Interacting Parastrings

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Interacting parastrings offer the alternative of constructing string theories directly in four (as well as a number of other) space-time dimensions. These theories are consistent with Lorentz invariance, Lovelace analyticity, and modular invariance. They are also derivable from actions in light-cone gauge. For supersymmetric parastrings, the corresponding actions are supersymmetric.

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String theories are presently regarded as leading candidates for a unified theory of all interactions. If so, then it is imperative that we understand a number of intriguing features of these theories at a fundamental level. Among these features are the critical dimensionalities and the restrictions on internal symmetries. For the currently popular models,[1-3] the critical dimensionality is either 26 or 10, and for the supersymmetric varieties the internal symmetries are limited to the groups SO(32) or $E_8 \otimes E_8$.[4]

The main objective of this Letter is to report on the possibility of constructing nontrivial interacting string theories which have critical dimensionalities different from 26 or 10. To put this possibility in its proper perspective, it is convenient to view the existence of critical dimensionalities as arising from the requirement of compatibility between quantum mechanics and Lorentz invariance. Then, to quantize the dynamical variables of the classical (super)string theories, one is faced with two distinct options: (a) To insist on standard Bose or Fermi quantization of the dynamical variables and let the consistency of the formalism determine the critical dimensionality of the corresponding theory. This leads to the critical dimensionalities 26 and 10. (b) To allow for the possibility of other critical dimsionalities, and hence other Lorentz groups, and see if there are quantization schemes which are compatible with them. In this way, one arrives at the paraquantization of strings[5] (parastrings). The physical basis of paraquantization can be made plausible by noting that if the spectrum of a given string theory does not exclusively correspond to the experimentally observed particles, then the relevant statistics need not be exclusively Fermi and Bose.

It might appear at first sight that the two possibilities stated above are diametrically opposite and have no common features. To stress that this is not the case, we would like to point out some of the properties which are common to both approaches: (i) Both approaches lead to an increase in the number of physical degrees of freedom over what would have been expected from a four-dimensional theory with standard Fermi or Bose statistics. (ii) In fact, the total number of degrees of freedom is the same in the two approaches. (iii) Considered from a generalized Kaluza-Klein point of view, the two approaches appear to be different means to the same end: More-dimensional string theories require compactification,[4,6] and this leads to internal symmetries. On the other hand, parastatistics are equivalent to some form of internal symmetries, and this is equivalent to the extra dimensions in a Kaluza-Klein setting. Therefore, it is reasonable to conclude that there are no fundamental issues which separate the two options. What distinguishes them is the physical basis upon which they rest and from which their degrees of freedom originate.

Parastatistics[7] are generalizations of standard Bose and Fermi statistics. They are solutions of the trilinear relations

$$[[a_j^\dagger, a_k]_\pm, a_l]_- = -2\delta_{jl} a_k, \tag{1}$$

$$[[a_j, a_k]_\pm, a_l]_- = 0, \tag{2}$$

in which, contrary to the Bose and Fermi case, the requirement that the (anti)commutator of two field operators be a *c*-number is relaxed. The paraquantization of string theories was carried out in Ref. 5. It was found that the closure of the Lorentz algebra leads to the critical dimensionalities

$$D = 2 + 24/Q, \tag{3}$$

for bosonic strings and

$$D = 2 + 8/Q, \tag{4}$$

for supersymmetric strings (Ramond-Neveu-Schwarz). In these expressions, Q is the order of parastatistics. For the supersymmetric varieties, parafermions and parabosons must have the same order Q. More recently, the derivation of the critical dimensionalities given by (3) and (4) has been put[8] on a more rigorous foundation by a reformulation of the theory in which one of the unnatural assumptions of the earlier approach was relaxed. The new formulation is based on the realization of the para operators in the form[9]

$$A=\sum_{\alpha=1}^{Q} e^{\alpha}A^{\alpha}, \tag{5}$$

where the components A^{α} are ordinary boson operators if A is paraboson and ordinary fermion operators if A is parafermion. The quantities e^{α} are elements of the real Clifford algebra

$$\{e^{\alpha},e^{\beta}\}=2\delta^{\alpha\beta},\quad \alpha,\beta=1\ldots,Q. \tag{6}$$

Moreover, $[e^{\alpha},A^{\beta}]=0$, $\alpha,\beta=1,\ldots,Q$. In this Letter, for simplicity, we will confine most of the discussion to the bosonic parastrings, although some results on supersymmetric parastrings will also be given.

