Amplitude for n-Gluon Scattering

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A nontrivial squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.

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Computations of the scattering amplitudes for the vector gauge bosons of non-Abelian gauge theories, besides being interesting from a purely quantum-field-theoretical point of view (determination of the S matrix), have a wide range of important applications. In particular, within the framework of quantum chromodynamics (QCD), the scattering of vector gauge bosons (gluons) gives rise to experimentally observable multijet production at high-energy hadron colliders. The knowledge of cross sections for the gluon scattering is crucial for any reliable phenomenology of jet physics, which holds great promise for testing QCD as well as for the discovery of new physics at present (CERN $Sp\bar{p}S$ and Fermilab Tevatron) and future (Superconducting Super Collider) hadron colliders.[1]

In this short Letter, we give a nontrivial squared helicity amplitude for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors. To our knowledge this is the first time in a non-Abelian gauge theory that a nontrivial, on-mass-shell, squared Green's function has been written down for an arbitrary number of external points. Our result can be used to improve the existing numerical programs for the QCD jet production, and in particular for the studies of the four-jet production for which no analytic results have been available so far. Before presenting the helicity amplitude, let us make it clear that this result is an educated guess which we have compared to the existing computations and verified by a series of highly nontrivial and nonlinear consistency checks.

For the n-gluon scattering amplitude, there are $(n+2)/2$ independent helicity amplitudes. At the tree level, the two helicity amplitudes which most violate the conservation of helicity are zero. This is easily seen by the embedding of the Yang-Mills theory in a supersymmetric theory.[2,3] Here we give an expression for the next helicity amplitude, also at tree level, to leading order in the number of colors in $SU(N)$ Yang-Mills theory.

If the helicity amplitude for gluons $1, \ldots, n$, of momenta $p_1, \ldots, p_n$ and helicities $\lambda_1, \ldots, \lambda_n$, is $\mathcal{M}_n(\lambda_1, \ldots, \lambda_n)$, where the momenta and helicities are labeled as though all particles are outgoing, then the three helicity amplitudes of interest, squared and summed over color, are

$$|\mathcal{M}_n(+ + + + + \cdots)|^2 = c_n(g,N)[0+O(g^4)], \tag{1}$$

$$|\mathcal{M}_n(- + + + + \cdots)|^2 = c_n(g,N)[0+O(g^4)], \tag{2}$$

$$\begin{aligned}|\mathcal{M}_n(- - + + + \cdots)|^2 = {} & c_n(q,N)[(p_1\cdot p_2)^4 \\ & \times \textstyle\sum_P [(p_1\cdot p_2)(p_2\cdot p_3)(p_3\cdot p_4)\cdots(p_n\cdot p_1)]^{-1} + O(N^{-2}) + O(g^2)],\end{aligned} \tag{3}$$

where $c_n(g,N) = g^{2n-4}N^{n-2}(N^2-1)/2^{n-4}n$. The sum is over all permutations P of $1, \ldots, n$.

Equation (3) has the correct dimensions and symmetry properties for this n-particle scattering amplitude squared. Also it agrees with the known results[4,5] for $n=4$, 5, and 6. The agreement for $n=6$ is numerical.[5,6] More importantly, this set of amplitudes is consistent with the Altarelli and Parisi[7] relationship for all n, when two of the gluons are made parallel. This is trivial for the first two helicity amplitudes but is a highly nontrivial statement for the last amplitude, as shown here:

$$|\mathcal{M}_n(- - + + + \cdots)|^2 \underset{1\parallel 2}{\longrightarrow} 0, \tag{4}$$

$$|\mathcal{M}_n(- - + + + \cdots)|^2 \underset{2\parallel 3}{\longrightarrow} 2g^2N\frac{z^4}{z(1-z)}\frac{1}{s}|\mathcal{M}_{n-1}(- - + + \cdots)|^2, \tag{5}$$

$$|\mathcal{M}_n(- - + + + \cdots)|^2 \underset{3\parallel 4}{\longrightarrow} 2g^2N\frac{1}{z(1-z)}\frac{1}{s}|\mathcal{M}_{n-1}(- - + + \cdots)|^2, \tag{6}$$

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where s is the corresponding pole and z is the momentum fraction. The result for particles 2 and 3 nearly parallel, Eq. (5), is only simple because $\mathcal{M}_{n-1}(-+++\cdots)$ is zero to this order in g so that there is no interference term and therefore azimuthal averaging is not required.

The surprise about this result is that all denominators are simple dot products of two external momenta. The Feynman diagrams for n-gluon $(n>5)$ scattering contain propagators $(p_i+p_j+p_k)^2$, $(p_i+p_j+p_k+p_m)^2$, ... These propagators must cancel for Eq. (3) to be correct; this occurs for $n=6$. Of course, Altarelli and Parisi have taught us that many cancellations are expected.

We do not expect such a simple expression for the other helicity amplitudes. Also, we challenge the string theorists to prove more rigorously that Eq. (3) is correct.

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[6]Another numerical fact worth mentioning is that to leading order in g but to all orders in N, the amplitude $|\mathcal{M}_{n=6}(--++++)|^2$ is permutation symmetric apart from the factor $(p_1\cdot p_2)^4$. This allows all permutations of this amplitude to be trivially calculated from one such permutation.

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