evaluated in two ways. First, following Partovi and Lomon[17] we compute the CSB of the terms in Figs. 1(c) and 1(d). The resulting class-IV force is mainly of the form of Eq. (2)[18] and there is a very severe cancellation between terms with $J=1$ and $J=2$. The outcome is the very small integrand shown in Fig. 2. For each of $J=1,2$ the magnitude of the TPEP is about $\frac{1}{10}$ that of OPEP. The class-IV part of TPEP is of extremely short range, and therefore is highly uncertain. To underscore this, one can study the exchange of a scalar-isovector meson (delta). This term plays the phenomenological role of the $\pi\pi$ term of TPEP in single-boson exchange calculations. Thus, use of delta exchange gives an alternative way of estimating the class-IV part of the medium-ranged attractive terms. Using the coupling constant and mass from the recent Bonn single-boson exchange potential[13] we obtain a result that is even smaller than the TPEP result given in Table I and Fig. 2.

(5) Quark effects.—In the past few years there has been considerable interest in a quark-level description of the short-range N-N force.[19–21] For example, one can replace the small-distance NN wave function by a six-quark state.[20] The gluon exchanges between these quarks depend on their (slightly charge-dependent) masses,[22] and so a class-IV force emerges.[18] However, the resulting value of γ_1 is at most $\frac{1}{15}$ of the dominant OPEP term. More details concerning this calculation will be given elsewhere.[18]

(6) Medium-range quark effects.—Nucleons with a radius of 1 fm overlap in the region where the OPEP integrand peaks. One might attempt to make a finite-size correction by using a smaller value of Λ, which would reduce the OPEP contribution. However, we already know that for the charge-dependent[14] and tensor pieces[23] of OPEP this is too simple. In both cases there are additional pion-quark interactions which tend to compensate for the softer form factor. Our estimates suggest that the same phenomenon occurs in this case,[18] and so we do not allow Λ to vary.

For completeness we observe that charge-dependent meson-nucleon coupling constants do not lead to class-IV forces. This is because one needs different operators for the right- and left-hand nucleon lines as in Fig. 1. The influence of pion production is also expected to be unimportant since the imaginary part of the phase shifts is small at our energies. While we do not expect the simultaneous exchange of a pion and a photon to be important[12] (because it should be of the same order as the CSB part of TPEP), it should be estimated in the near future. The charge dependence of the exchange of a pion and a rho (and other combinations) is of shorter range and we expect it to be even smaller.

Let us finish with a summary of the situation at our three energies. As we see from Table I, the results for ΔA at 477 and 350 MeV are fairly similar—basically because of the slow variation of the strong interaction in this region. Clearly OPEP dominates while vector-meson exchange tends to make the agreement with data a little bit worse. On the other hand, at 188 MeV the result for ΔA is much smaller. At this energy the centrifugal barrier strongly reduces the $J=2$ contributions. In addition, the strong amplitude [which interferes with $f(\theta)$ to give ΔA] is smaller because of the decreasing magnitude of the s- and p-wave shifts.

As we have discussed, there is considerable model dependence in our estimate of ΔA, particularly the vector-meson contribution. We estimate the theoretical uncertainty as about 10% at 477 MeV, reducing to maybe a few percent at 188 MeV. While we do agree with the existing measurements within our respective errors, we regard it as extremely important that our predictions at the lower energies be tested as soon as possible. Any significant deviation from our predictions would provide a dramatic challenge to our present understanding of the nuclear force.

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