Consider the paraquantization of the bosonic open string in the light-cone gauge. Using the realization (5), we write the transverse string variable as

$$y^{i}(\tau,\sigma)=\sum_{\beta=1}^{Q} e^{\beta}y^{i\beta}(\sigma,\tau), \tag{7}$$

where

$$y^{i\beta}(\tau,\sigma)=x^{i\beta}+2\alpha' p^{i\beta}\tau+(2\alpha')^{1/2}\sum_{n=1}^{\infty} n^{-1/2}[a_n^{i\beta}e^{-in\tau}+a_n^{i\beta\dagger}e^{in\tau}]\cos n\sigma. \tag{8}$$

The variables $y^{i\alpha}(\tau,\sigma)$ and their (para) canonical momenta satisfy the equal time commutation relations

$$[y^{i\alpha}(\sigma),\pi^{j\beta}(\sigma')]=i\delta^{ij}\delta_{\alpha\beta}\delta(\sigma-\sigma'). \tag{9}$$

The rest of the development parallels that of the bosonic string in light-cone gauge.[10] One of the main differences which arise is that for parastrings, the c-number term in Virasoro algebra becomes dependent on the order of parastatistics Q:

$$[T_m,T_n]=(n-m)T_{n+m}+[Q(D-2)/12](n^3-n)\delta_{m+n,0}. \tag{10}$$

As discussed in Refs. 5 and 7, it is this dependence on Q which leads to the critical dimensionalities given by Eqs. (3) and (4).

Next, we turn to interactions. Following the analogy with open bosonic string theory, we write down a ground-state vertex operator for the open bosonic parastring of order Q in the form

$$V(k)=g{:}e^{i\{\mathbf{k}\cdot\mathbf{Y}(0)\}/2}{:}, \tag{11}$$

where the momenta are taken to be transverse, colons indicate normal ordering, and

$$\{\mathbf{k}\cdot\mathbf{y}(0)\}=\mathbf{k}\cdot\mathbf{Y}(0)+\mathbf{Y}(0)\cdot\mathbf{k}. \tag{12}$$

From this and the propagator $\Delta=(P^2-M^2)^{-1}$, where

$$M^2=2p^+p^--\mathbf{p}^2=\frac{1}{\alpha'}\sum_{\beta=1}^{Q}\sum_{n=1}^{\infty} n\mathbf{a}_n^{\beta\dagger}\cdot\mathbf{a}_n^{\beta}-1, \tag{13}$$

dual tree amplitudes can be constructed in the form

$$B_N=\langle 0,k_N|V(k_{N-1})\Delta\cdots V(k_2)|0,k_1\rangle. \tag{14}$$

Not surprisingly, the four-point amplitude is a β function.

In any nontrivial string theory, it is essential that interactions be compatible with the critical dimensionalities (3) or (4). It will be recalled that in dual models, critical dimensionalities first arose in connection with the analytic behavior of the one-loop corrections to the dual amplitudes.[11,12] For example, in the open bosonic parastring, in the notation of Ref. 12, a typical planar one-loop amplitude with M external lines is given by

$$E=\frac{1}{\pi}g^M\int_0^1\left[\prod_{I=1}^{M-1}\theta(\nu_{I+1}-\nu_I)\,d\nu_I\right]\int_0^1 dq\, q^{-1-Q(2-D)/12}W^{-1-Q(2-D)/24}\times\left(\frac{-2\pi^2}{\ln q}\right)^M[f(q^2)]^{(2-D)Q}\prod_{I<J}(\psi_{IJ})^{k_I\cdot k_J}. \tag{15}$$

Here θ is a step function, ψ_{IJ} is related to a Jacobi function, and W is related to q by the relation $\ln q\,\ln W=2\pi^2$. In this expression, for $D\leqslant 26$, the integrand is meromorphic at $q=0$ only at critical dimensionalities given by Eq. (3). Similar statements hold for other loop amplitudes. These results lend further support to the nontriviality of these critical dimensions.

We conclude this Letter by discussing a number of additional properties of parastring models:

(i) It was pointed out in Ref. 5 that a parastring model of order Q behaves in many respects like a string model which possesses an SO(Q) symmetry. In contrast to Chan-Paton–type symmetries which the string might also carry, the SO(Q) symmetry is distributed over the entire length of the string for both open and closed boundary conditions. For example, the massless vector paraboson of the open bosonic parastring belongs to the Q-dimensional representation of SO(Q).

(ii) In the spectrum of a given parastring, there are states which obey ordinary statistics. For example, the closed bosonic parastring has a massless spin-2 state[8] which is a boson and which can be identified with the graviton.

