Fracture Behavior of a Solid with Random Porosity

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We report the results of an experimental study of the fracture stress, S, and the Young's modulus, E, of a two-dimensional solid (network) which has undergone random dilution to the point where the solid becomes geometrically disconnected. In the scaling region, $E \sim (p - p_c)^f$ with $f = 3.1 \pm 0.1$ and $S \sim (p - p_c)^F$ with $F = 1.7 \pm 0.1$. We find that appropriately modified formulations of the Griffith relation can account for the fracture stress in the entire range of bond dilution.

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In this Letter, we report the results of experimental studies of the Young's modulus and fracture strength of a solid which has undergone random dilution to the point where the solid becomes geometrically disconnected. The behavior of the elastic moduli of such solids has been the focus of a number of recent theoretical investigations[1-9] and two experimental studies.[10,11] A principal result of these studies is that the elasticity exponent, f, which governs behavior near p_c, the percolation threshold, can be significantly higher than the corresponding electrical conductivity exponent of elastic systems with both central and bond-bending forces. The theoretical studies yield values of f in 2D of approximately 3.5 for the discrete lattice models[2,4,6,9] and $f + \frac{3}{2}$ for the "Swiss cheese" continuum model.[7] Benguigi[10] performed experiments on a 0.2-mm-thick metal sheet in which holes were punched at random on sites of a 20×20 square lattice and reported a value of f in 2D of 3.5 ± 0.4. On the other hand, very little attention has been given to the fracture properties of dilute solids. The conventional area of interest in fracture has been in the regime where the density of cracks is such that they are not considered to interact. The Weibull statistics has been used to treat the brittle fracture problem in this regime.[12] To date, we know of only one study in which the fracture properties of dilute brittle solids were investigated. Ray and Chakrabarti[13] reported results of molecular-dynamic simulations on a 20×20 square lattice of atoms interacting with the 6-12 Lennard-Jones potential. Bonds were removed at random and the fracture stress of the resultant structure was determined. They found that $S \sim |p - p_c|^F$ with $F \approx 1$, where S is the fracture stress and p is the fraction of bonds present.

Here we report results of the Young's modulus, E, and fracture stress, S, for a system composed of a 2-mm-thick plate of aluminum with holes drilled at positions corresponding to a triangular lattice of 21 rows and 20 columns as shown in Fig. 1. The sample contains 1230 ligaments (or "bonds") which link the voids arranged in the triangular lattice. For each realization each bond was assigned a random number, n, between 0 and 1. The bonds for which $n > p$ are cut where p is decreased successively from unity and a measurement of the elastic modulus and fracture stress is made. The tests were performed on a Instron model 4202 universal testing machine which is inter-

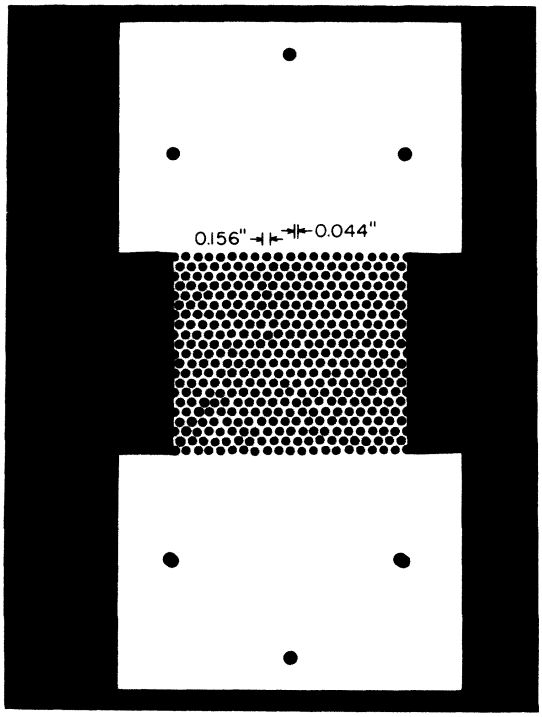

FIG. 1. The configuration of the 2D triangular network. The ligaments connecting the voids are randomly cut forming cracks as described in the text.