(iii) The planar one-loop amplitude with M external closed parastring tachyon states is, in the notation of Ref. 12, given by

$$E' = \int d^2\tau (\mathrm{Im}\tau)^{-2} C(\tau) F_\nu(\tau), \tag{16}$$

where $F_\nu(\tau)$ has the same form as that given in Ref. 12, and

$$C(\tau) = 4(\tfrac{1}{2}\mathrm{Im}\tau)^{-Q(D-2)/2} e^{4\pi \mathrm{Im}\tau} |f(e^{2\pi i\tau})|^{-2Q(D-2)}. \tag{17}$$

The expression for E' is modular invariant at the critical dimensionalities given by (3).

(iv) For supersymmetric parastrings, the critical dimensionalities given by Eq. (4) are in one-to-one correspondence with the space-time dimensionalities in which supersymmetric Yang-Mills theories can be constructed.[13] Given the intimate connection between the zero mass sector of superstring theories and supersymmetric Yang-Mills theories, this is of course not surprising. On the other hand, the critical dimensionalities (4) as well as other properties of the supersymmetric parastrings were first studied within the framework of the original Ramond-Neveu-Schwarz formalism. It would therefore be of interest to see if these parastring theories can be reformulated in terms of the new superstring formalism of Green and Schwarz. This turns out to be the case. Here we give an illustrative example, leaving the details to a forthcoming paper. Thus, consider the $D=4$, $Q=4$ supersymmetric parastrings. In the notation of Green and Schwarz,[2] the analog of the Green-Schwarz light-cone action for this case is

$$S = \frac{1}{4\pi}\int d\sigma\, d\tau [-\alpha'^{-1}\partial_\beta Y^i \partial^\beta Y^i + i\bar{\psi}\gamma^- \rho^\beta \partial_\beta \psi]. \tag{18}$$

Here $Y^i(\sigma,\tau)$ is not a ten- but a four-dimensional transverse vector paraboson of order four, and ψ is not a ten- but a four-dimensional Majorana (not Majorana-Weyl) parafermion of order four. ψ is also a spinor in σ-τ space. It is straightforward to check that this action is invariant under the supersymmetry transformations

$$\delta Y^i = C\bar{\epsilon}\gamma^i\psi, \qquad \delta\psi = (2\alpha')^{-1} C\gamma_-\gamma^\mu(\rho^\beta\partial_\beta Y_\mu)\epsilon, \tag{19}$$

where C is a normalization factor, and ϵ is a constant four-dimensional Majorana fermion as well as a two-spinor in σ-τ space. Similar actions can be written down for supersymmetric parastrings in $D=6$ and $D=3$ space-time dimensions. A more detailed description of the properties of these theories is in preparation and will be reported elsewhere.

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[1]Y. Nambu, unpublished; L. N. Chang and F. Mansouri, Phys. Rev. D **5**, 2535 (1972).

[2]P. Ramond, Phys. Rev. D **3**, 2415 (1971); A. Neveu and J. H. Schwarz, Nucl. Phys. **B31**, 1109 (1971); M. B. Green and J. H. Schwarz, Phys. Lett. **109B**, 444 (1982), and **149B**, 117 (1984).

[3]D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **55**, 502 (1985), and Nucl. Phys. **B256**, 253 (1985).

[4]For attempts to understand these symmetries as arising from the compactification of the bosonic string, see P. G. O. Freund, Phys. Lett. **151B**, 387 (1985); A. Casher, F. Englert, H. Nicolai, and A. Taormina, Phys. Lett. **162B**, 121 (1985).

[5]F. Ardalan and F. Mansouri, Phys. Rev. D **9**, 3341 (1974).

[6]P. Candelas, G. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985).

[7]H. S. Green, Phys. Rev. **90**, 270 (1953); O. W. Greenberg and A. M. L. Messiah, Phys. Rev. **138**, B1155 (1965).

[8]F. Ardalan and F. Mansouri, to be published.

[9]O. W. Greenberg and K. I. Macrae, Nucl. Phys. **B219**, 358 (1983).

[10]P. Goddard, J. Goldstone, C. Rebbi, and C. B. Thorn, Nucl. Phys. **B56**, 109 (1973).

[11]C. Lovelace, Phys. Lett. **34B**, 500 (1971).

[12]J. H. Schwarz, Phys. Rep. **89**, 223 (1982).

[13]L. Brink, J. H. Schwarz, and J. Scherk, Nucl. Phys. **B121**, 77 (1977); F. Gliozzi, J. Scherk, and D. I. Olive, Nucl. Phys. **B122**, 253 (1977).