faced to a Hewlett-Packard 86B desk-top computer. Measurements of Young's modulus were made in the linear-elastic portion of the load-displacement curve. Elastic behavior was confirmed by unloading and checking for resultant plastic strains. Each datum point for the Young's modulus represents the average of at least three separate load-displacement curves. The fracture stress, S, was determined by obtaining the full load-displacement curve for the sample to failure. The fracture stress is defined at the maximum in load of the load-displacement curve.[14] The behavior of the Young's modulus was examined by use of two realizations. A single sample was used for the first realization and E was determined for successive descending values of p. In order to implement the second realization eighteen identical samples ($p=1$) were prepared from a template. Bonds were cut in each sample according to a prescribed value of p and the Young's modulus determined for that sample. Following the Young's modulus determination, the load-displacement curve of the sample to failure was measured.

The results for the Young's modulus and the fracture stress are shown in Fig. 2. E^0 and S^0 are the measured values of these quantities when all the bonds are present ($p=1$). In the region far away from p_c the modulus behavior is well described by $E/E^0=1-3\phi$ (where $\phi=1-p$) which is in good agreement with effective-medium theories.[15] Figure 3 is a log-log plot of E and S vs $p-p_c$ for $p_c=0.63$, the average value of p at which the solid became geometrically disconnected. The measured value of p_c in the first realization was 0.625 and in the second realization 0.630. The slopes represent estimates of the exponents and were determined by use of the points with values of $p-p_c$ such that the percolation correlation length is smaller than the macroscopic sample size, L. The elasticity exponent f is 3.1 ± 0.1.[16] The fracture stress also displays power-law behavior in this regime and the fracture stress exponent, F, is 1.7 ± 0.1.

Consider the fracture stress behavior, first in the region far away from p_c. For $\phi=N_b^{-1}$, i.e., one "crack," $d(S/S_0)/dp\rightarrow\infty$ (N_b is the number of bonds). This is to be expected since any cracklike flaw in an otherwise homogeneous sample results in a discontinuous change in the fracture stress of the sample. As ϕ increases there are at first a small number of short cracks (formed by a single cut bond). The crack density is so low that the interaction between cracks is negligible and the fracture stress behavior is described by the weakest-link concept and Weibull statistics. Here the fracture is very sensitive to structural flaws. That is, minor deviations in ligament width resulting from inevitable machining inaccuracies (of the template) in conjunction with the crack locations determine the location of the fracture initiation site and the fracture stress. Our experiments show that in this region the data are subject to considerable scatter. For example, the normalized fracture stress, S/S^0, was 0.75 for samples with 1 and 9 broken bonds. (This is indicated by a single point in Fig. 2, as a result of the scale of the figure.)

As the crack density increases the crack interactions become more significant. Each crack may be con-

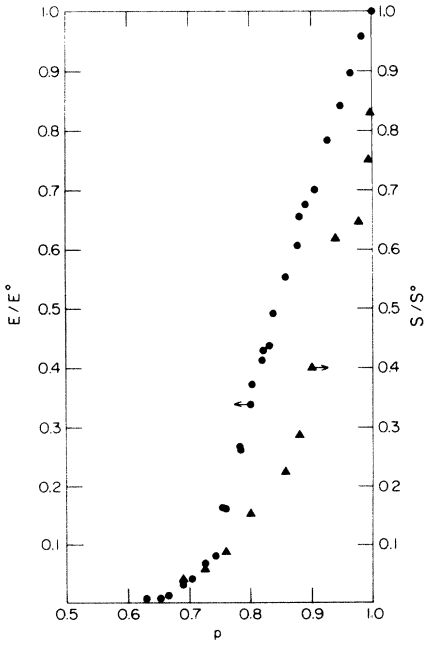

FIG. 2. Normalized Young's modulus, E/E^0, and fracture stress, S/S^0, as functions of p.

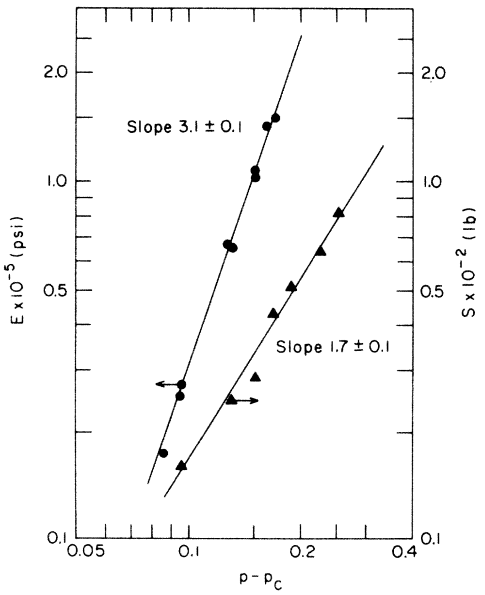

FIG. 3. Double logarithmic plot of E and S vs $p-p_c$, $p_c=0.63$.

sidered to occupy a region of average areal size given by $A/N^*\phi$ where A is the sample area (4.0×3.6 in.^{2}) and N^* is the effective number of bonds oriented perpendicular to the tensile axis. In order to evaluate N^* we consider that two adjacent diagonal bonds have a projected length perpendicular to the tensile axis equivalent to a single horizontal bond so that $N^*=820$. This tesselation forms $N^*\phi$ miniature tensile samples each with central crack of length $2l=0.356$ in., such that the tensile axis is oriented perpendicular to the crack. The width, W, of each block may be taken to be $(A/N^*\phi)^{1/2}=0.13\phi^{-1/2}$ in. W is bounded by the finite sample size ($W<L$) so that the expression developed for W is applicable in the range $0.04\leq\phi\leq\phi_s$ where $\phi_s\approx0.14$. In our block scheme ϕ_s represents the crack density for which $W=2l$. Near ϕ_s the average crack size is no longer represented by a crack formed by a single cut bond, crack condensation and overlap is common, and the fracture behavior crosses over to the power-law regime. A modified form of the Griffith equation which accounts for finite-sample-size effects in a plate of width W containing a central crack of length $2l$ oriented perpendicular to the tensile axis is given by[17]

$$S=[(2E\gamma/\pi l)\cos(\pi l/W)]^{1/2}, \quad (1)$$

where 2γ is the work per unit area associated with forming the new surfaces. The modulus behavior in this regime is described by $E=E^0(1-3\phi)$ and substitution into Eq. (1) yields

$$S/S^0=[(1-3\phi)\cos(\pi l/W)]^{1/2}, \quad (2)$$

where

$$S^0=[2E^0\gamma/\pi l]^{1/2}.$$

In this region where our definition of W is applicable, $l/W=1.37\phi^{1/2}$, and the magnitude of the slope, $d(S/S^0)/d\phi$, evaluated by use of Eq. (2) is ~6. Examination of Fig. 2 indicates that this result is in reasonable agreement with our measurements.

In order to understand the behavior of the fracture stress in the scaling region we have examined the behavior of the average crack-spanning length perpendicular to the loading axis; $l_{av}=\sum_i n(l_i)l_i^2/\sum_i n(l_i)l_i$ as shown in Fig. 4. Here l_i is the spanning length perpendicular to the loading axis of crack i and $n(l_i)$ is the number of cracks of length i. Our measurements indicate that $l_{av}\sim(p-p_c)^{-m}$ with $m=0.9\pm0.1$. Figure 4 shows that as the average crack length increases the total number of cracks, n, decreases according to $n\sim(p-p_c)^b$ with $b=0.5\pm0.05$. We define an effective crack length, l_{eff}, as (the number of cracks per row)$\times l_{av}$ so that $l_{eff}\sim(p-p_c)^{-0.4}$. In effect, as a result of the high linear density of cracks, we treat collinear cracks as connected to form a single crack of length l_{eff}. These relations in conjunction with the Griffith relation, $S=(2E\gamma/\pi l)^{1/2}$, provide us with an understanding of the fracture stress in the scaling region. Substituting $E\sim(p-p_c)^{3.1}$ and, for l, $l_{eff}\sim(p-p_c)^{-0.4}$ into the Griffith relation yields $S\sim(p-p_c)^{1.75}$ in good agreement with the measured result.

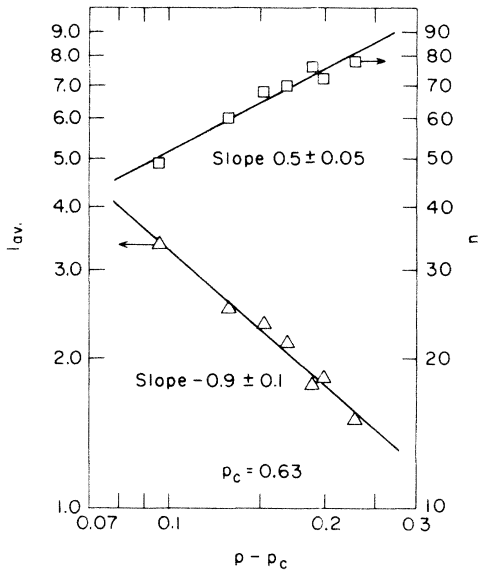

FIG. 4. Double logarithmic plot of l_{av} and the number of the cracks, n, vs $p-p_c$; $p_c=0.63$.

In summary, we have reported the results of experimental studies of the Young's modulus and the fracture strength of a solid network which has undergone random dilution. We find that appropriately modified formulations of the Griffith relation can account for the fracture stress in the entire range of bond dilution, i.e., $p_c<p<1$. In the scaling regime $E\sim(p-p_c)^f$ with $f=3.1\pm0.1$ and $S\sim(p-p_c)^F$ with $F=1.7\pm0.1$.

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[14]It has been pointed out to us that there is an analogy between the breakage of the first fuse in a fuse network and the fracture of the first bond in the elastic network. The load at which fracture of the first bond occurs does not in general coincide with the maximum load in the load-displacement curve.

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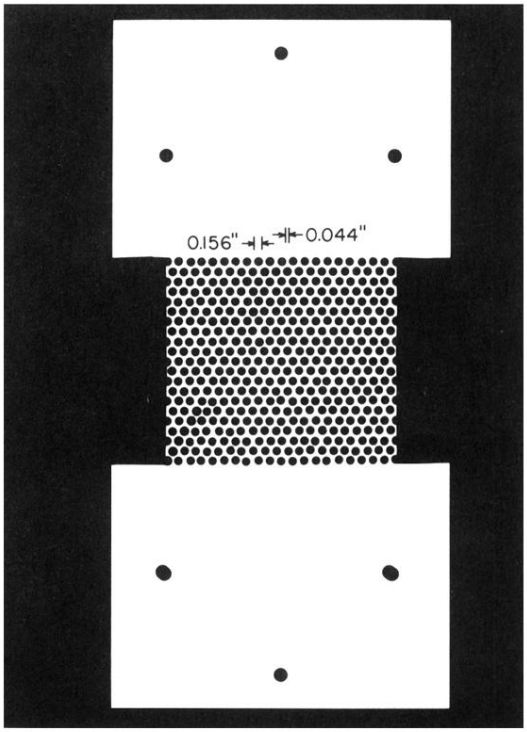
[16]In his Letter, Benguigi (Ref. 10) states that he considers his measurement of the elasticity exponent on Al to be "less dispersed" than the measurement on Cu and reports a value for Al of $f=3.3\pm0.2$. Our estimate of f reported herein seems in reasonable agreement with this value.

[17]The finite-size correction on the Griffith equation is actually made on the stress intensity factor, k, which for a crack of length $2l$ in an infinite sample is given by $S(\pi l)^{1/2}$. For a sample of finite width, W, $k=S[\pi l\sec(\pi l/W)]^{1/2}$. In terms of k the Griffith equation is expressed as $k^2=2E\gamma$. See, for example, J. R. Rice, in *Fracture,* edited by H. Liebowitz (Academic, New York, 1968), Vol. 2, p. 221.



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